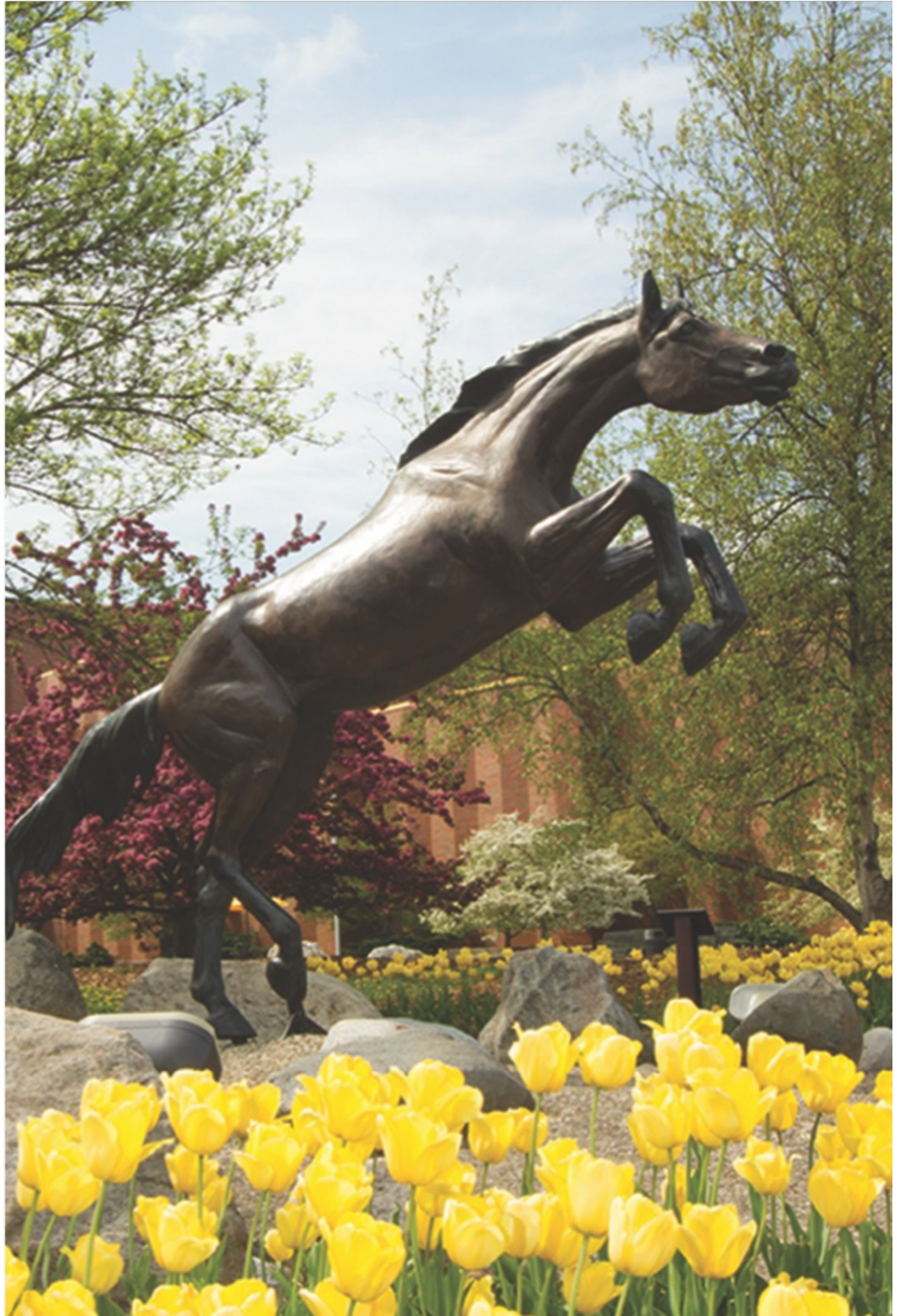
Western Michigan University

CS 5541 – Computer Systems

"Based on lecture notes developed by Randal E. Bryant and David R. O’Hallaron in conjunction with their textbook “Computer Systems: A Programmer’s Perspective”



Module 1

Representing Numbers

Part 1 – Integers

From: Computer Systems, Chapter 2

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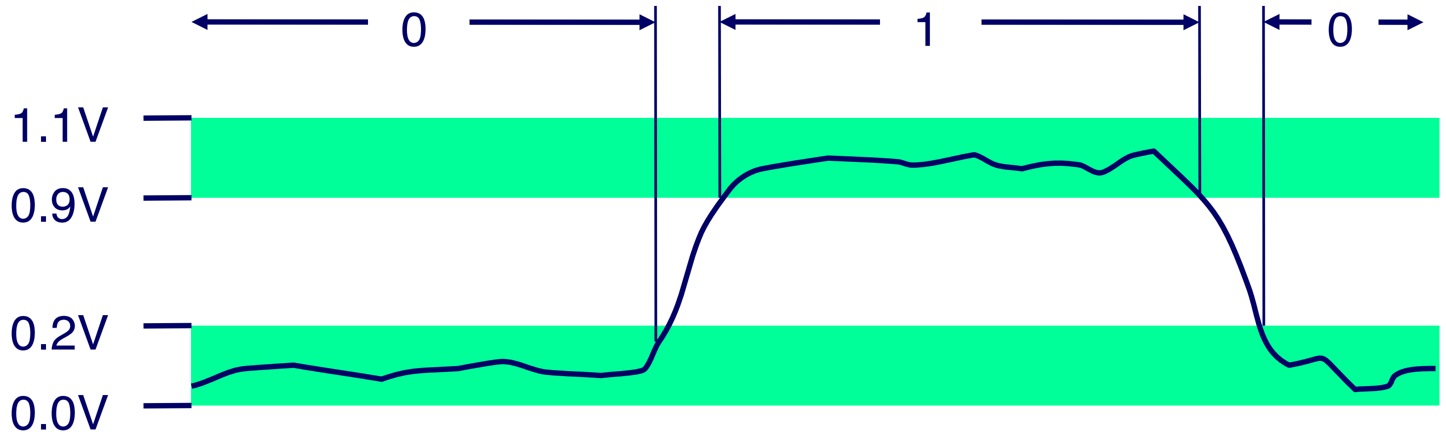
Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
`long double`	–	–	10/16
pointer	4	8	8

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Boolean Algebra

- **Operate on Bit Vectors**
  - **Operations applied bitwise**

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Representing & Manipulating Sets

- **Representation**
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$
    - `01101001 { 0, 3, 5, 6 }`
    - `76543210`
    - `01010101 { 0, 2, 4, 6 }`
    - `76543210`
- **Operations**
  - `&  Intersection       01000001   { 0, 6 }`
  - `|  Union              01111101   { 0, 2, 3, 4, 5, 6 }`
  - `^  Symmetric diff.    00111100   { 2, 3, 4, 5 }`
  - `~  Complement         10101010   { 1, 3, 5, 7 }`

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any “integral” data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise

- **Examples (Char data type)**
  - ~0x41 ➝ 0xBE
    - $\sim01000001_2 \rightarrow 10111110_2$
  - ~0x00 ➝ 0xFF
    - $\sim00000000_2 \rightarrow 11111111_2$
  - 0x69 & 0x55 ➝ 0x41
    - $01101001_2 \;\&\; 01010101_2 \rightarrow 01000001_2$
  - 0x69 | 0x55 ➝ 0x7D
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

- **Contrast to Logical Operators**
  - &&, ||, !
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41 → 0x01

  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

Contrast: Logic Operations in C

- **Contrast to Logical Operators**
  - &&, ||, !
    - View 0 as "False"
    - Anything nonzero
    - Always ret
    - Early term
- **Examples (ch**
  - !0x41 ➙ 0x
  - !0x00 ➙ 0x
  - !!0x41 ➙ 0x

  - 0x69 && 0x55 ➙ 0x01
  - 0x69 || 0x55 ➙ 0x01
  - p && *p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)… one of the more common oopsies in C programming

Shift Operations

- **Left Shift: x << y**
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
  - Fill with 0's on right
- **Right Shift: x >> y**
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument **x**	`01100010`
<< 3	`00010000`
Log. >> 2	`00011000`
Arith. >> 2	`00011000`

Argument **x**	`10100010`
<< 3	`00010000`
Log. >> 2	`00101000`
Arith. >> 2	`11101000`

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C short 2 bytes long**

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Encoding Integers

```
x =       15213: 00111011 01101101
y =      -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

Unsigned Values

$UMin = 0$

000…0

$UMax = 2^w - 1$

111…1

Two's Complement Values

$TMin = -2^{w-1}$

100…0

$TMax = 2^{w-1} - 1$

011…1

Other Values

Minus 1

111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

$|TMin| = TMax + 1$

Asymmetric range

$UMax = 2 * TMax + 1$

C Programming

#include <limits.h>

Declares constants, e.g.,

ULONG_MAX

LONG_MAX

LONG_MIN

Values platform specific

Unsigned & Signed Numeric Values

X	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - **Same encodings for nonnegative values**
- **Uniqueness**
  - **Every bit pattern represents unique integer value**
  - **Each representable integer has unique bit encoding**
- $\Rightarrow$ **Can Invert Mappings**
  - **U2B(x) = B2U-1(x)**
    - **Bit pattern for unsigned integer**
  - **T2B(x) = B2T-1(x)**
    - **Bit pattern for two's comp integer**

Module 1 (Part 1)
Summary

- Explain bistable elements
- Perform simple logic operations
- Perform Shift operations
- Explain Integer encoding rules and numeric ranges