

# Redes Neurais e Aprendizagem Profunda

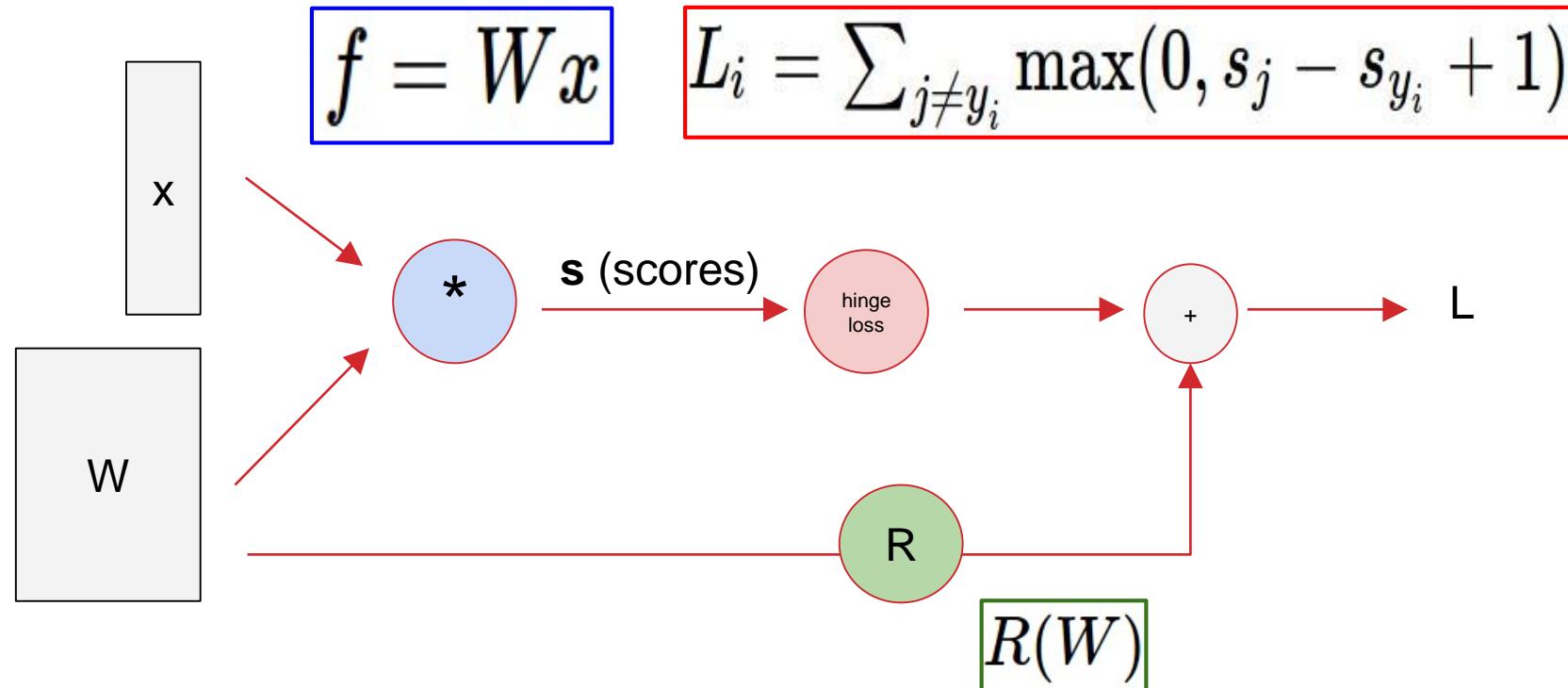
## REDES NEURAIS ARTIFICIAIS PROPAGAÇÃO RETRÓGRADA (I)

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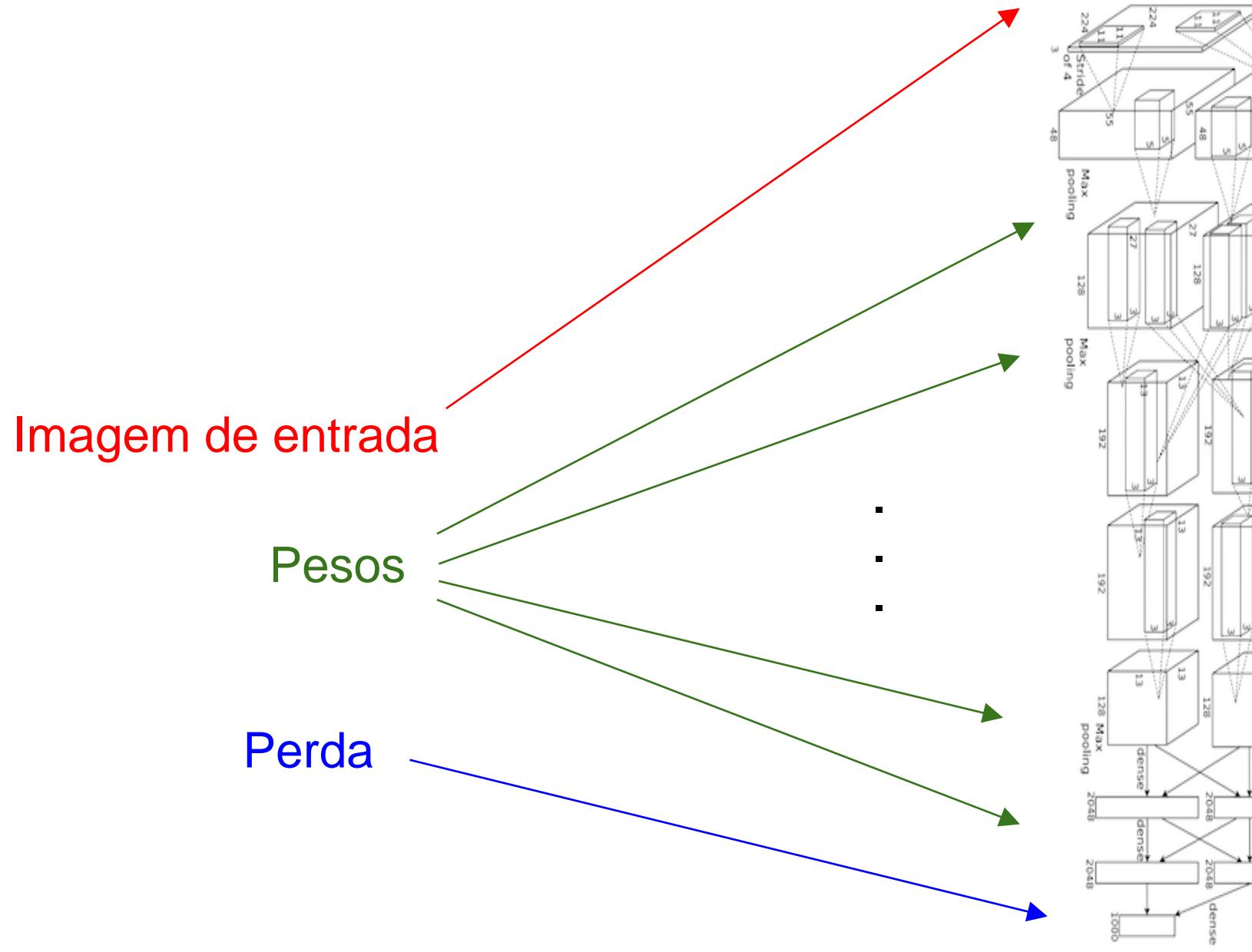
Zenilton K. G. Patrocínio Jr

[zenilton@pucminas.br](mailto:zenilton@pucminas.br)

# Grafo de Computação da Função de Perda



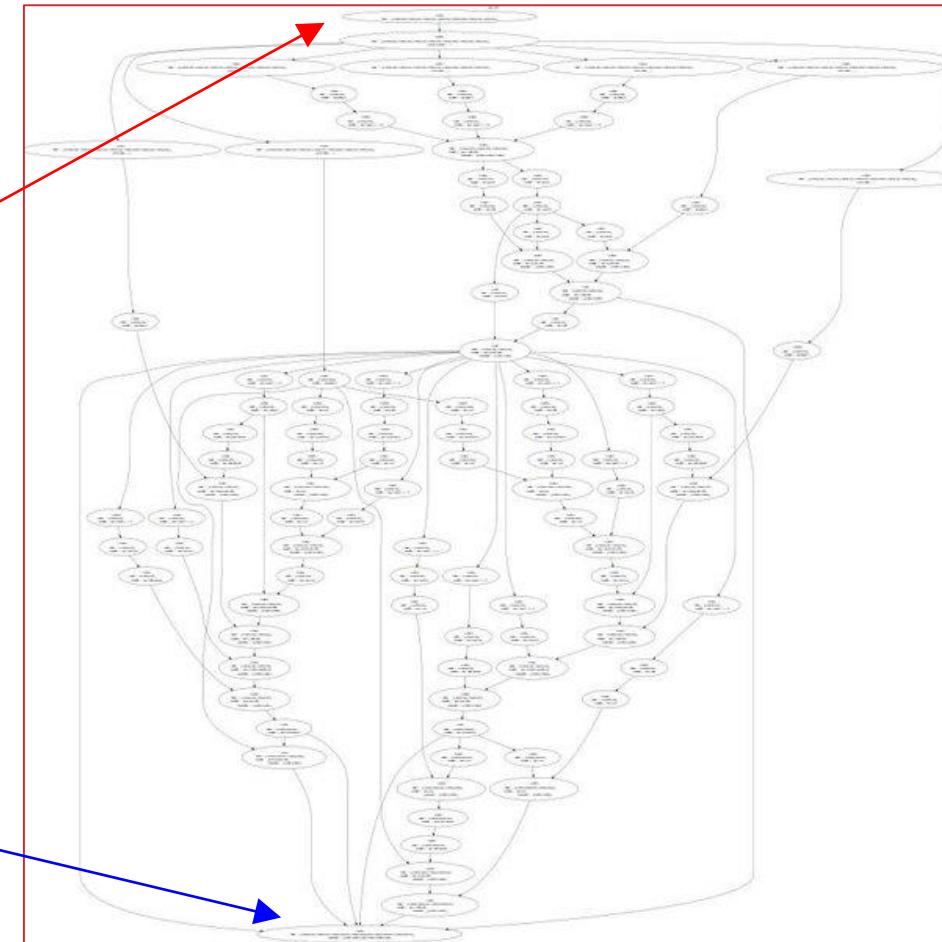
# Rede Convolucional (AlexNet)



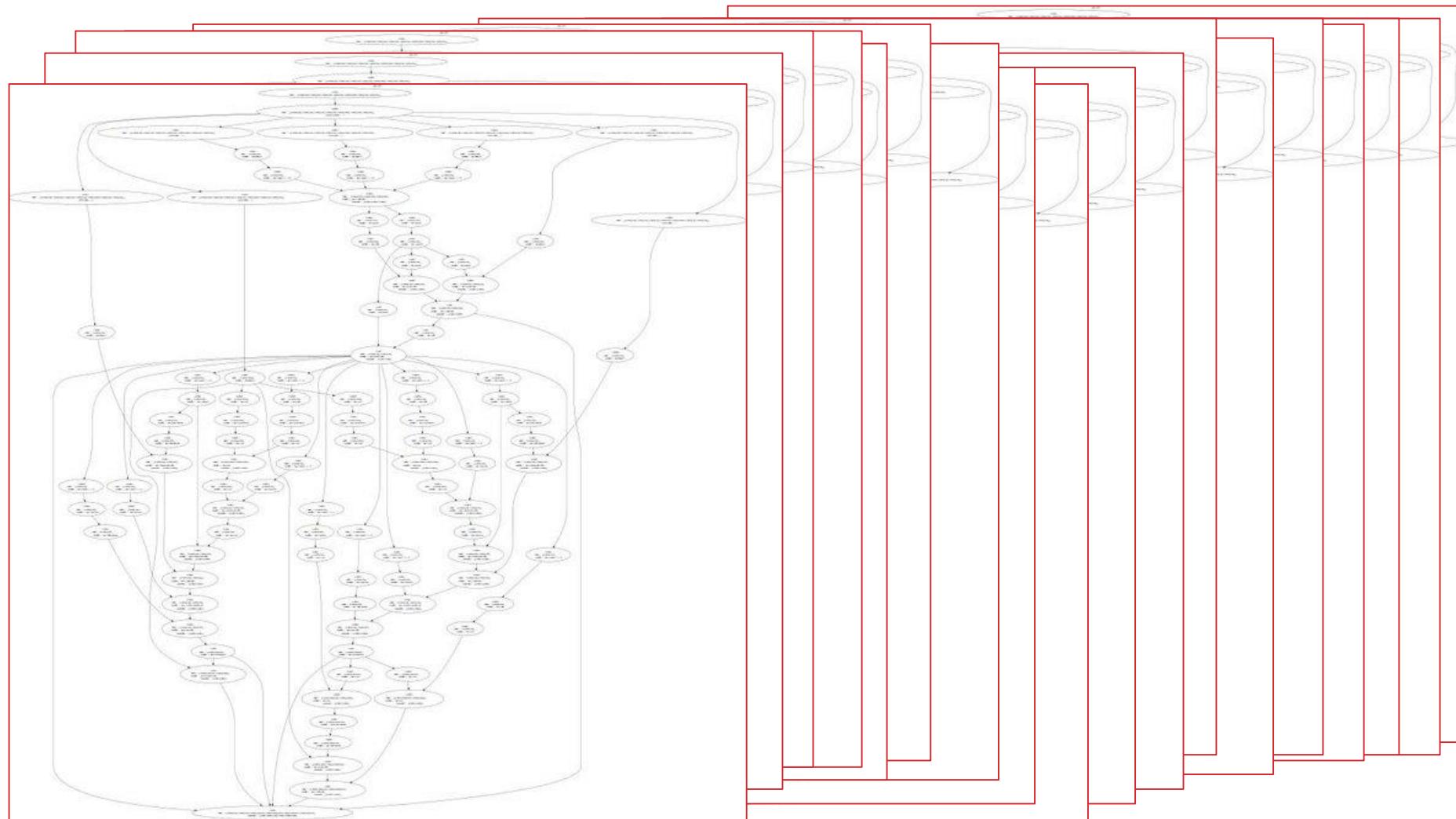
# Máquina Neural de Turing

Fita de entrada

Perda

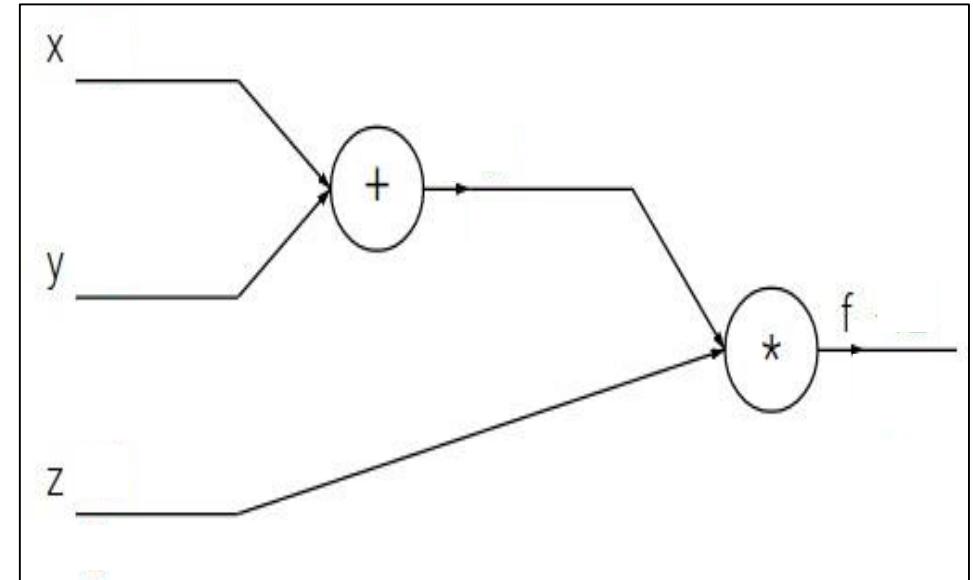


# Neural Turing Machine



# Diferenciação de um Grafo de Computação

Seja  $f(x, y, z) = (x + y) \times z$

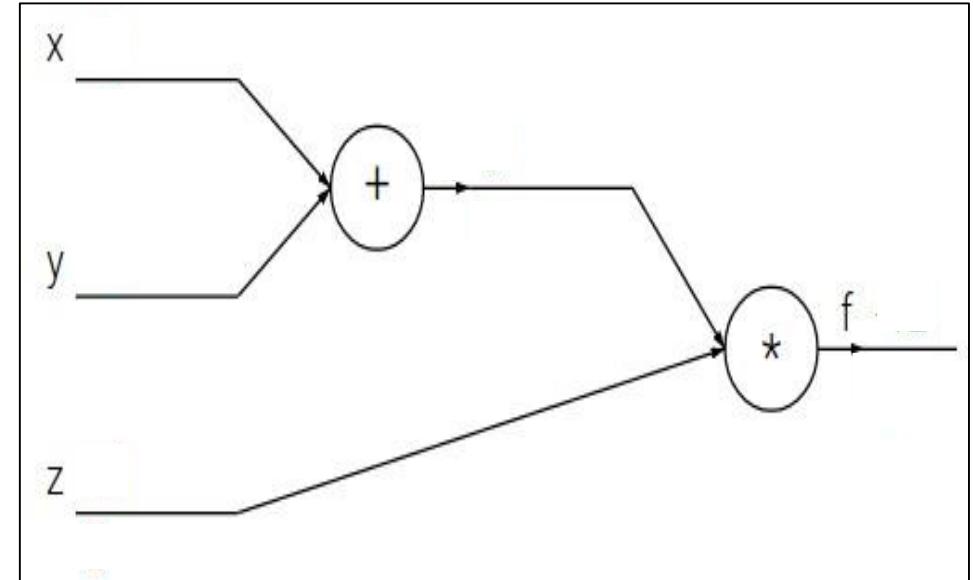


# Diferenciação de um Grafo de Computação

Seja  $f(x, y, z) = (x + y) \times z$

Então, pode-se dizer que

$$f(x, y, z) = g(h(x, y), z)$$



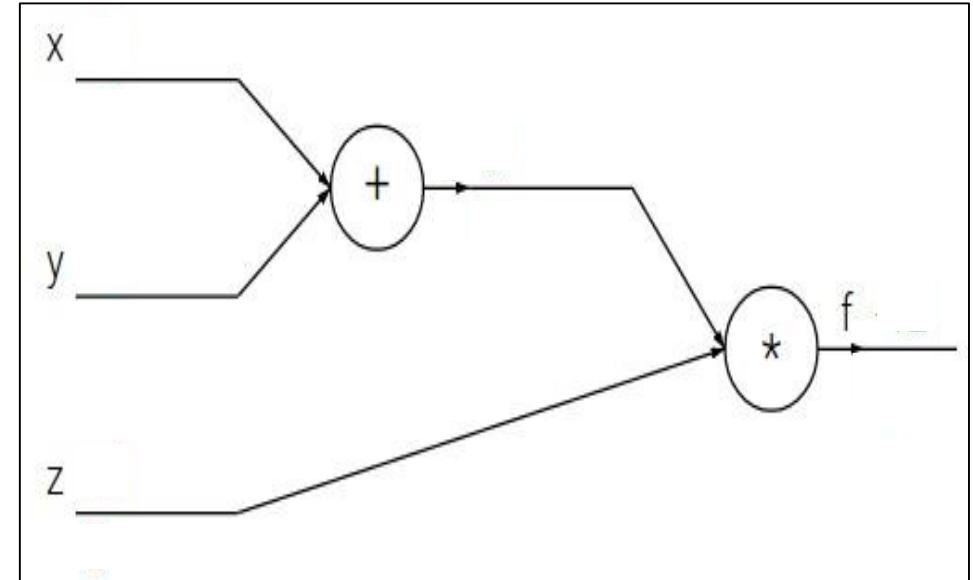
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em  $h(x, y) = x + y$



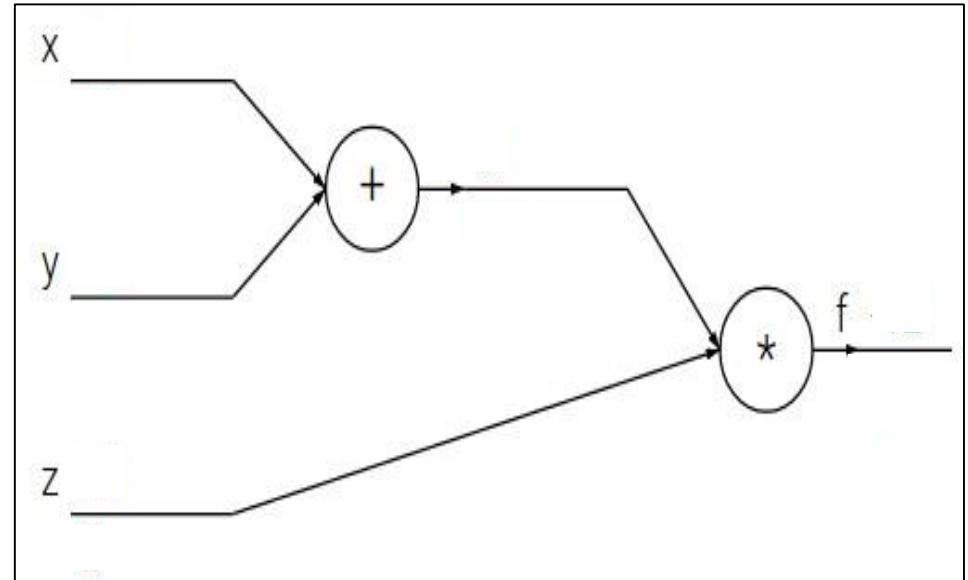
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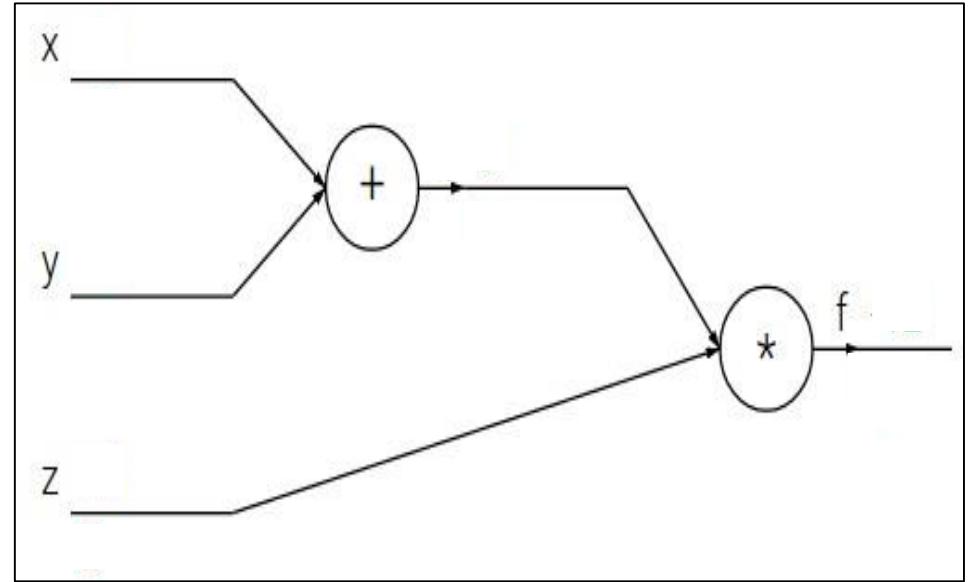
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Pela **regra da cadeia**, sabe-se que

$$\frac{df}{dx} = \frac{dg}{dh} \times \frac{dh}{dx}$$



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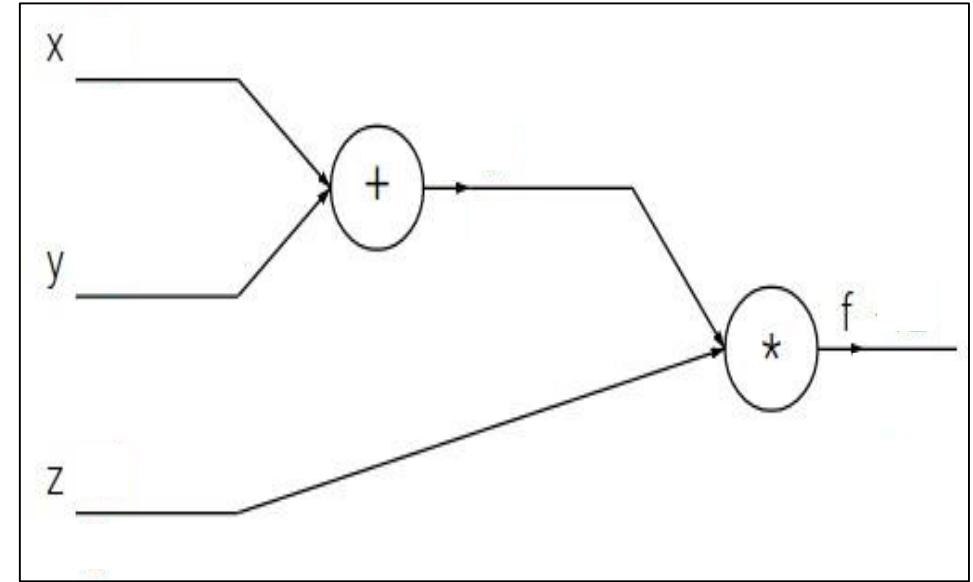
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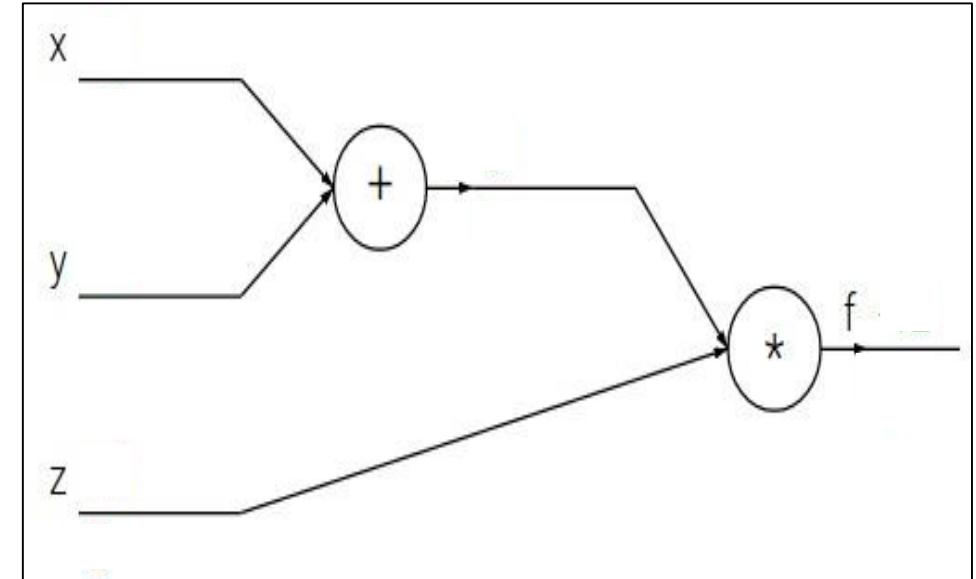
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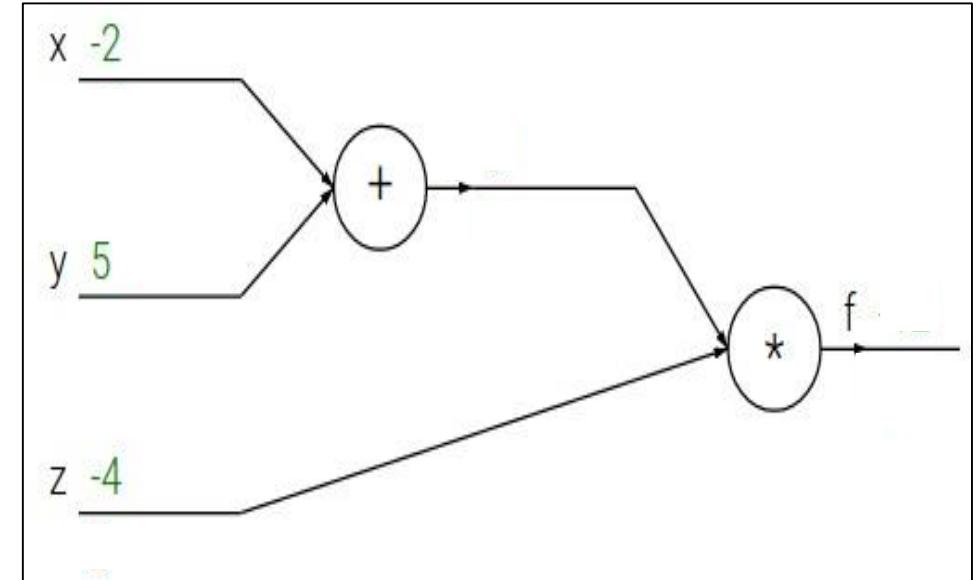
Por exemplo:  $x = -2, y = 5, z = -4$



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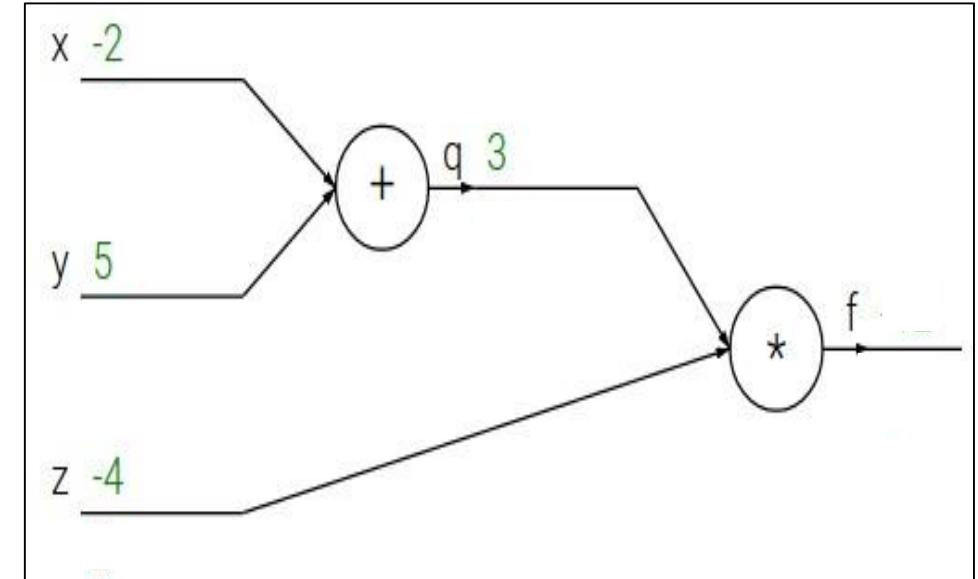


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$$q = x + y$$



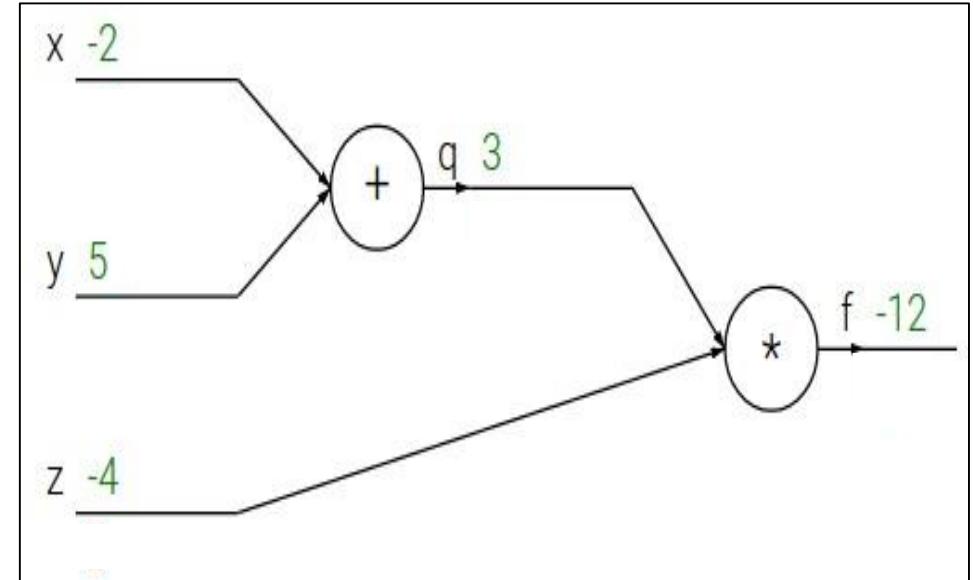
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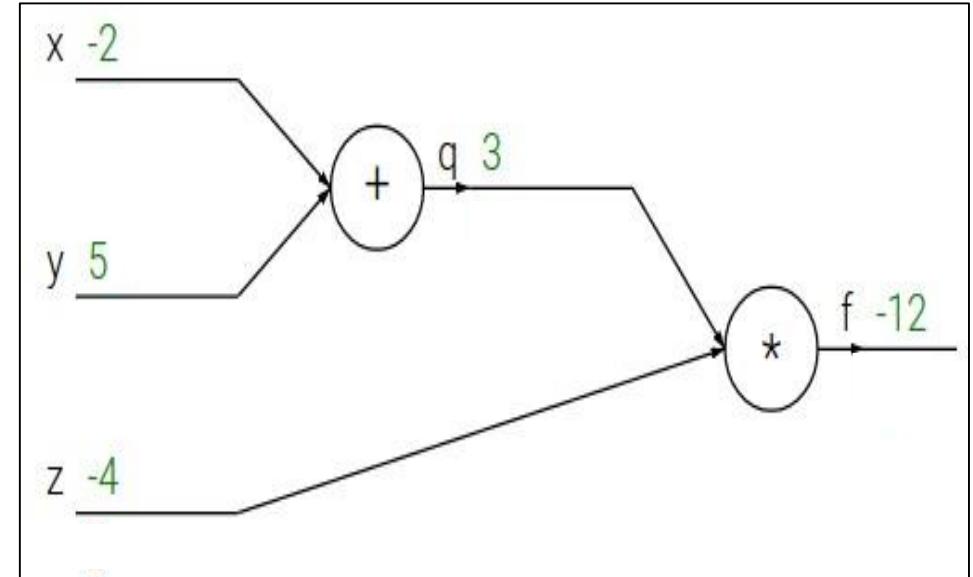
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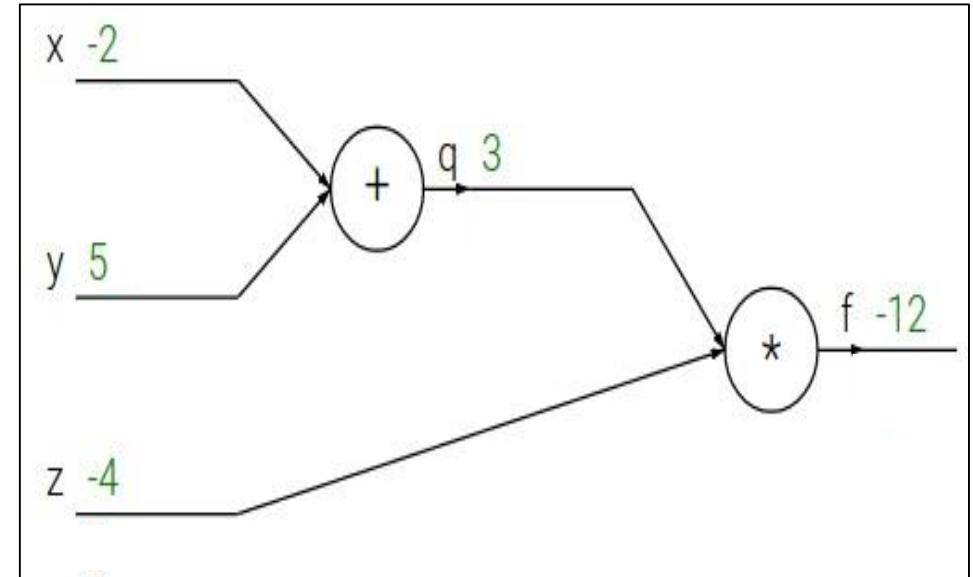
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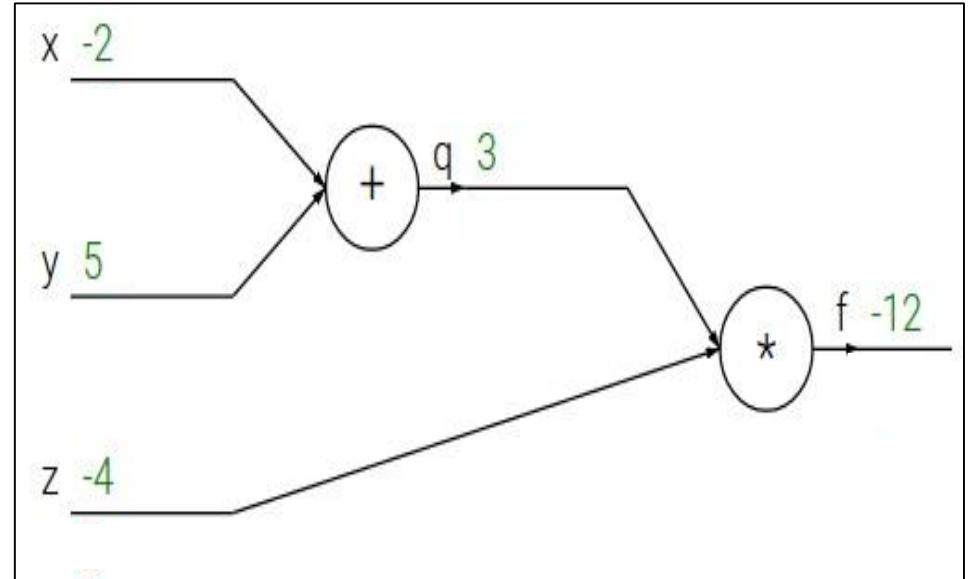
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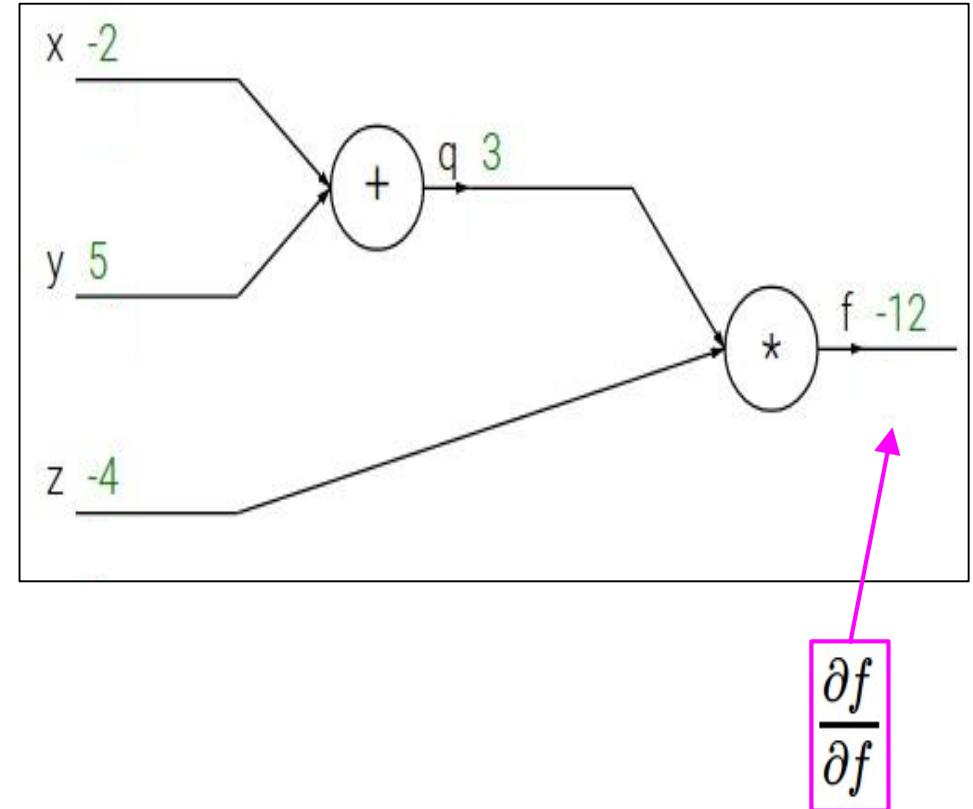
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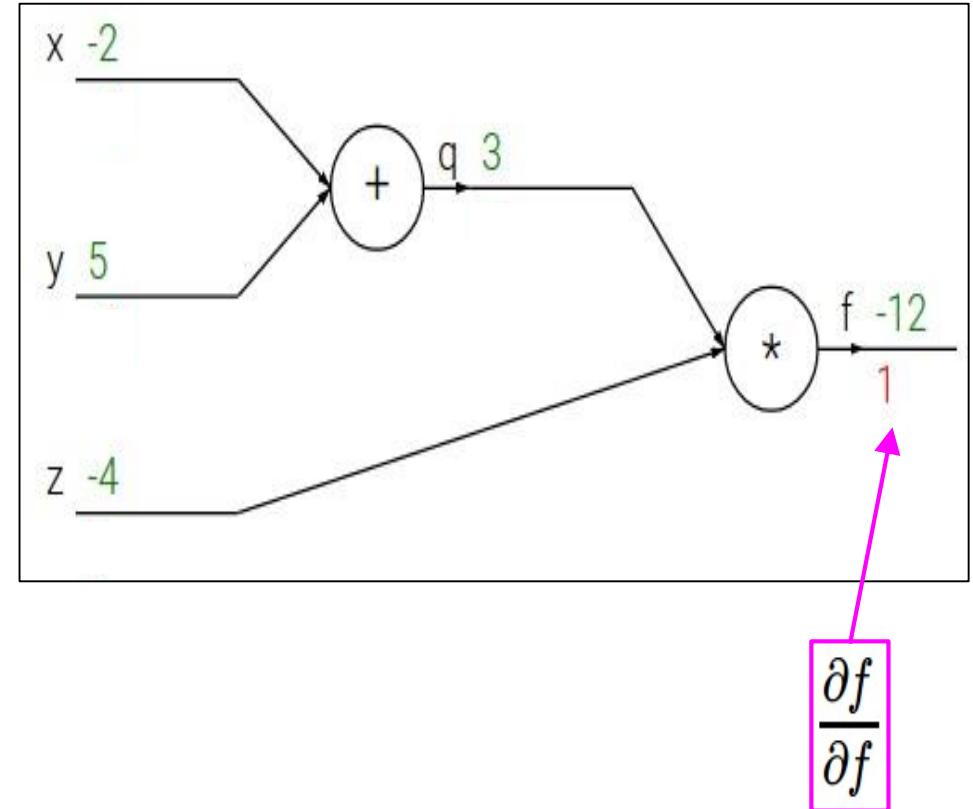
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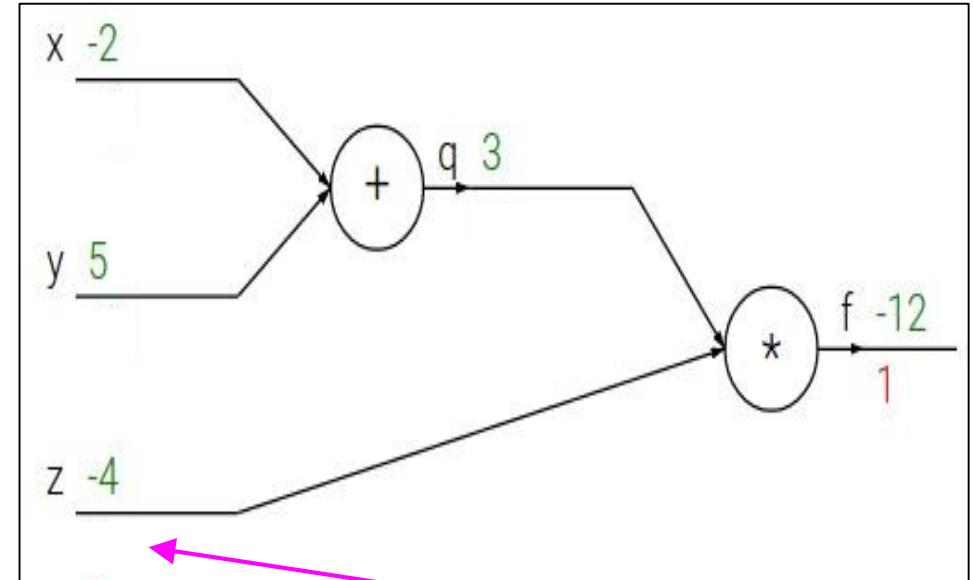
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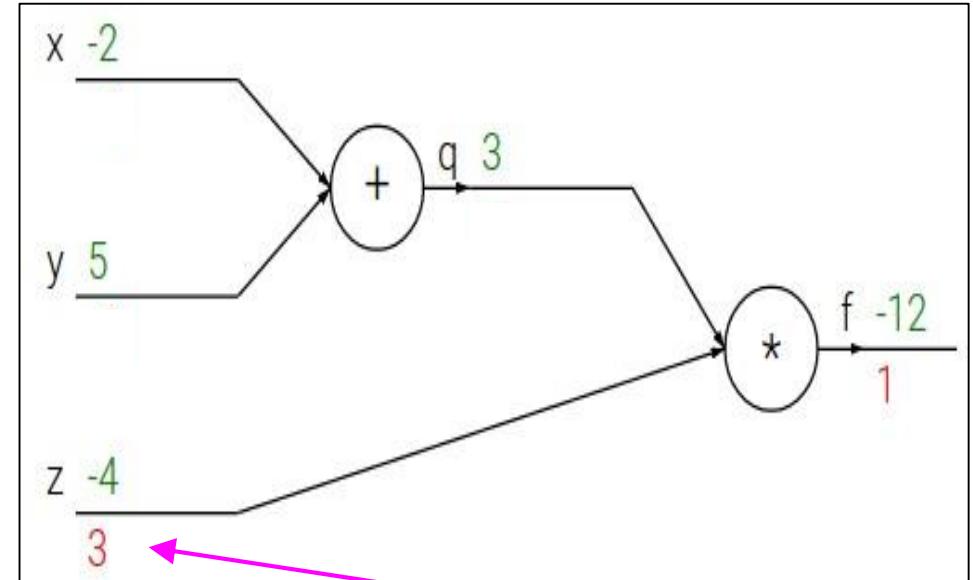
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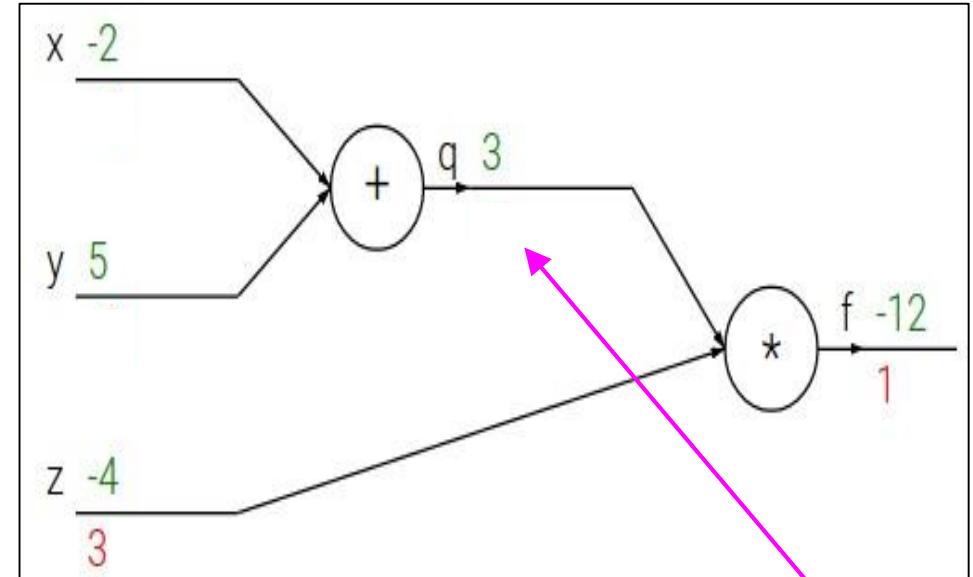
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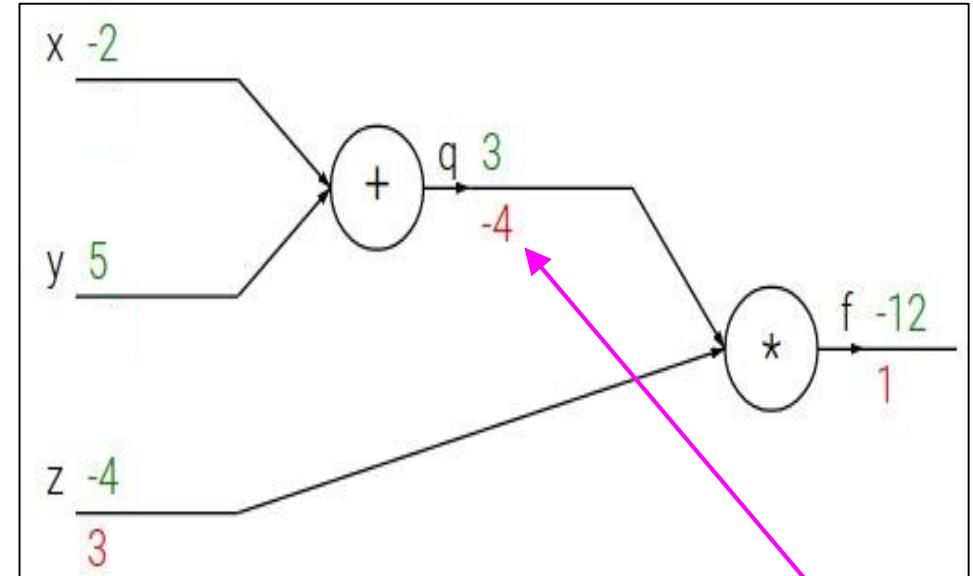
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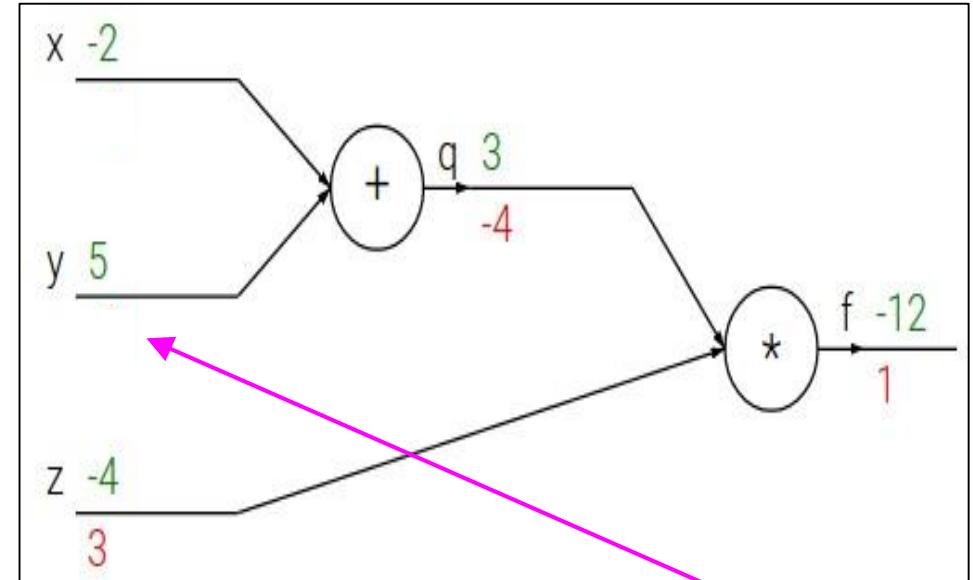
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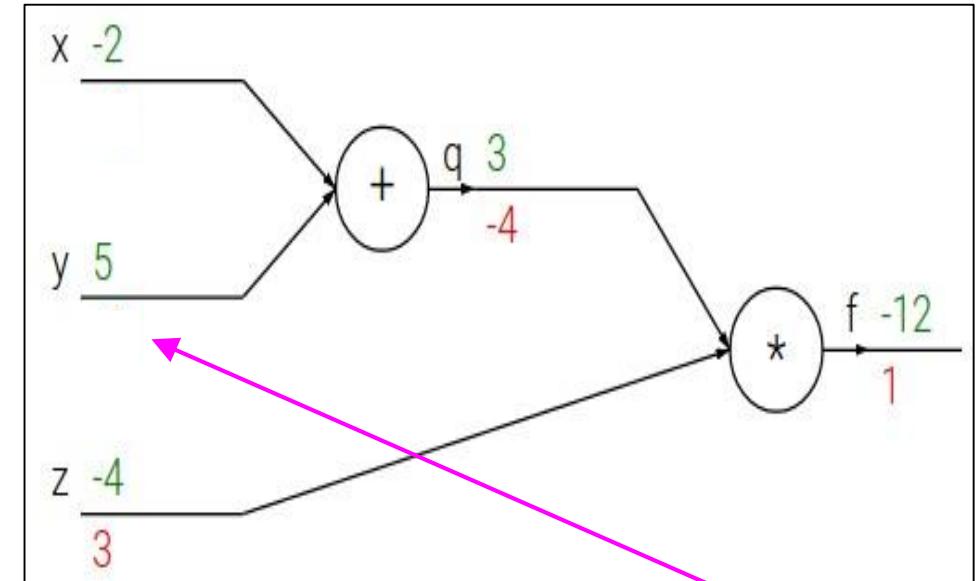
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Regra da Cadeia:

$$\frac{\partial f}{\partial y}$$

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# Diferenciação de um Grafo de Computação

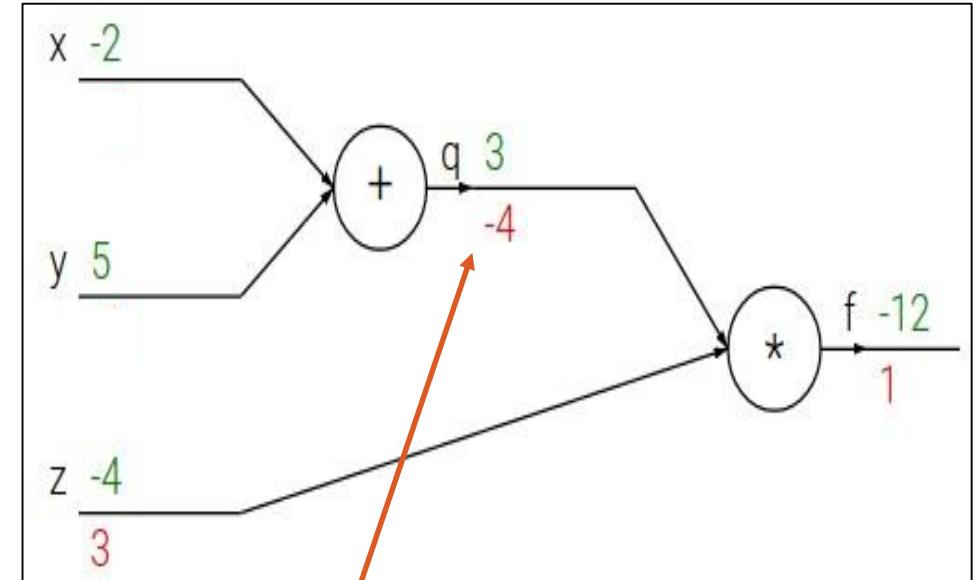
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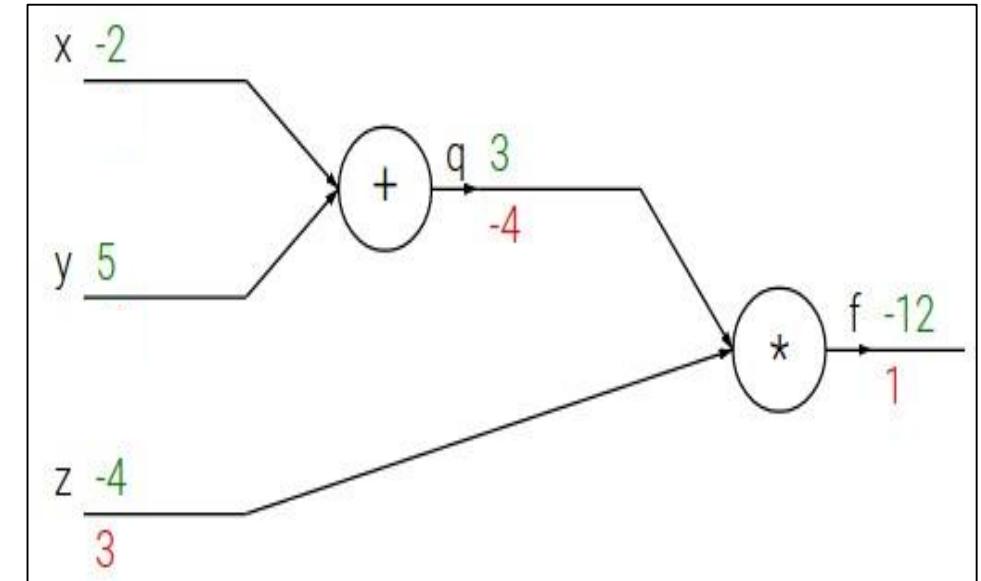
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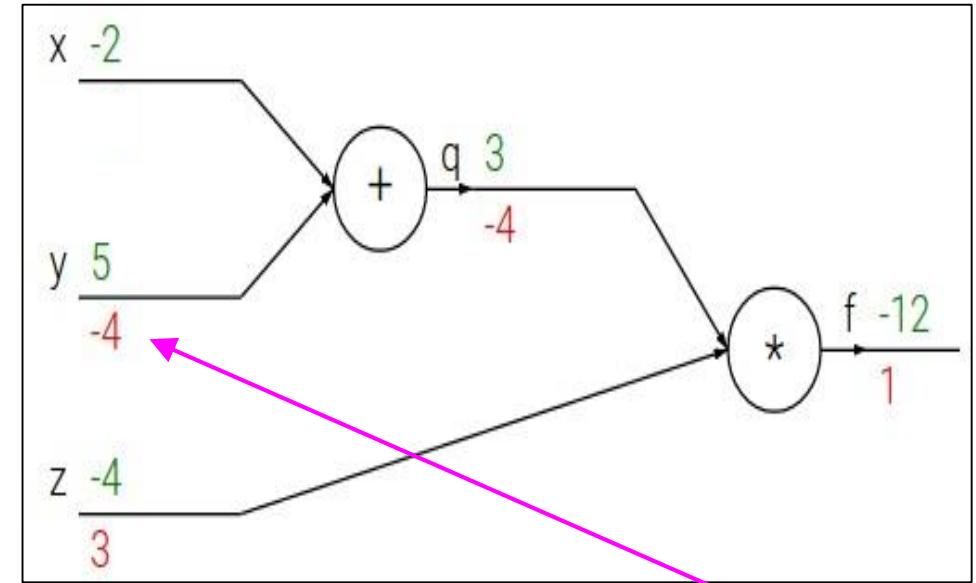
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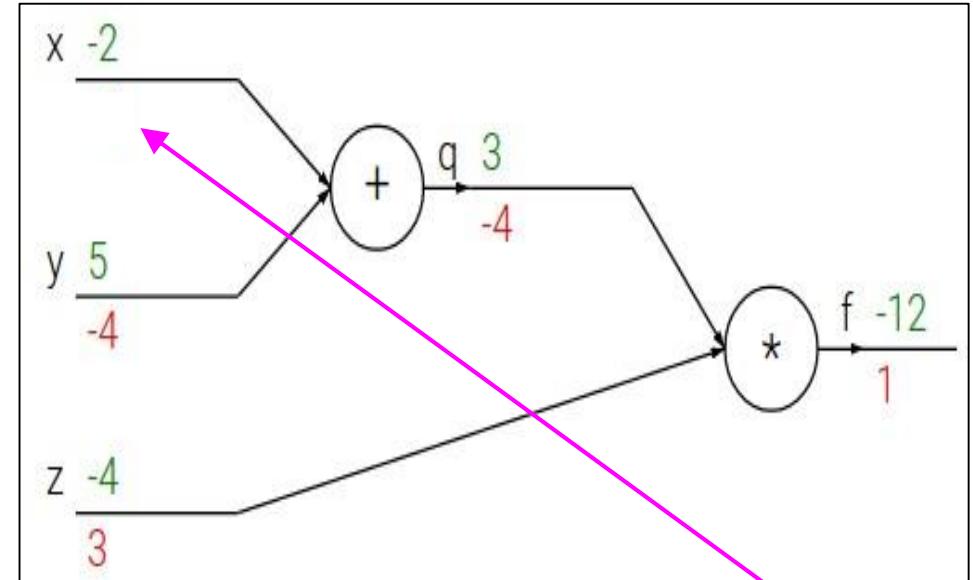
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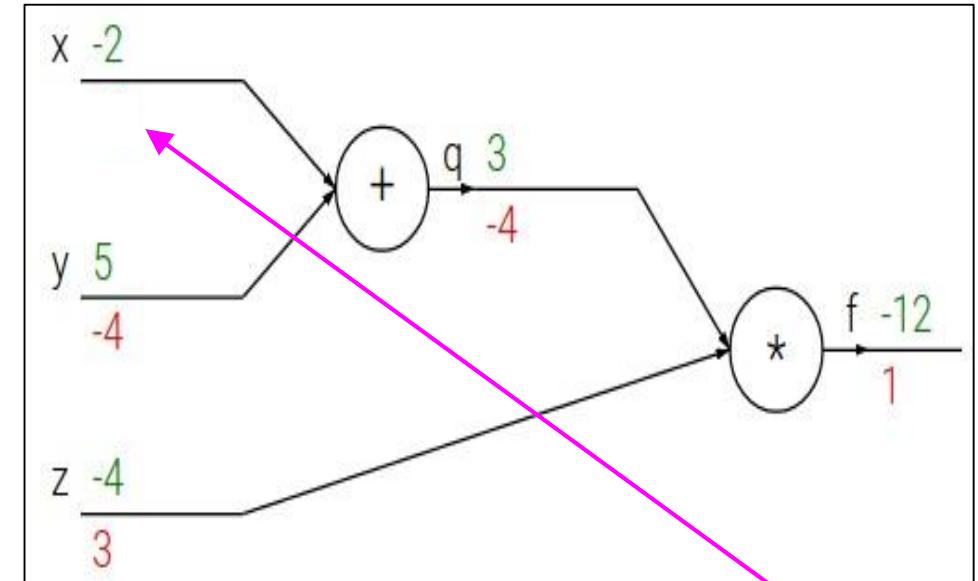
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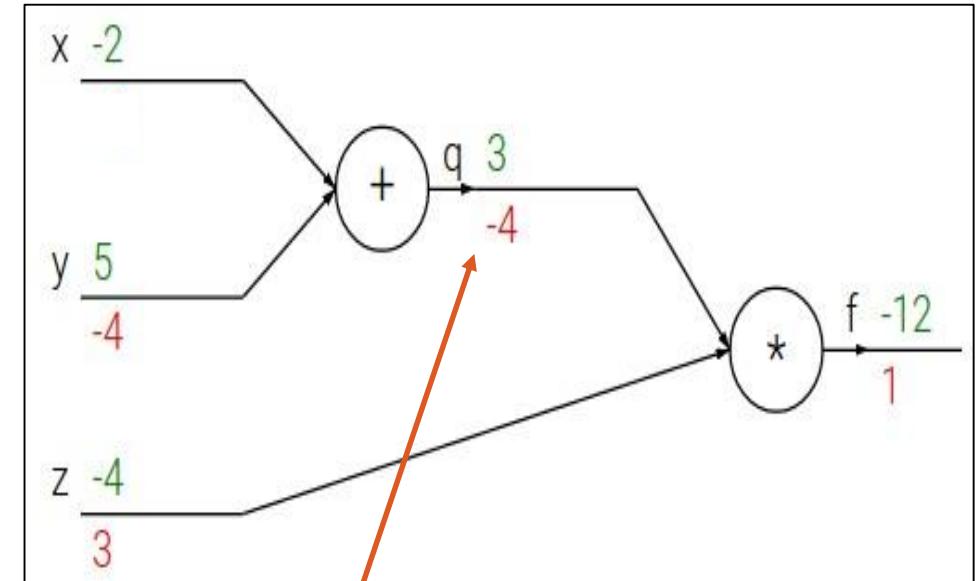
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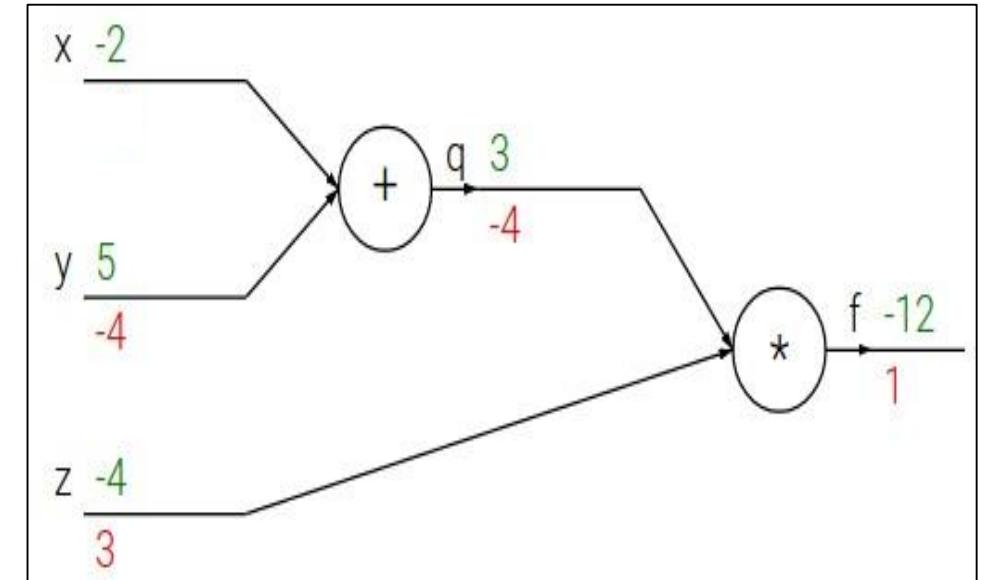
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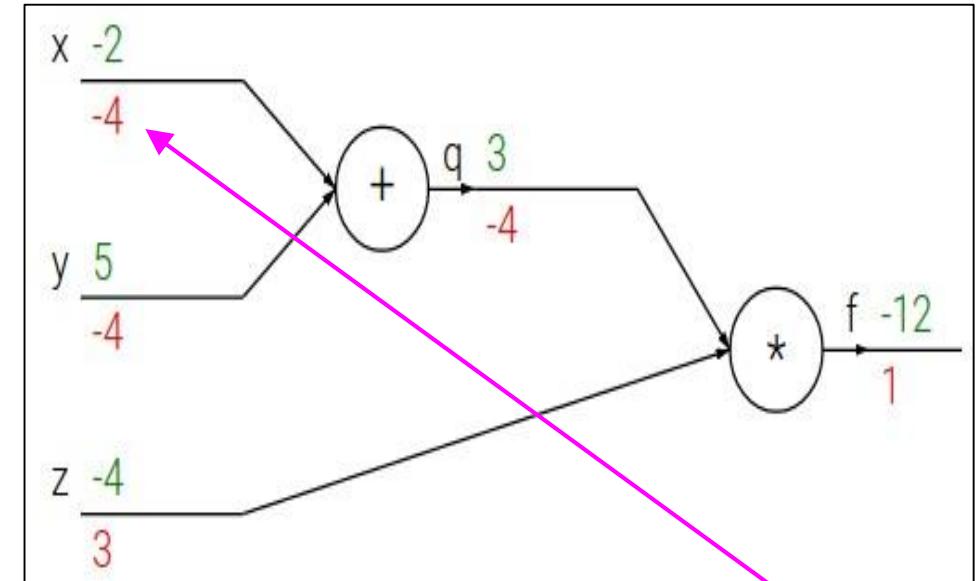
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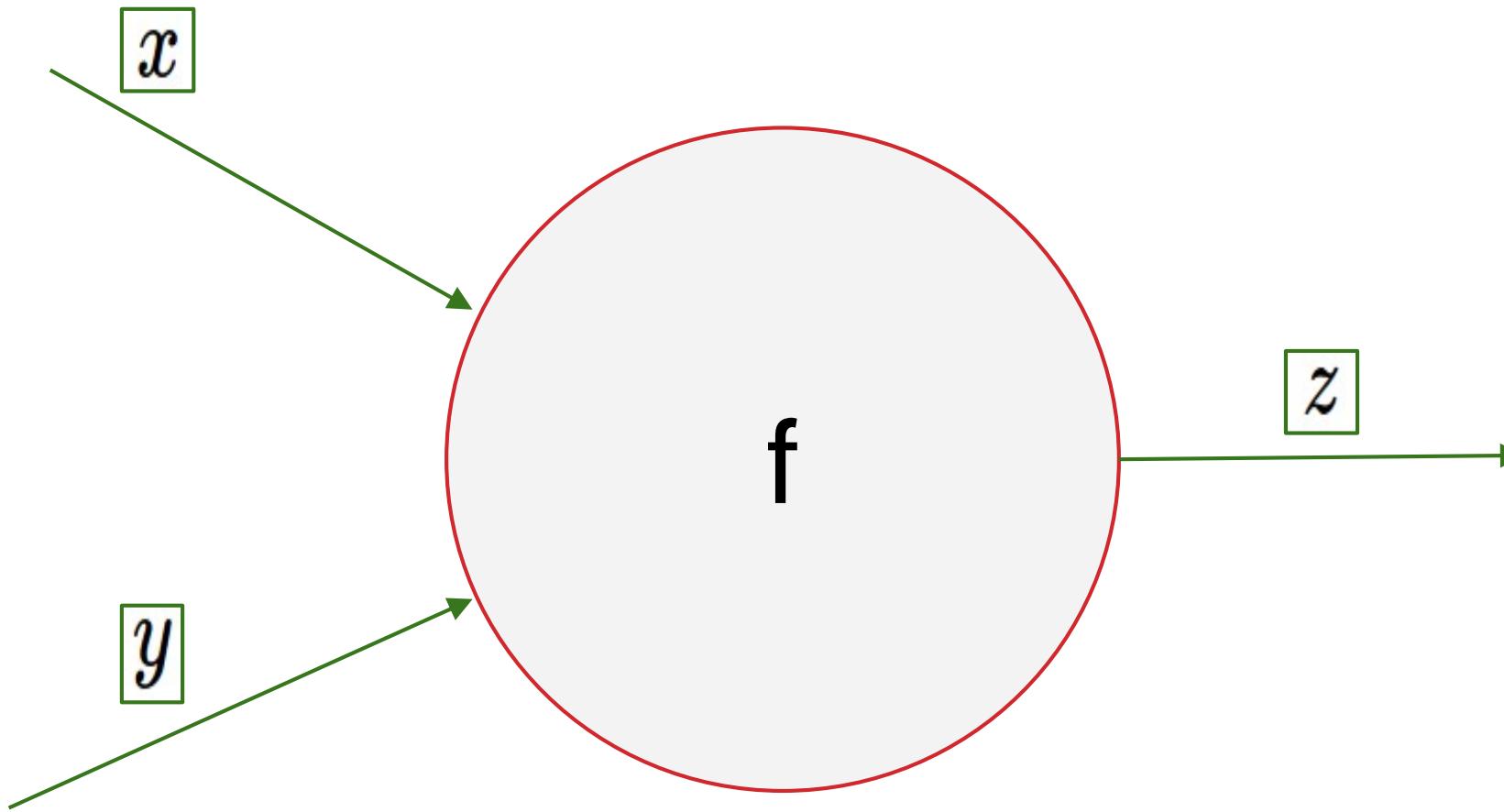


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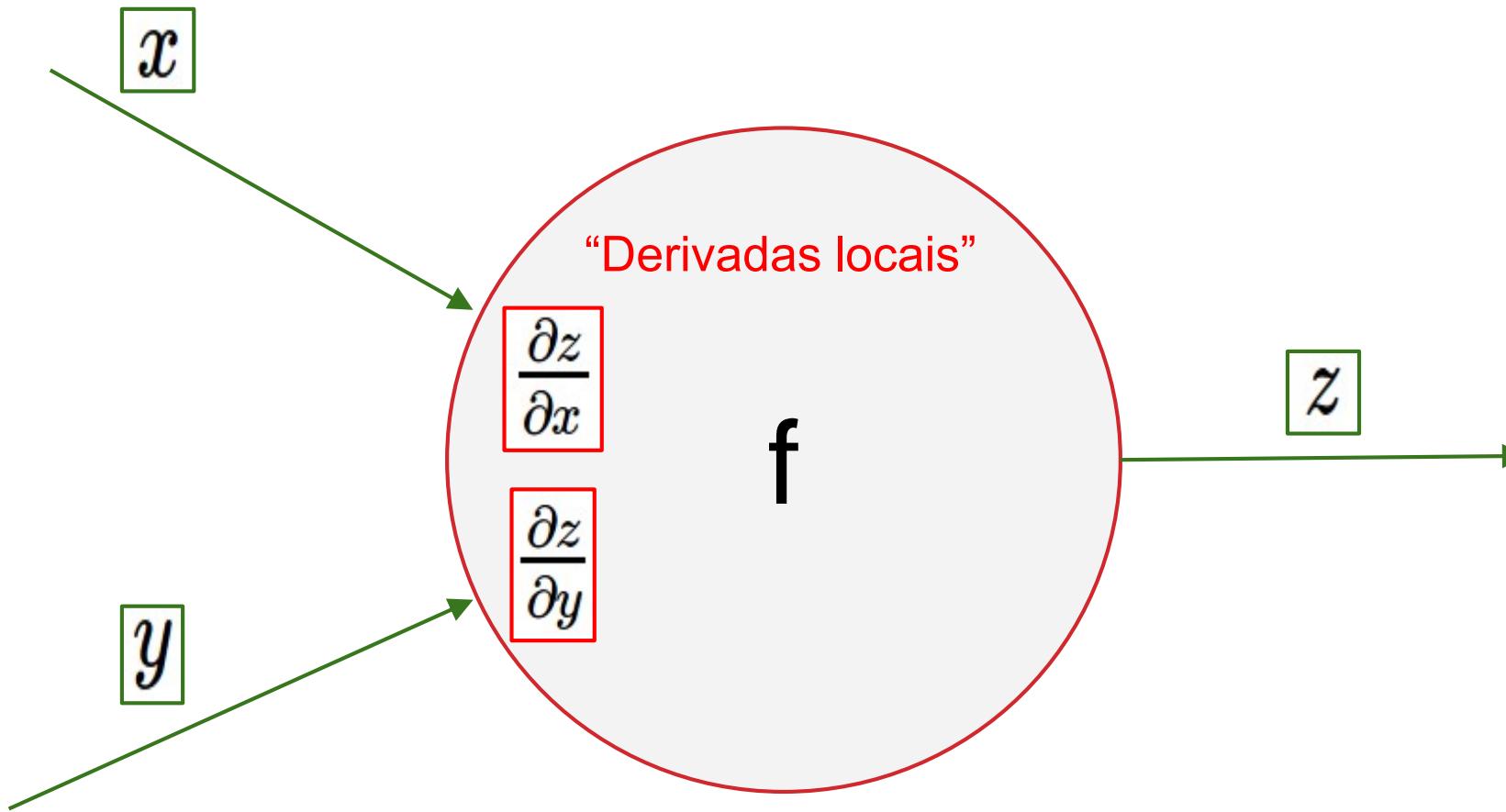
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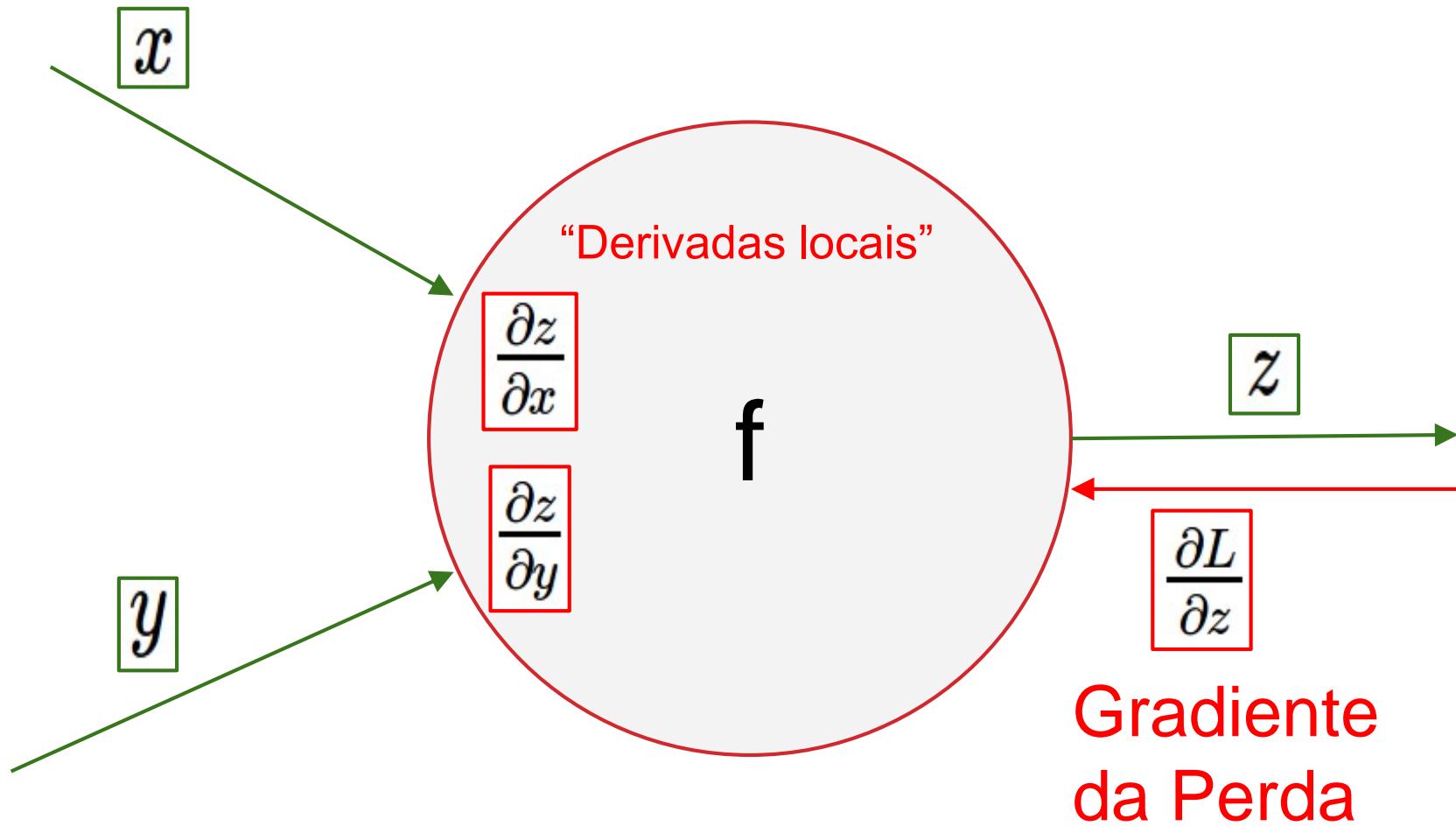
# Passo Retrógrado (*Backward Pass*)



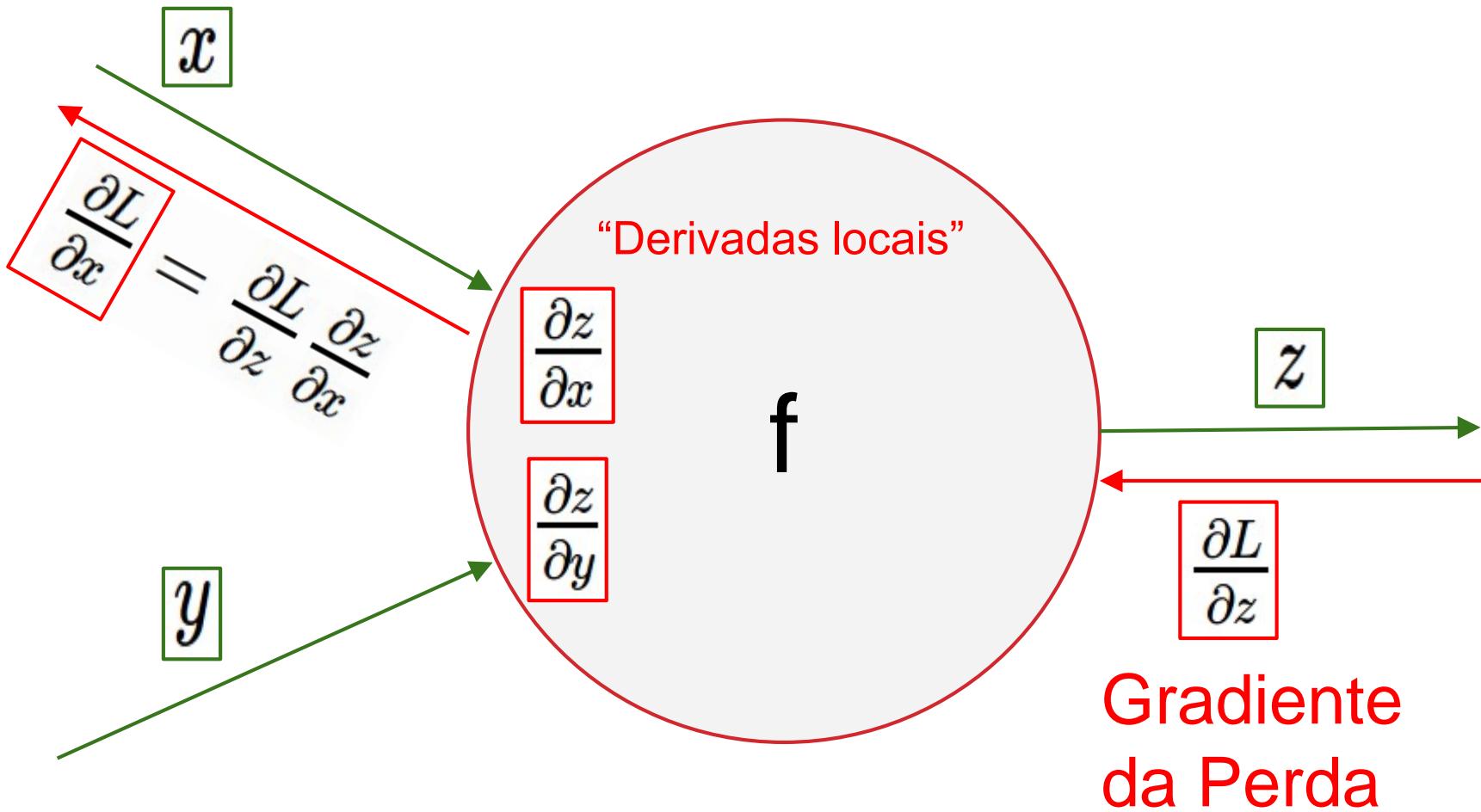
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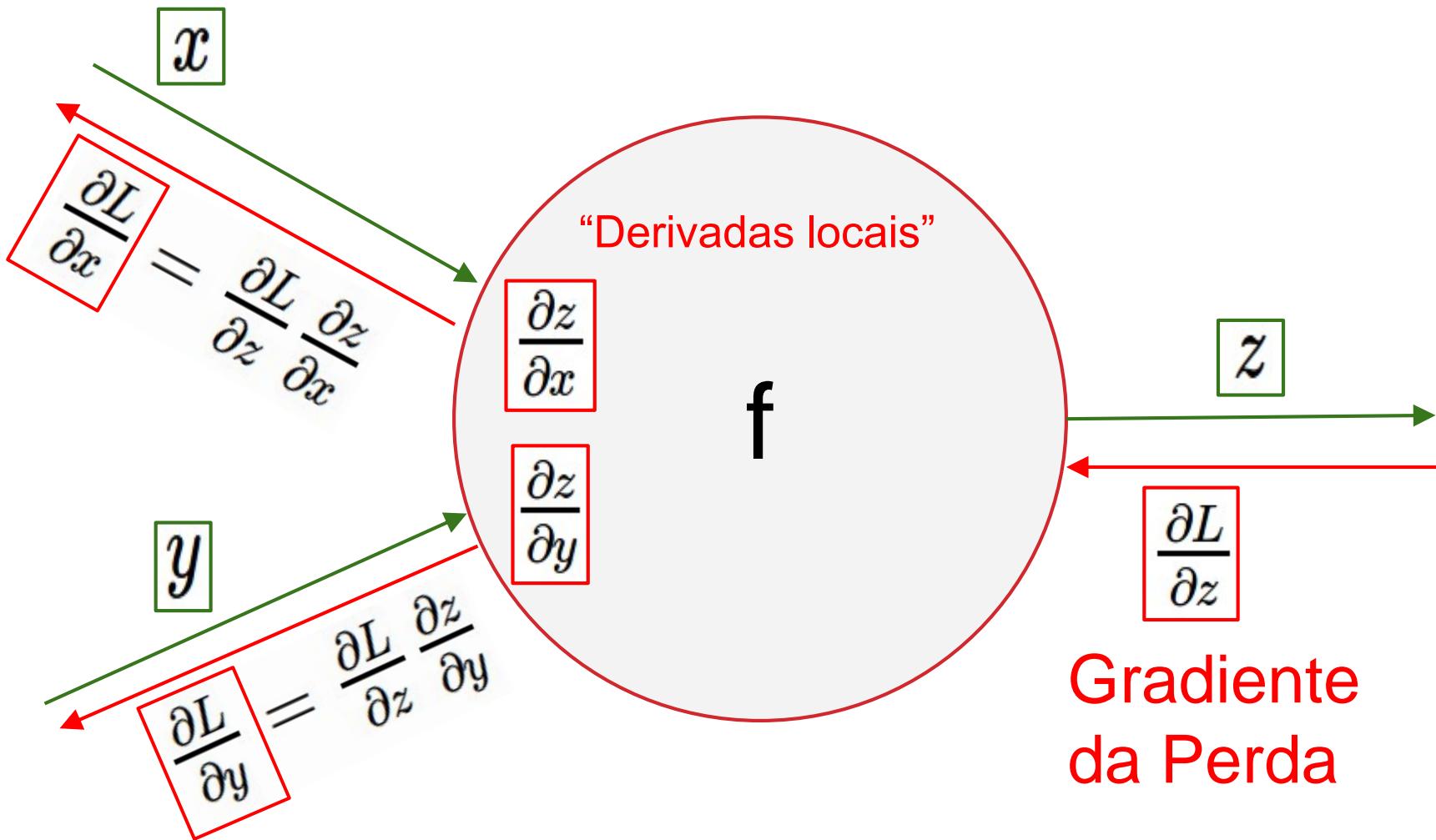
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