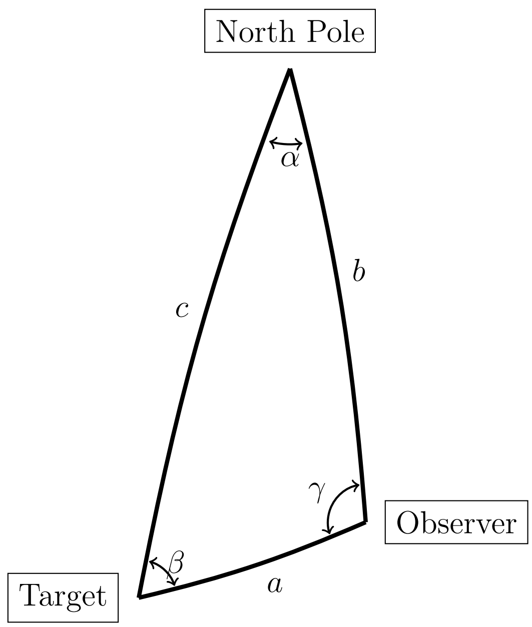
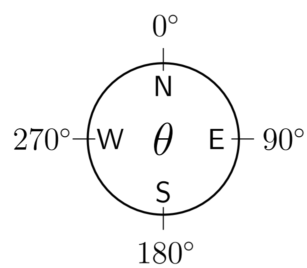
Spherical Geometry

What we want to do is: Given $(LAT, LONG)$ coordinates of an observer, and a given angle and distance from compass degrees and mils, find the $(LAT, LONG)$ coordinates of a target.





The spherical law of cosines is

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(\gamma)$$

α, β, γ are regular angles on the sphere, and a, b, c are the side-lengths of the triangle in terms of angular distance, which means to normalize the distance by dividing by the radius of the sphere (earth, in this case). So if the side length corresponding to a is one mile on the surface of the earth, then you'd have $a = \frac{1}{3960}$.

Since a value of a like this is pretty small, and $\cos(\theta)$ causes a lot of roundoff error in numerical computations when θ is small, it is standard to rewrite the law of cosines using a bunch of trig identities as

$$\boxed{\mathrm{hav}(c) = \mathrm{hav}(a-b) + \sin(a)\sin(b)\,\mathrm{hav}(\gamma)}$$

where $\mathrm{hav}(\theta) = \sin^2\left(\frac{\theta}{2}\right)$, so that no cosines are used in the computation. The variables that we know are

a	distance to target, from mils
b	from latitude of observer
γ	from compass degrees

From this we first compute c. This is the distance from the target to the north pole, which is used to find the latitude of the target.

$$c = \mathrm{hav}^{-1}\left(\mathrm{hav}(a-b) + \sin(a)\sin(b)\,\mathrm{hav}(\gamma)\right)$$

where $\mathrm{hav}^{-1}(x) = 2\arcsin(\sqrt{|x|})$. $\gamma = 2\pi(1-\theta/360)$ radians, where θ is the compass reading in degrees.

Then to get the longitude of the target we need to find α. The law of cosines gives

$$\mathrm{hav}(a) = \mathrm{hav}(b-c) + \sin(b)\sin(c)\,\mathrm{hav}(\alpha)$$

so

$$\mathrm{hav}(\alpha) = \frac{\mathrm{hav}(a) - \mathrm{hav}(b-c)}{\sin(b)\sin(c)}.$$

Now to get the coordinates of the target, you just convert c from angular distance to the north pole to angular distance from the equator:

$$\begin{aligned} LAT_T &= \frac{\pi}{2} - c \quad \text{(radians)} \\ &= 90 - \left(\frac{180}{\pi}\right) c \quad \text{(degrees)}. \end{aligned}$$

To get the longitude of the target you either add or subtract α in degrees from the angle that gives longitude of the observer. Longitude is increasing from West to East, so if the target is to the east of the observer ($0 \le \theta \le 180$), you add α. If the target is to the west of of the observer ($180 \le \theta \le 360$), you subtract α. So the formula for longitude is

$$LONG_T = LONG_O + \mathrm{sign}(180-\theta)\left(\frac{180}{\pi}\right)\alpha \quad \text{(degrees)},$$

where α is computed by taking hav^{-1} of the fraction above.