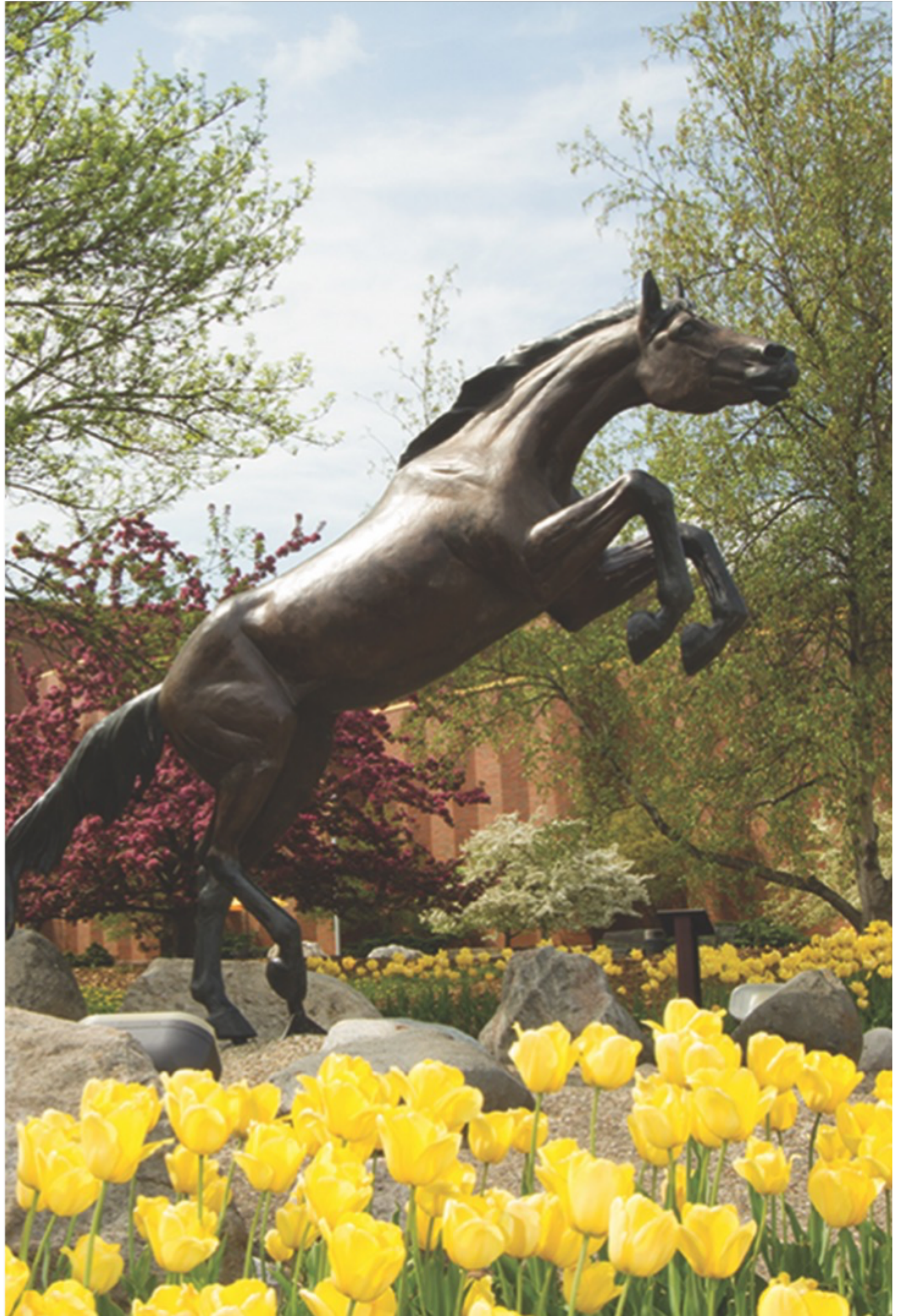
Western Michigan University

CS 5541 – Computer Systems

"Based on lecture notes developed by Randal E. Bryant and David R. O’Hallaron in conjunction with their textbook “Computer Systems: A Programmer’s Perspective”



Module 1

Representing Numbers

Part 7 – Extras

From: Computer Systems, Chapter 2

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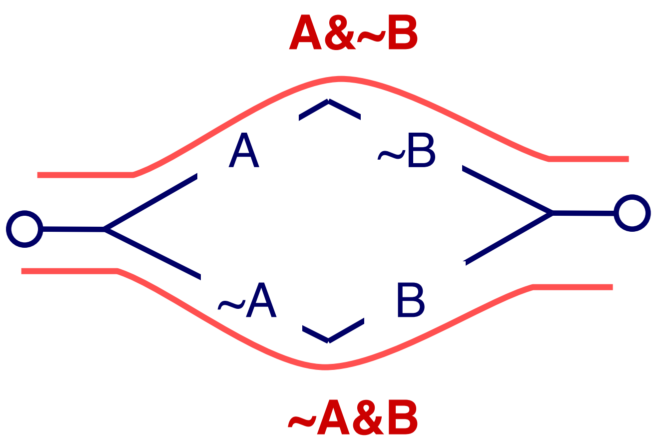
Application of Boolean Algebra

- **Applied to Digital Systems by Claude Shannon**
  - 1937 MIT Master's Thesis
  - Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Binary Number Property

Claim

$$1 + 1 + 2 + 4 + 8 + \ldots + 2^{w\text{-}1} = 2^w$$

$$1 + \sum_{i=0}^{w-1} 2^i \quad = \quad 2^w$$

- **w = 0:**
  - $1 = 2^0$
- **Assume true for w-1:**
  - $\underbrace{1 + 1 + 2 + 4 + 8 + \ldots +}_{=\ 2^w} 2^{w\text{-}1} + 2^w \quad = \quad 2^w + 2^w \quad = \quad 2^{w+1}$

Code Security Example

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

- **Similar to code found in FreeBSD's implementation of getpeername**
- **There are legions of smart people trying to find vulnerabilities in programs**

Typical Usage

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, MSIZE);
    printf("%s\n", mybuf);
}
```

Malicious Usage

```
/* Declaration of library function memcpy */
void *memcpy(void *dest, void *src, size_t n);
```

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, -MSIZE);
    . . .
}
```

Mathematical Properties

- **Modular Addition Forms an *Abelian Group***
  - **Closed** under addition

    $0 \leq \mathrm{UAdd}_w(u\,,v) \leq 2^w - 1$
  - **Commutative**

    $\mathrm{UAdd}_w(u\,,v) = \mathrm{UAdd}_w(v\,,u)$
  - **Associative**

    $\mathrm{UAdd}_w(t, \mathrm{UAdd}_w(u\,,v)) = \mathrm{UAdd}_w(\mathrm{UAdd}_w(t, u\,), v)$
  - **0** is additive identity

    $\mathrm{UAdd}_w(u\,,0) = u$
  - Every element has additive **inverse**
    - Let $\mathrm{UComp}_w(u\,) = 2^w - u$

      $\mathrm{UAdd}_w(u\,,\mathrm{UComp}_w(u\,)) = 0$

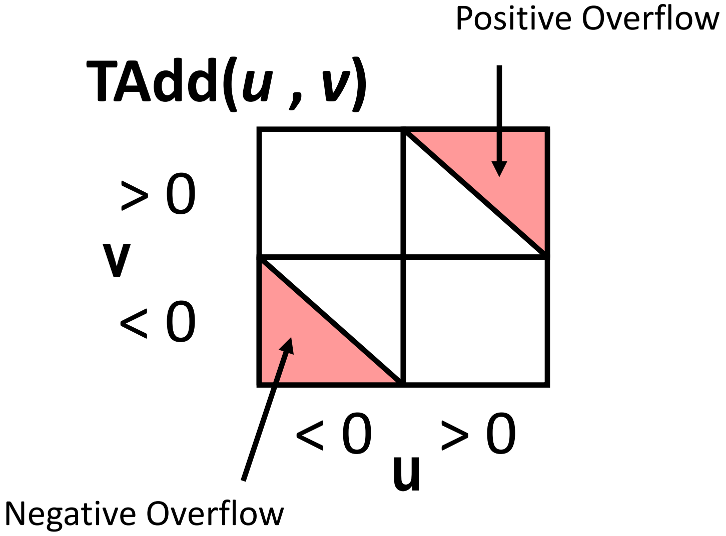
Mathematical Properties of TAdd

- **Isomorphic Group to unsigneds with UAdd**
  - $\text{TAdd}_w(u\,,\,v) = \text{U2T}(\text{UAdd}_w(\text{T2U}(u\,),\,\text{T2U}(v)))$
    - Since both have identical bit patterns
- **Two's Complement Under TAdd Forms a Group**
  - Closed, Commutative, Associative, 0 is additive identity
  - Every element has additive inverse

$$TComp_w(u) \quad = \quad \begin{cases} -u & u \neq TMin_w \\ TMin_w & u = TMin_w \end{cases}$$

Characterizing TAdd

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \quad \textbf{(NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \quad \textbf{(PosOver)} \end{cases}$$

Negation: Complement & Increment

- **Claim: Following Holds for 2's Complement**

  `~x + 1 == -x`

- **Complement**
  - Observation: `~x + x == 1111…111 == -1`

	x	1	0	0	1	1	1	0	1
+	~x	0	1	1	0	0	0	1	0
	-1	1	1	1	1	1	1	1	1

- **Complete Proof?**

Complement & Increment Examples

x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
y	-15213	C4 93	11000100 10010011

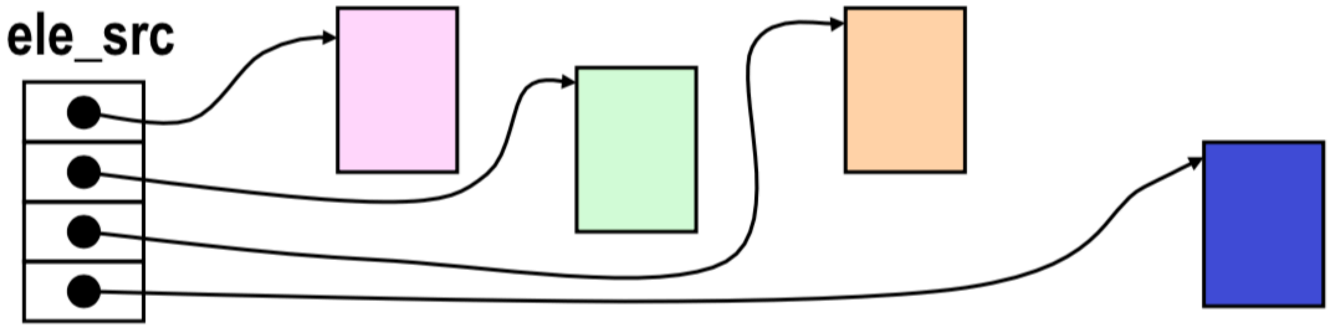
x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

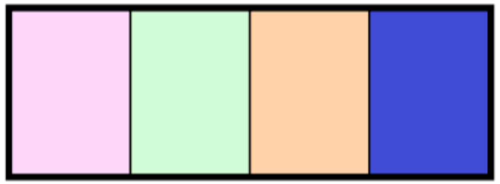
Code Security Example #2

- **SUN XDR library**
  - Widely used library for transferring data between machines

```
void* copy_elements(void *ele_src[], int ele_cnt, size_t ele_size);
```



malloc(ele_cnt * ele_size)



XDR Code

```
void* copy_elements(void *ele_src[], int ele_cnt, size_t ele_size) {
    /*
     * Allocate buffer for ele_cnt objects, each of ele_size bytes
     * and copy from locations designated by ele_src
     */
    void *result = malloc(ele_cnt * ele_size);
    if (result == NULL)
        /* malloc failed */
        return NULL;
    void *next = result;
    int i;
    for (i = 0; i < ele_cnt; i++) {
        /* Copy object i to destination */
        memcpy(next, ele_src[i], ele_size);
        /* Move pointer to next memory region */
        next += ele_size;
    }
    return result;
}
```

XDR Vulnerability

malloc(ele_cnt * ele_size)

- **What if:**
  - `ele_cnt` = $2^{20} + 1$
  - `ele_size` = 4096 = 2^{12}
  - Allocation = ??
- **How can I make this function secure?**

Compiled Multiplication Code

C Function

```
long mul12(long x)
{
  return x*12;
}
```

Compiled Arithmetic Operations

```
  leaq (%rax,%rax,2), %rax
  salq $2, %rax
```

Explanation

```
  t <- x+x*2
  return t << 2;
```

- C compiler automatically generates shift/add code when multiplying by constant

Compiled Unsigned Division Code

C Function

```
unsigned long udiv8
         (unsigned long x)
{
   return x/8;
}
```

Compiled Arithmetic Operations

```
   shrq $3, %rax
```

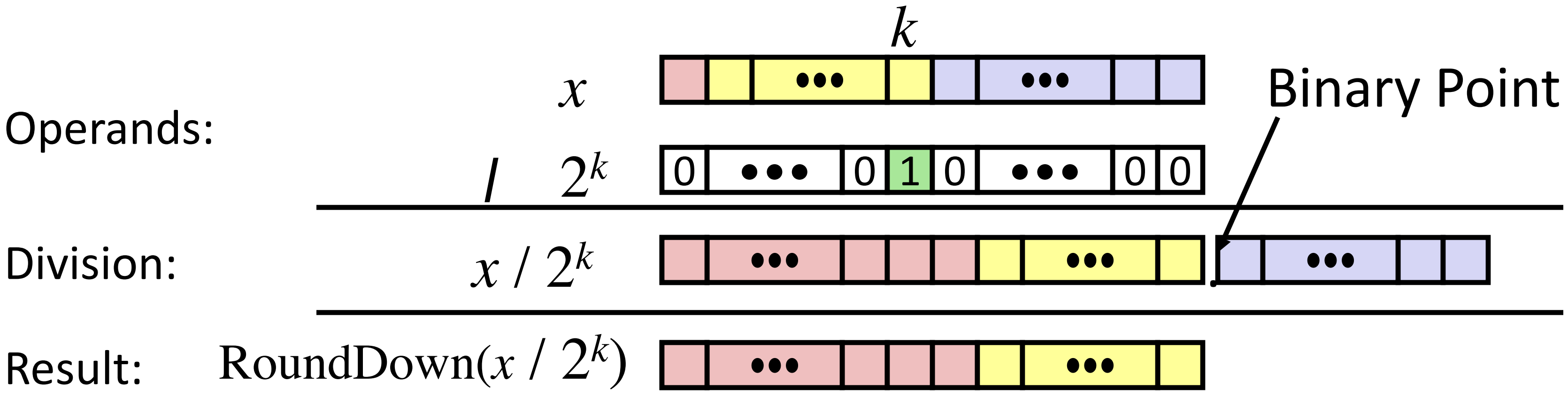
Explanation

```
   # Logical shift
   return x >> 3;
```

- **Uses logical shift for unsigned**
- **For Java Users**
  - Logical shift written as >>>

Signed Power-of-2 Divide with Shift

- **Quotient of Signed by Power of 2**
  - **`x >> k`** gives $\lfloor$ **`x / 2`**^{k} $\rfloor$
  - Uses arithmetic shift
  - Rounds wrong direction when **`u`** `< 0`

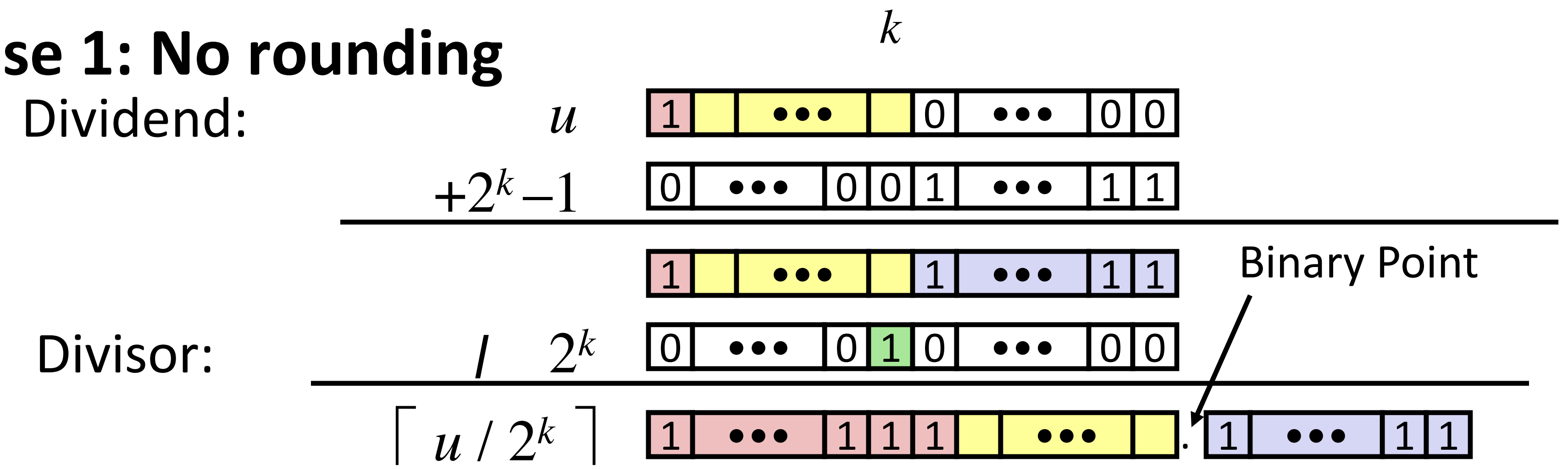


	Division	Computed	Hex	Binary
`y`	-15213	-15213	`C4 93`	`11000100 10010011`
`y >> 1`	-7606.5	-7607	`E2 49`	`11100010 01001001`
`y >> 4`	-950.8125	-951	`FC 49`	`11111100 01001001`
`y >> 8`	-59.4257813	-60	`FF C4`	`11111111 11000100`

Correct Power-of-2 Divide

- **Quotient of Negative Number by Power of 2**
  - Want $\lceil$ `x / 2`^{k} $\rceil$ (Round Toward 0)
  - Compute as $\lfloor$ `(x+2`^{k}`-1)/ 2`^{k} $\rfloor$
    - In C: `(x + (1<<k)-1) >> k`
    - Biases dividend toward 0

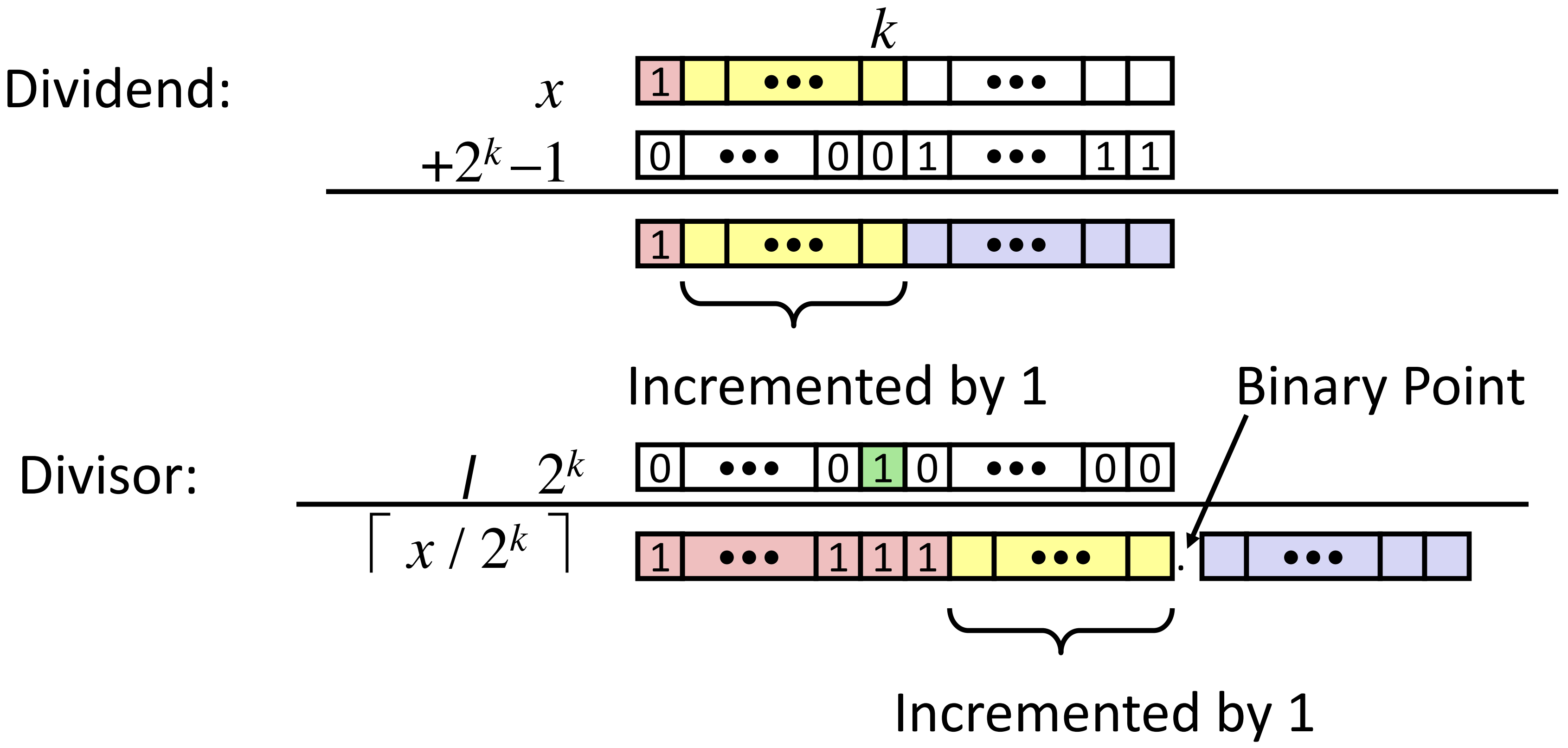
Case 1: No rounding



Biasing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding



Biasing adds 1 to final result

Compiled Signed Division Code

C Function

```
long idiv8(long x)
{
  return x/8;
}
```

Compiled Arithmetic Operations

```
  testq %rax, %rax
  js    L4
L3:
  sarq $3, %rax
  ret
L4:
  addq $7, %rax
  jmp  L3
```

Explanation

```
  if x < 0
    x += 7;
  # Arithmetic shift
  return x >> 3;
```

- **Uses arithmetic shift for int**
- **For Java Users**
  - Arith. shift written as >>

Arithmetic: Basic Rules

- **Unsigned ints, 2's complement ints are isomorphic rings: isomorphism = casting**

- **Left shift**
  - Unsigned/signed: multiplication by 2^k
  - Always logical shift

- **Right shift**
  - Unsigned: logical shift, div (division + round to zero) by 2^k
  - Signed: arithmetic shift
    - Positive numbers: div (division + round to zero) by 2^k
    - Negative numbers: div (division + round away from zero) by 2^k

Properties of Unsigned Arithmetic

- **Unsigned Multiplication with Addition Forms Commutative Ring**
  - Addition is commutative group
  - Closed under multiplication

    $0 \leq \mathrm{UMult}_w(u, v) \leq 2^w - 1$
  - Multiplication Commutative

    $\mathrm{UMult}_w(u, v) = \mathrm{UMult}_w(v, u)$
  - Multiplication is Associative

    $\mathrm{UMult}_w(t, \mathrm{UMult}_w(u, v)) = \mathrm{UMult}_w(\mathrm{UMult}_w(t, u), v)$
  - 1 is multiplicative identity

    $\mathrm{UMult}_w(u, 1) = u$
  - Multiplication distributes over addtion

    $\mathrm{UMult}_w(t, \mathrm{UAdd}_w(u, v)) = \mathrm{UAdd}_w(\mathrm{UMult}_w(t, u), \mathrm{UMult}_w(t, v))$

Properties of Two's Comp. Arithmetic

- **Isomorphic Algebras**
  - Unsigned multiplication and addition
    - Truncating to *w* bits
  - Two's complement multiplication and addition
    - Truncating to *w* bits
- **Both Form Rings**
  - Isomorphic to ring of integers mod 2^w
- **Comparison to (Mathematical) Integer Arithmetic**
  - Both are rings
  - Integers obey ordering properties, e.g.,

    $u > 0 \quad \Rightarrow \quad u + v > v$

    $u > 0, v > 0 \quad \Rightarrow \quad u \cdot v > 0$
  - These properties are not obeyed by two's comp. arithmetic

    TMax `+ 1` `==` *TMin*

    `15213 * 30426 == -10030` (16-bit words)

Reading Byte-Reversed Listings

- **Disassembly**
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- **Example Fragment**

Address	Instruction Code	Assembly Rendition	
8048365:	5b	pop	%ebx
8048366:	81 c3 ab 12 00 00	add	$0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl	$0x0,0x28(%ebx)

- **Deciphering Numbers**
  - Value: 0x12ab
  - Pad to 32 bits: 0x000012ab
  - Split into bytes: 00 00 12 ab
  - Reverse: ab 12 00 00

Module 1 (Part 7)

Summary

- Bonus content