Abstract

Abstract stuff

1 Introduction

1.1 LHC - Large Hadron Collider

The Large Hadron Collider (LHC) is the world's largest particle collider ever built.

2 Theory

2.1 RICH Detector

Particle identification is a fundamental requirement at the LHCb experiment. Meaningful CP-violation measurements are only possible if hadron identification is available hence the ability to distinguish between kaons and pions is essential.

2.2 Cherenkov-Radiation

The speed of light in vacuum, **c**, is a universal physical constant. According to Einstein's special theory of relativity, c is the maximum speed at which all matter (or information) in the universe can travel. The speed at which light propagates in a medium, however, can be significantly less can c.

Cherenkov radiation results when a charged particle travels through a dielectric medium with a speed greather than the speed of light through said medium. Moreover, the velocity that must be exceeded is the phase velocity (v_{Phase} or short v_{P}) and not the group velocity $v_{\text{Group}} = \frac{\partial \omega}{\partial k}$.

$$v_{\text{Phase}} = \frac{\lambda}{T} \quad \text{or} \quad \frac{\omega}{k}$$

As a charged particle travels through the medium, it disrupts the local electromagnetic field. If the particle travels slowly then the disturbance elastically relaxes to the mechinal equilibrium as the particle passes. However, if the particle travels fast enough, the limited response speed of the medium means that a disturbance is left in the wake of the particle, and the energy in this disturbance radiates as coherent shockwave.

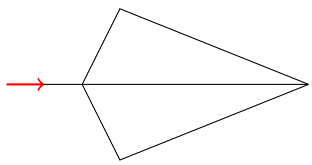


FIG. 2.1 – Cherenkov radiation

$$x_p = v_p \cdot t = \beta c t$$
$$x_{\text{em}} = v_{\text{em}} \cdot t = \frac{c}{n} t$$
$$\cos\theta = \frac{x_{\text{p}}}{x_{\text{em}}} = \frac{\frac{c}{n} t}{\beta c t} = \frac{1}{n\beta}$$

which is independent from the angle θ.

2.3 Hough Transform

The Hough-Transform is a feature extraction technique used in image analysis, computer vision and digital image processing.

the purpose is to find imperfect instances of objects within a certain cass of shapes by a voting procedure. This voting procedure is carried out in a parameter space from which object candidates are obtained as local maxima in a so called accumlator space that is explicitly constructed by the algorithm for computing the Hough-Transform.

Initially the Hough-Transform was concerned with finding straight lines but has been extended to identifying positions of arbitrary shapes, such as circles and ellipses.

2.3.1 Linear Hough Transform

A linear function is normally defined as the following:

$$f(x) = m \cdot x + b$$

where m is the slope of the line and b the intercept. For the Hough-Transform however, this representation is not ideal. For a vertical line m would go to infinity which gives us an unbound transform space for m. For this reason Duda and Hart suggested the ρ-θ parametrization [1].

$$r = x \cos\theta + y \sin\theta$$

where r is the distance from the origin to the closest point in the line and θ is the angle between the x-axis and the line connecting the origin with that closest point.

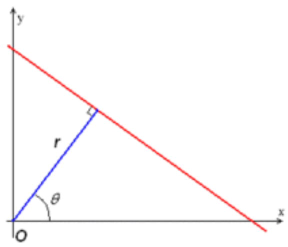
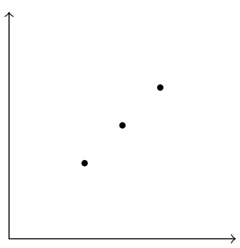


FIG. 2.2 – ρ-θ parametrisation

This means given a single point in the plane, the set of all lines going through this point form a sinusoidal curve in ρ-θ space. Another point that lies on the same straight line in the plane will produce a sinusoidal curve that intersects with the other at (ρ-θ).

Example of a Linear HT



Tab. 2.1 – Angle vs Distance

Angle	Distance
0	2.334

2.3.2 Circle Hough Transform

For this thesis we are interested in circle detection so we need to adapt our linear Hough Transform in order to find circles. In a two dimensional space, a circle can be described by:

$$(x - c_x)^2 + (y - c_y)^2 = r^2 \tag{2.1}$$

Where (c_x, c_y) is the center of the circle and r the radius. The possible parameters for the parameters space are now c_x, c_y and r. This means if we know the center of the circle the parameter space is one-dimensional and if we know the radius of the circle our parameter space is two-dimensional and of course if we know nothing the parameter space is three-dimensional.

2.3.3 Combinatorial Approach

This approach is slightly different from the usual Hough Transforms. In this approach we use the fact that a circle is uniquely defined by 3 points.

3 Methods

3.1 Conventional Hough-Transforms

In the following subsections we discuss the conventional Hough-Transforms for the case of a one, two and three dimensional parameter space.

3.1.1 1D: Known Center - Find Radius

For this case we assume that we know the center of each circle and just need to find the radius. Our parameter space has thus only dimension 1. Since we know the center points we of course also know the number of circles that we need to find

3.1.2 2D: Known Radius - Find Center

3.1.3 3D: Nothing is Known - Find Everything

3.2 Combinatorial approach

The combinatorial approach relies on the fact that a circle is uniquely defined by 3 points. With 2 arbotrary points we couldn't tell which side the circle is going to go. A third point gives us all the information we need. The general idea then is the following:

1. Build all possible triples of points given the data points
2. For all the point triples calculate the center and the radius of the potential circle
3. Due to constraints in the radius we can drop many of the circles with a radius bigger than a certain threshold
4. Create a histogram with the radius distribution. Peaks in the radius distribution hint to a circle.
5. We scan the radius histogram for peaks and look at the center point histogram for the given radius of a peak. If we have also a peak in the center point histogram the set of the points of the triples lie on a circle with a radius and center given by the histogram peaks.

3.2.1 Calculating the Circle given 3 points

Let (A, B, C) be a triple of points in a 2D plane and a, b, c the length of the sides opposite to the respective corner. The semiperimeter is defined as

$$s = \frac{a+b+c}{2} \tag{3.1}$$

using this we can calculate the radius R of the circumcircle of triangle $\overline{ABC}$:

$$R = \frac{abc}{4\sqrt{s(a+b-s)(a+c-s)(b+c-s)}} \tag{3.2}$$

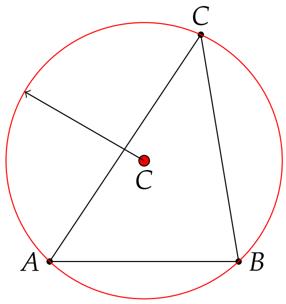


FIG. 3.1 – The circumradius and the circumcenter of a circle defined by the points

We have $\lambda_1, \lambda_2, \lambda_3$ as the barycenteric coordinates of the circumcenter:

$$\lambda_1 = a^2 \cdot (b^2 + c^2 - a^2) \tag{3.3}$$
$$\lambda_2 = b^2 \cdot (a^2 + c^2 - b^2) \tag{3.4}$$
$$\lambda_3 = c^2 \cdot (a^2 + b^2 - c^2) \tag{3.5}$$

Multiplying a matrix consisting of the column vectors of A, B, C with a column vector of $\lambda_1, \lambda_2, \lambda_3$ and dividing the resulting vector by the sum of the barycentric coordinates (for noramlization) leads to the circumcenter of the triangle $\overline{ABC}$

$$\begin{pmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \boldsymbol{P'} \tag{3.6}$$

$$\frac{\boldsymbol{P'}}{\lambda_1 + \lambda_2 + \lambda_3} = \boldsymbol{P} \tag{3.7}$$

3.2.2 Drawback

There is also a drawback with this method:

- The combinatorics blow up with a high number of data points $\binom{N}{3}$

So for example with 200 data points (circle data and background) the number of triplets is

$$\binom{200}{3} = 1313400$$

and for 300:

$$\binom{300}{3} = 4455100$$

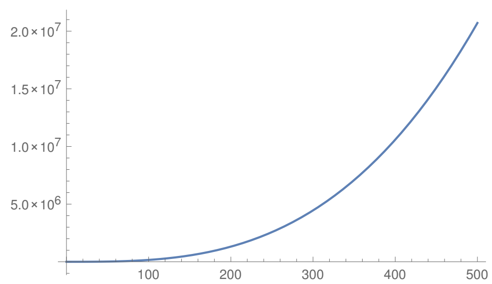


FIG. 3.2 – Binomial Growth with $\binom{N}{3}$

4 Results

5 Conclusions

Bibliography

[1] Richard O. Duda and Peter E. Hart. "Use of the Hough Transformation to Detect Lines and Curves in Pictures". In: *Commun. ACM* 15.1 (Jan. 1972), pp. 11–15. ISSN: 0001-0782. DOI: 10.1145/361237.361242. URL: http://doi.acm.org/10.1145/361237.361242.