# 6447 Digital Content Protection

Dan is working for a digital content protection company, which is responsible for the content protection of blu-ray discs based on a standard called Anti Content Misuse (ACM).

The ACM standard works as follows. Assume there are  $2^n$  blu-ray drives/players. We represent these  $2^n$  drives as the leaves of a complete binary tree of height n, so that each root-to-leaf path consists of n edges. Each node u in this binary tree is assigned



Picture from Wikimedia Common

an identifier number and contains a random key  $k_u$ . The identifier numbers are assigned as follows. The root, r, is assigned 1. In addition, the left and right children of an internal node having number i are assigned numbers 2i and 2i+1, respectively. This scheme assigns a distinct number to each node in the tree. The keys contained in the nodes are unknown to blu-ray users, but they are available to blu-ray drive manufacturers. Each blu-ray player is assigned the identifier number i ( $2^n \le i \le 2^{n+1}-1$ ) of its corresponding leaf in the tree. A manufacturer of blu-ray drives embeds the keys associated with the nodes in the path from the root to leaf number i in player number i.

To encrypt the content of a blu-ray disc, the company in charge creates a random key k called the master key. First, they encrypt k with the key  $k_r$  (recall r is the root node of binary tree) and write it on the disc as a header. Then, they encrypt the content with k, and write the encrypted data on the blu-ray disc. A blu-ray drive first decrypts the header using key  $k_r$  embedded in it and recovers the master key k and then, decrypts the content using the key k.

Unfortunately, the keys embedded in a set of blu-ray drives, R, are exposed by hackers and published on the web. As a result, we cannot encrypt the master key k using any of these exposed keys. For example, since all blu-ray drives contain  $k_r$ , the encryption scheme above does not work any more. There is a solution oversaw for this situation in the ACM standard. At the cost of a larger header, the industry can safely encrypt the content of a new blu-ray disc. They carefully choose a subset of unexposed keys K in the binary tree such that all blu-ray drives, except for drives in R, have at least one of the keys in K. They encrypt the master key k with each key  $k' \in K$  and put the result in the header (i.e., there are |K| ciphertexts in the header). Now, each active blu-ray drive can decrypt at least one of the ciphertexts in the header and can recover the master key k. Dan needs your help to determine a subset of keys K with minimum cardinality (which results in the smallest header) given the identifiers of hacked drives.

#### Input

The input file contains several test cases, each of them as described below.

A test case consists of two lines. The first line contains two integers n and |R|, where  $1 \le n \le 62$  and  $1 \le |R| \le 1000$ . |R| is the cardinality of R, the set of exposed drives. The second line contains |R| integers, which are the identifiers of exposed blu-ray drives. You can assume that there is at least one blu-ray drive not hacked.

#### Output

For each test case, display, on a line by itself, the identifiers of nodes corresponding to the keys in K, satisfying the above requirements and having minimum cardinality, in increasing order and separated

with single spaces.

## Sample Input

2 1

5

3 3

10 11 12

### **Sample Output**

3 4

4 7 13