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REVIEW ARTICLE

A Review of Multi-objective Optimization: Methods and Algorithms in Mechanical Engineering Problems

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Abstract

The optimization problems that must meet more than one objective are called multi-objective optimization problems and may present several optimal solutions. This manuscript brings the most important concepts of multi-objective optimization and a systematic review of the most cited articles in the last years in mechanical engineering, giving details about the main applied multi-objective optimization algorithms and methods in this field. Some of the applications that can be found in this study are: (i) problems in design optimization, (ii) problems in manufacturing: welding, machining and molding and (iii) problems in structural health monitoring. It can be seen that classic optimization methods had their importance in the past, but lost space for new algorithms that emerged with the advancement of computing, better able to deal with a greater number of variables, objectives and nonlinearities. These powerful algorithms, still little used in Mechanical Engineering, showed significant improvement where they were applied. Meta-heuristics with a *posteriori* decision-making techniques proved to be a modern trend in solving multi-objective problems, although it is not limited due to the constant battle of new algorithms more adapted to specific problems.

Abbreviations

MOP	Multi-objective Problems
PF	Pareto Front
ZDT	Zitzler-Deb-Thiele test function
DM	Decision Maker
NBI	Normal Boundary Intersection
GA	Genetic Algorithm
MOEA	Multi-objective Evolutionary Algorithm
VEGA	Vector Evaluation GA
MOGA	Multi-objective GA
NSGA (I and II)	Non-dominated Sorting GA
SPEA (I and II)	Strength Pareto Evolutionary Algorithm
MOPSO	Multi-objective Particle Swarm Optimization
MOGWO	Multi-objective Grey Wolf Optimizer
MOALO	Multi-objective Ant Lion Optimizer
OM	Orthogonal Method
FG	Functionally Graded
TOPSIS	Technique to Order of Preference by Similarity to Ideal Solution
ORC	Organic Rankine Cycles
DMS	Direct Multi-Search
SAW	Submerge Arc Welding
FCAW	Flux Cored Arc Welding
GMAW	Gas Metal Arc Welding
FEM	Finite Element Method
DOE	Design of Experiments
ANOVA	Analysis of Variance
FSW	Friction Stir Welding
ANN	Artificial Neural Network
NSTLBO	Non-dominated Teaching–Learning Based Algorithm
TLBO	Teaching–Learning Based Optimization
HMOGWO	Hybrid grey wolf optimizer
SHM	Structural Health Monitoring

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1 Introduction

Optimization can be described as a process of searching for the best solution within a set of possible solutions [7]. As in practical engineering problems most applications are nonlinear, have complicated or nonexistent analytical solutions and must often serve more than one objective or function that may even be in conflict with each other, they require sophisticated optimization tools to be determined [65, 179].

In optimizing a single objective, it is possible to determine given one set of solutions, which is better than others. As result, a single solution is usually obtained. However, in multi-objective optimization there is no direct method to determine if one solution is better than another because the answer is a set of solutions that involve multiple conflicting objectives considered simultaneously [66].

There are several techniques, between algorithms and decision-making techniques, for dealing with a multi-objective optimization problem and even more recently. The methods can be divided into optimization processes in which an operator can participate at any time: not participating, participating at the beginning of the optimization process, during or only at the end. The solutions found depend heavily on when the problem operator acts [31].

Regardless of how the operator works, there are several optimization algorithms that can work with him according to the methodology adopted. Understanding the basic concepts of multi-objective optimization that are available in the literature is extremely important to situate the operator of the problem and provide support for him to choose the best possible approach to his problem.

Mechanical engineering is a vast and complex area with numerous possible applications for multi-objective optimization. Nowadays, according to the author's best knowledge, there is no work in the literature that compiles the main applications and allows the researcher an overview of the subject in this area. The review works found, and some are even out of date, usually focus on the (i) multi-objective optimization itself: Long et al. [102], Gunantara [68], Wang et al. [175], Marler and Arora [108] focusing on engineering in general, (ii) in algorithms and where they have already been applied: Liu et al. [101] in meta-heuristics for discrete optimization, Mane and Rao [107] in Evolutionary Algorithms, Tamaki et al. [165] in Genetic Algorithms, Song and Gu [110] in Particle Swarm Optimization, Leguizamón and Coello [93] in Ant Colony Optimization,or (iii) specific cases: Ridha et al. [148] in photovoltaic system, Kumar et al. [90] in machining, Ojstersek et al. [123] in production scheduling, Liu et al. [100, 101] in wind energy, Rangaiah et al. [140] in chemical process engineering, Afshari et al. [2] in concrete structures, Cui et al. [34] in energy saving, Fadaee et al. [46] in renewable energy.

This manuscript summarizes the most cited applications in the last five years and some others which, although not so recent, have great relevance in mechanical engineering. The main focus is to highlight the most modern and efficient trends on algorithms and decision-making techniques used, indicating precisely the decision variables and objective functions in real multi-objective optimization problems in mechanical engineering.

Some of the applications that can be found in this work are: (i) problems in design optimization: meta-materials, functionally graded plates, airfoils, wind turbines, fan and pumps, security and support structures, suspensions, exchangers and expanders, beams, composites, etc. (ii) problems in process engineering: welding, machining and molding and (iii) problems in structural health monitoring.

So this paper review has two purposes: (i) to synthesize the main concepts of multi-objective optimization: multi-objective problem; Pareto front and its types; decision-making techniques—no preference, a *priori*, *interactive* and a *posteriori* techniques and algorithms—enumerative, deterministic (gradient-based) and especially stochastic (evolutionary and others meta-heuristics) with examples and a critical discussion of its drawbacks and (ii) situate which techniques and algorithms within this vast area have been applied to problems in mechanical engineering, pointing out modern and more efficient trends.

This manuscript is organized as follows: Sect. 2 presents a general theoretical background review about multi-objective optimization. Section 3 presents a systematic review of the literature on the main applications of multi-objective optimization in Mechanical Engineering detailing which algorithms, techniques, decision variables and objectives were used in each of the problems and Sect. 4 brings conclusions.

2 Backgrounds

The optimization problems that must meet more than one objective are called Multi-objective Optimization Problems (MOPs) and present several optimal solutions [28]. The solution is the determination of a vector of decision variables $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ (variable decision space) that optimizes the vector of objective functions $F(\boldsymbol{X}) = \{f_1(x), f_2(x), \ldots, f_n(x)\}$ (objective function space) within a feasible region of solutions subject to equality $h_i(x)$ or inequalities $g_i(x)$ constraints where x_{min} and x_{max} are the limits that determine the search space for each of the variables, or vector of variables [17]. As described in Eq. (1) [67].

$$\min\ F(\mathbf{X}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})\}$$
$$\text{subject to}: \ h_i(\mathbf{x}) = 0,\ i = 1, 2, \ldots, p$$
$$g_i(\mathbf{x}) \leq x0,\ i = 1, 2, \ldots, q$$
$$x_{min} \leq \mathbf{x} \leq \mathbf{x}_{max} \tag{1}$$

This leads to a set of solutions called Pareto-optimal, which according to Rao [147] is a feasible region $\boldsymbol{X}$ that there is no other feasible region $\boldsymbol{Y}$ such that $f_i(\boldsymbol{Y}) \leq f_i(\boldsymbol{X})$ *for* $i = 1, 2, \ldots, k$ with $f_j(\boldsymbol{Y}) \leq f_j(\boldsymbol{X})$ for at least one j. That is, there is no other feasible solution $\boldsymbol{Y}$ that would reduce some objective function without causing an increase in at least one other objective function at the same time. The method most commonly used to compare solutions is Pareto Dominance Relationship, which instead of determining a single optimal solution, leads to a set of optimal alternatives between the objectives. These solutions are also called non-dominated solutions or Pareto Front (PF) [78] and any of these solutions are optimal and it is up to the operator of the problem to choose the best one according to his preference. See the variable decision space and the objective or solution space with examples of non-dominated solutions in blue (PF) in Fig. 1, where n is the number of design variables and k is the number of objective functions of the problem.

Possible solutions in variable decision space generate solutions in the objective space and dominance relations are analyzed to eliminate those that are not Pareto-optimal. As seen, this front is built through iterations and before having a final Pareto front, there are local Pareto fronts behind this. Each MOP has a characteristic PF, which can be continuous or discontinuous (disconnected) and convex or concave. A MOP will be considered convex if the viable set and the individual objective functions are convex (as in Fig. 1) and this leads to a convex PF. If the viable set is not convex, or at least one of the functions is non-convex, the problem will be considered concave. In general, for non-convex MOP, PF can be concave and disconnected [36]. Not every region of the objective space, including those above the PF, is a feasible region. There may be voids without solutions, which contribute to the discontinuity of the problem [17, 31]. Figure 2 illustrates this paragraph well.

In mono-objective optimization, the specific parameters of each meta-heuristic must be correctly chosen for each problem to have the best exploration and exploitation response. This is respectively, being able to escape from local minimums and still being able to improve the precision of the solutions already found [125]. In MOP, this alone is not enough. A good multi-objective optimization algorithm when looking for a set of possible solutions forming the Pareto front, must be able to find a PF with precision (convergence) and good distribution of possible solutions throughout the solution space (coverage) [22, 114, 115], as shown in Fig. 3.

Accurately determine the set of non-dominated solutions to a problem is a hard task made possible by the development of better computers and new techniques and algorithms. Zitzler et al. [193] interested in testing and comparing evolutionary multi-objective optimization algorithms has created six test

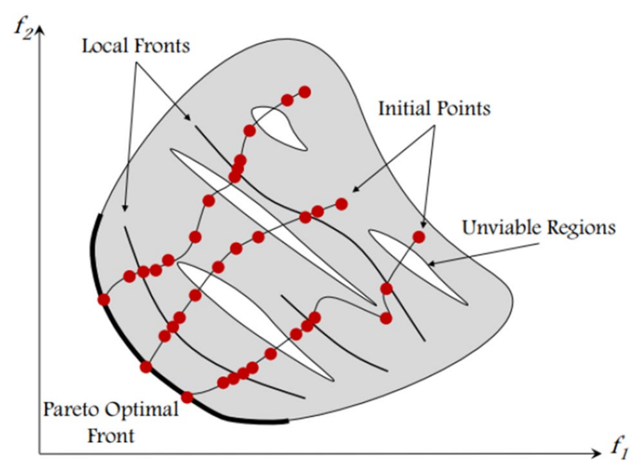
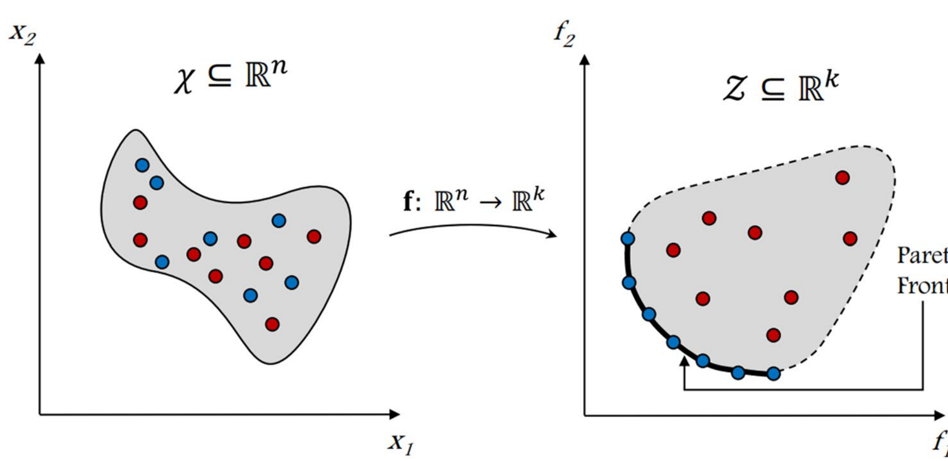


Fig. 2 Regions of a design problem with two-variable and two objective functions [66]

Fig. 1 Regions of a design problem with two-variable and two objective functions [66]



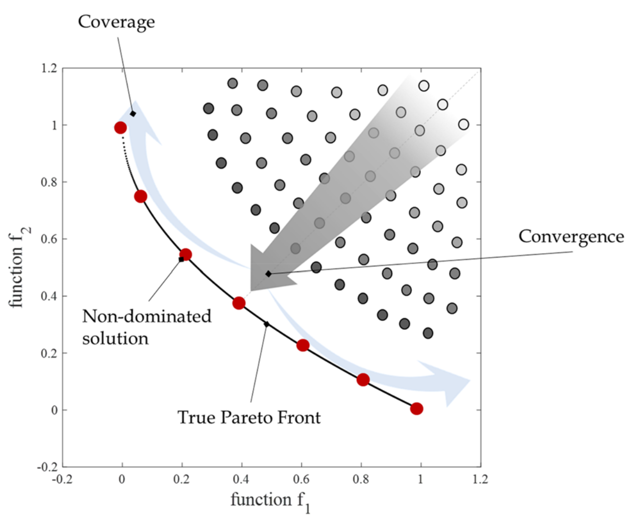


Fig. 3 Capabilities that a meta-heuristic must have to be successful in a MOP

functions that are still one of the most popular ones for testing new algorithms [114, 115]. The ZDT (Zitzler-Deb-Thiele) test functions are a set of problems with very diverse Pareto fronts in which the best algorithm is the one that has the Pareto front found closest to the true Pareto front. Figure 4 shows the true Pareto front for some of these functions.

Note that the Pareto fronts of the ZDT1 (Fig. 4a) and ZDT2 (Fig. 4b) functions are continuous, however the first is convex and the second is concave. The ZDT3 (Fig. 4c) represents the discreteness feature; its Pareto-optimal front consists of several noncontiguous convex parts and the ZDT4 (Fig. 4d) contains 21^9 local Pareto-optimal front and, therefore, tests for the algorithm´s ability to deal with multi-modality. The first three have thirty design variables and the last ten. Those functions that are well defined can check how well a new algorithm has convergence and coverage capabilities. At the same time, it shows how diverse the Pareto fronts of a real engineering problem can be.

2.1 Main Methods to Approach Multi-objective Problems

There are several techniques for dealing with MOPs. In general, they are classified according to when the preferences of the problem operator (or decision maker, DM) are inserted in the problem: a *priori*, *interactive*, a *posteriori* methods or no preference method [32].

2.1.1 No Preference Methods

In no preference methods the DM is not needed. Only one solution is computed and is usually as close as possible to the ideal point.

2.1.1.1 Global Criterion Method The ideal point or ideal vector is the utopia solution that contains each separately minimum objective function value achieved at the same point; see Eqs. (2) and (3) [31]. The Nadir point is the exact opposite, is the vector that contains each separately maximum objective function value found.

$$f_i^0(x^{0i}) = \min f_i(x) \tag{2}$$

$$f^0 = \left[f_1^0, f_1^0, f_1^0, \ldots, f_k^0\right]^T \tag{3}$$

Therefore, the best solution found in this method is seen in Eq. (4) [113]:

$$L_p = \min\left(\sum_{i=1}^{k} \left|f_i(x) - f_i^0\right|\right)^{1/p} \tag{4}$$

where $f_i(x)$ is the objective function i evaluated for all x with $i = 1, 2, \ldots, k$ the number of objectives and f_i^0 the minimum value found in the minimization separately of function i. The solution obtained depends greatly on the value chosen for p, widely used choices are 1, 2 or ∞. This is the function that provides the shortest distance between the PF and the ideal point. If p is one, there is a linear distance and if 2, a Euclidean distances [113].

The main drawback of these types of methods is to neglect all other solutions.

2.1.2 A Priori Preference Articulation

In a *priori* methods the DM need to input his preference before optimization starts. This method have some difficulties: (i) The DM, when initially taking his preferences, can neglect important aspects of the problem and consequently arrive at ineffective or even confusing results [167], (ii) An algorithm like this should be run multiple times to determine the PF and (iii) some special PF´s can not be determined with this approach [36, 111]. The main drawbacks of these methods are that: (i) an algorithm should be run multiple times to determine the Pareto optimal set, (ii) there is a need to consult with an expert because an inexperienced DM can select bad regions for exploration and neglect better ones by inducing the optimization process in the wrong direction and (iii) some special Pareto optimal fronts cannot be determined with this approach [114].

2.1.2.1 Lexigraphic Method In Lexigraphic method the DM must arrange the objective function according to their absolute importance (best to worst). After, the most important objective function is minimized subject to the original constraints. If this problem has a unique solution, is the solution of the whole MOP. Otherwise, the second most important objective function is minimized. Now, in addition

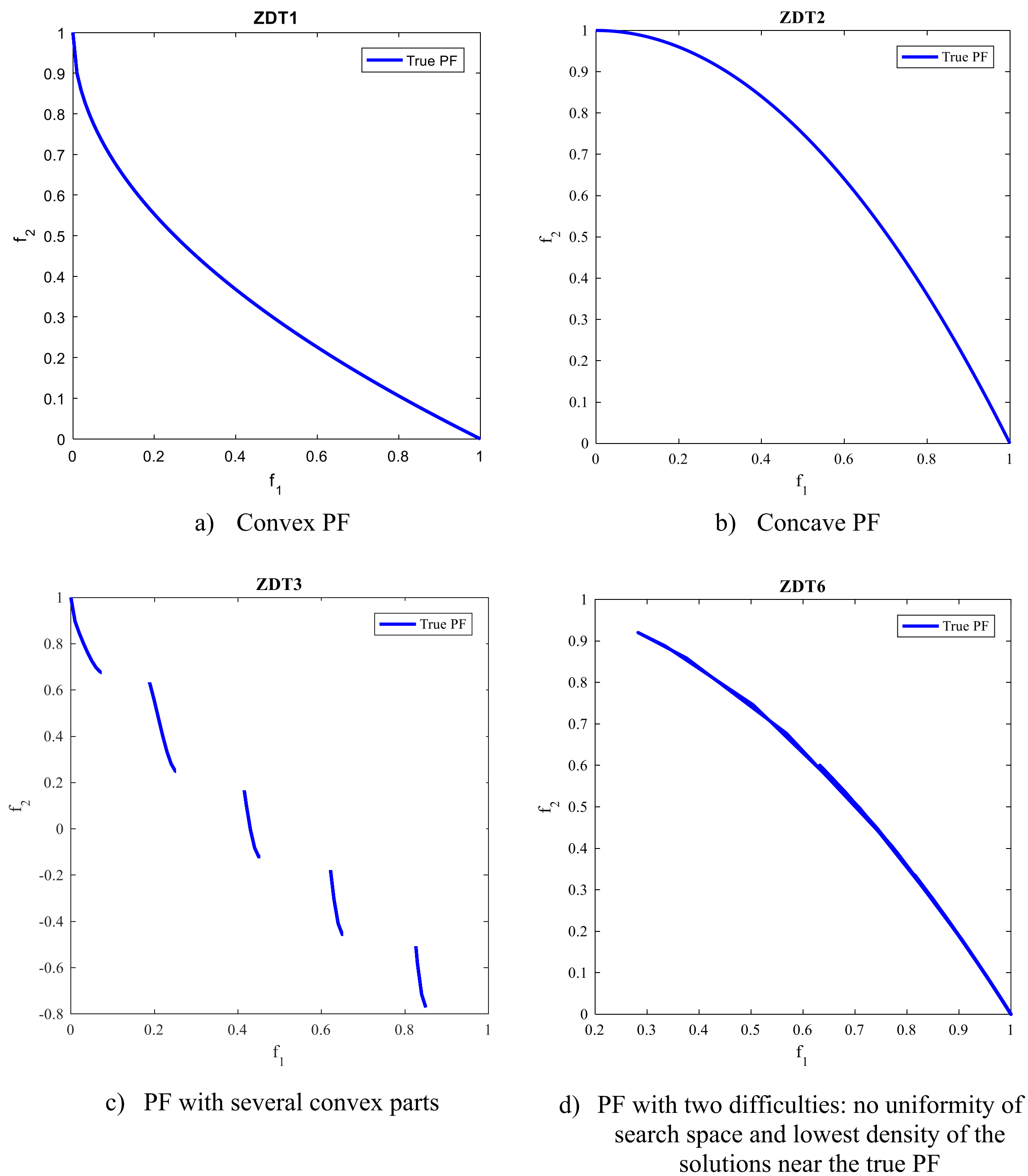


Fig. 4 Main ZDT test functions for MOP

to the original constraints, a new constraint is added to guarantee that the most important objective function preserves its optimal value. If this problem has a unique solution, is the solution of the original problem. Otherwise, the process goes on as above [145]

2.1.2.2 Goal Programming In this method, DM has to assign targets or a goal that wishes to achieve for each objective. These values are incorporated into the problem as additional constraints and then the objective function tries to minimize the absolute deviations from the targets to the objectives. The simplest form of this method is in Eq. (5) [31, 42]

$$\min \sum_{i=1}^{k} \left| f_i(x) - T_i \right| \tag{5}$$

where T_i denotes the target or goal set by the DM for the i-th objective function $f_i(x)$.

Other less used a priori methods and references for knowing hands are: Min–Max Optimization [127, 146], Multi-attribute Theory [121], ELECTRE (elimination and choice translating algorithm) [19] and it is derivations; PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) [21], among others.

2.1.3 An Interactive Preference Articulation

In *Interactive* methods the DM can to articulate his preferences during the optimization process, usually based on the domain knowledge acquired during the optimization [50, 82]. These techniques normally operate in three stages: (i) find a non-dominated solution, (ii) get the reaction of the DM regarding this solution and modify the preferences of the objectives according to his need and (iii) repeat the two previous steps until the DM is satisfied [32].

The main algorithms that use this technique are: (i) Probabilistic Trade-Off Development Method (PROTRADE), (ii) STEP Method and (iii) Sequential Multi-objective Problem Solving Method, (iv) Interactive Surrogate Worth Trade-Off Method (ISWT), (v) Geoffrion–Dyer–Feinberg Method (GDF), (vi) Sequential Proxy Optimization Technique (SPOT), (vii) Tchebycheff Method, (viii) Reference Point Method, among others [63, 31, 33, 113, 118].

2.1.4 A *Posteriori* Preference Articulation

In a *posteriori* methods, a set of representatives Pareto optimal is obtained and the DM can analyze the trade-off relationships between the objectives [32]. This method is the most used in the literature to solve real problems, since one of the advantages is to find PFs that no other method can find and with just one program run [114, 115].

2.1.4.1 Weighting Method The idea is associate each objective function with a weighting coefficient and minimize the weighted sum of the objectives. The multiple objective functions are transformed into a single objective functions. The weights w_i are positive real numbers for al $i = 1,\ldots,k$ objective functions. The weights are normalized, that is, $\sum_{i=1}^{k} w_i = 1$. So, here is the Eq. (6) [184]:

$$\min \sum_{i=1}^{k} w_i f_i(x) \tag{6}$$

However, the main drawbacks of this approach are the need to run an algorithm multiple times to find multiple Pareto optimal solutions, dealing with all the challenges in every run, lack of information exchange between Pareto optimal solutions during optimization cause weights are always positive and concave PF can not be found and the need to consult with an expert to find the best weights [116]. According to Miettinen [113], this method can be an a priori method if the DM defines the weight he wants to transform a MOP into a mono-objective problem.

2.1.4.2 ∈-Constraint Method This method was introduced by Haimes et al. [70] and only one of the objective functions is selected to be optimized and all other objective functions are converted into constraints by setting an upper bound to each of them. The problem to be solved has the form in Eq. (7):

$$\begin{aligned} &\text{minimize} \quad f_l(x) \\ &\text{subject to} \quad f_j(x) \le \varepsilon_j \end{aligned} \tag{7}$$

where for all $j = 1, \ldots, k, j \neq l, l \in \{1, \ldots, k\}$.

2.1.4.3 Hybrid Method This method combines the Weighting Method and the ∈-Constraint Method [113].

2.1.4.4 Normal Boundary Intersection The Normal Boundary Intersection (NBI) is proposed by Das and Dennis [36] and according to the author, the method is independent of the relative scales of the functions and is successful in producing an evenly distributed set of points in the Pareto set given an evenly distributed set of parameters, a property which the popular method of minimizing weighted combinations of objective functions lacks. However, according to Brito et al. [24], this method is extremely sensitive to the presence of correlation between objective functions that are used in the construction of PF.

This method starts with the determination of the payoff matrix (Φ). The vector of decision variables that minimizes the objective function $f_i(x)$ is represented by x_i^* and consequently the minimum value of $f_i(x)$ at this point is $f_i^*(x_i^*)$. When replacing the individual point x_i^* in the other functions, there is $f_i(x_i^*)$, this is a non-optimal value of this function. Repeating the algorithm for all m functions, is obtained the payoff matrix represented in Eq. (8):

$$\Phi = \begin{bmatrix} f_1^*(x_1^*) & \cdots & f_1(x_i^*) & \cdots & f_1(x_m^*) \\ \vdots & \ddots & \vdots & ⋰ & \vdots \\ f_i^*(x_1^*) & \cdots & f_i^*(x_i^*) & \cdots & f_i^*(x_m^*) \\ \vdots & ⋰ & \vdots & \ddots & \vdots \\ f_m^*(x_1^*) & \cdots & f_m(x_i^*) & \cdots & f_m^*(x_m^*) \end{bmatrix} \tag{8}$$

Each line of Φ is composed of minimum and maximum values of $f_i(x)$. These sets of extreme points are used to normalize objective functions. Considering a set of weights w_i, the $\Phi^* w_i$ will represent a point on the utopia line. Since η is a unit vector in the direction of origin and normal to the utopia line at points $\Phi^* w_i$, is obtained that $\Phi^* w + D^* \eta$ will represent the set of points in that normal. The point where the normal intersects the boundary of the viable region closest to the origin will be the point corresponding to the maximization of the distance between the utopia line and the PF [36, 24]. Therefore, the NBI method can be written as a constrained nonlinear maximization problem defined as in Eq. (9):

$$\begin{aligned} &\mathrm{Max}_{(x,t)}\mathrm{D} \\ &\text{s.t. } : \Phi * w + \mathrm{D} * \eta = F(x) \end{aligned} \tag{9}$$

This problem must be solved iteratively for different values of w to generate an equally spaced PF.

2.1.4.5 Technique for Order of Preference by Similarity to Ideal Solution The Technique for the Order of Preference by Similarity to Ideal Solution (TOPSIS) was introduced by Hwang and Yoon [76] and became a classic multiple attribute decision making method with more than 4500 citations [183]. TOPSIS determines the positive ideal solution (A+) (Utopia point) as well as the negative ideal solution (A−) (Nadir Point) and normalizes each of the objectives according to itself and multiplies them by the weight assigned to each objective, being the sum of all weights w_i always one. Then it calculates the Euclidian distance of each solution (A_i) in the Pareto front to the utopia point (originating S_i^+) and to the Nadir point (originating S_i^-) and calculates the score P_i using Eq. (10):

$$P_i = \frac{S_i^-}{S_i^+ + S_i^-} \tag{10}$$

If $P_i = 1$, $A_i = \mathrm{A}+$ and if $P_i = 0$, $A_i = \mathrm{A}$ (that is, the higher Pi, the better the solution). In few words, TOPSIS determines the best compromised solution, which was the closest to (A+) and the furthest from (A−) based on the Pareto set according to the objective weights and the normalization of these solutions. Note that with the Score P_i of each solution, it is possible to rank all solutions on the Pareto front from best to worst according to TOPSIS.

2.2 Main Algorithms to Approach Multi-objective problems

Regardless of the way the DM approaches MOP, this as an optimization problem and must have some technique of general search that should lead to the maximization or minimization of the functions or their compositions. Just like optimizing a single objective, The MOP can be classified through 3 categories according to the applied search technique: enumerative, deterministic and stochastic.

Enumerative is the simplest search strategy where each possible solution is evaluated. However, this technique is inefficient or even unfeasible as search space becomes large, making the MOP solution extremely costly and slows even computationally [31]. Deterministic methods are those based on gradient or derivatives and the most used in MOP are: (i) Greedy, (ii) Hill Climbing algorithms, (iii) Branch and Bound, (iv) Depth-First and (v) Breadth-First, (vi) best-first and (vii) calculations-based [23, 64]. However according Parkinson et al. [136] and Coello et al. [31], deterministic algorithms have difficulty for optimization problems with: (i) discrete-valued design variables,(ii) large number of design variables; (iii) multiple local minima, maxima, and saddle points (multimodal); (iv) no differentiable objectives and constraints; (vi) discontinuities of functions or regions (vi) analysis programs which crash for some designs. Therefore, enumerative and deterministic search techniques are unsuitable in real-world and engineering MOPs. Therefore they will not be addressed in this paper.

Stochastic techniques have demonstrated great potential for the solution of complex MOPs and are increasingly gaining space with the increase in the speed and processing capacity of computers. Today this is the main technique for engineers and designers and although there are several algorithms, the basis of all of them consists in the initialization of the optimization process with a set of random candidate solutions for a given problem and improves them over a predefined number of steps. To address the real-world issues, these algorithms should be equipped with different operators [114, 115].

The literature shows that almost all stochastic algorithms used in multi-objective optimization were inspired by some optimal phenomena found in nature and are commonly called meta-heuristics. In general, there are four main groups that divide meta-heuristics according to the inspiration for their creation: (i) based on evolution, (ii) based on physical phenomena, (iii) based on behaviors related to humans and (iv) based on swarms [72]. For Yang [179], there is also a classification for meta-heuristic algorithms that can be based on trajectories or population.

In the literature, an immense variety of meta-heuristics can be found that are capable of solving mono-objective optimization problems, but the number of algorithms capable of solving a MOP is much less.

2.2.1 Evolutionary Algorithms

Evolutionary algorithms were the first meta-heuristics created to deal with multi-objective optimization problems and they drastically broke (and at the same time can be used together) with most of the classic methods presented earlier. They deal simultaneously with a set of possible solutions (the so-called population). This allows finding several members of the Pareto optimal set in a single run of the algorithm and are less susceptible to the shape or continuity of the PF [31].

In short, they use paradigms from natural evolution, such as selection, recombination and mutation to lead a population (set) of individuals (decision vectors) towards optimal or near-optimal solutions [15]. The first meta-heuristic created in this sense was by Holland [73], it dealt with mono-objective problems and is called Genetic Algorithm (GA). The first to be developed to deal with MOP´S is now called multi-objective evolutionary algorithm (MOEA) or Vector Evaluation Genetic Algorithm (VEGA) [159]. VEGA was mainly aimed for solving problems in machine learning [31].

After this, many other derivations with attempts at improvement came. The main ones are: (i) VEGA, (ii) Lexigraphic Ordering GA (LOGA- a *priori* preference) [52], (iii) Vector Optimized Evolution Strategy (VOES) [91], (vi) Weight-Based GA (WBGA) [71], (v) Multiple Objective GA (MOGA) [50], (vi) Niched Pareto GA (NPGA, NPGA 2 [74, 75], (vii) Non-dominated Sorting GA (NSGA, NSGA II) [38, 163], (viii) Strength Pareto Evolutionary Algorithm (SPEA, SPEA II [193, 194], (ix) Multi-objective Evolutionary algorithm Based on Decomposition [189], (x) Pareto Archived Evolution Strategy (PAES) [89], among others.

With some exceptions, the distinction between all evolutionary multi-objective algorithms is mainly due to the differences in the paradigms used to define the selection operators, whereas the choice of the variation operators is generic and dependent on the problem. As example, one of the most popular is the NSGA II which can be applied to continuous search spaces as well as to combinatorial search spaces, whereas the selection operators stay the same, the variations operators (mutation and combination) must be adapted to the representations of solutions in the decision space [45]. All population-based multi-objective algorithms are almost identical. They start the optimization process with multiple candidate solutions and such solutions are compared using the Pareto dominance operator. The most well regarded ones are: SPEA, NSGA-II, MOEA/D and PAES [114, 115]

Although most of these algorithms are divided between a *priori*, *interactive* or a *posteriori* methods, most of them are like the latter. All are stochastic. According to the no-free-lunch theorem, none of them can be excellent at solving any type of problem. Zitzler et al. [193] applied several evolutionary algorithms to identify the PF of the ZDT3 function shown in Fig. 4c. Figure 5 has the result for each of these algorithms and also that of a random search. As evidenced, any evolutionary algorithm can be better than a rand, but there are significant differences between them and these differences can be very different from problem to problem.

2.2.2 Others Meta-heuristics

Although there are good evolutionary algorithms for certain types of problems, that are population-based and the inspiration is natural evolution, it has been shown that there is still room for improvement for specific problems. Based on this, other important multi-objective optimization algorithms found in the literature are: (i) Simulated Annealing for Multi-objective Optimization (SAMO) [157],(ii) Multi-objective Tabu Search (MOTS) [56]; (iii) Multi-objective Ant-Q (MOAQ) [109], (iv) Vector Evaluated Particle Swarm (VEPSO) [56]; (v) MOPSO [120]; (vi) Adaptive Weighted Particle Swarm Optimization (AWPSO) [106]. 2004); (vii) Artificial Immune Systems (AIS) [30]; (viii) Multi-objective Water Cycle Algorithm (MOWCA) [150], (ix) Multi-Objective Grey Wolf Optimizer (MOGWO) [114, 115],(x) Multi-objective Imperialist Competitive Algorithm (MOICA) [20], (xi) Self-adaptive Multi-objective Brain Storm Optimization (SMOBSO) [69], (xii) Multi-objective Ant Lion Optimization (MOALO) [115], (xiii) MOGOA [116], (xiv) Multi-objective Sine–Cosine algorithm (MO-SCA) [166], (xv) Multi-objective Stochastic Fractal Search (MOSFS) (Khalilpourazary et al. 2019), (xvi) Multi-objective Seagull Optimization Algorithm (MOSOA) [39], (xvii) Evolutionary MOSOA (EMOSOA) [39], (xviii) Differential Evolution-Crossover Quantum Particle Swarm Optimization (DE-CQPSO) [171] (xix) Multi-objective Sunflower Optimization [53], etc. As well as the evolutionary algorithms, all of them have their internal parameters that regulate their search according to their inspiration and then, these solutions are compared using a Pareto dominance operator. All are stochastic algorithms and most of them has a *posteriori* preference.

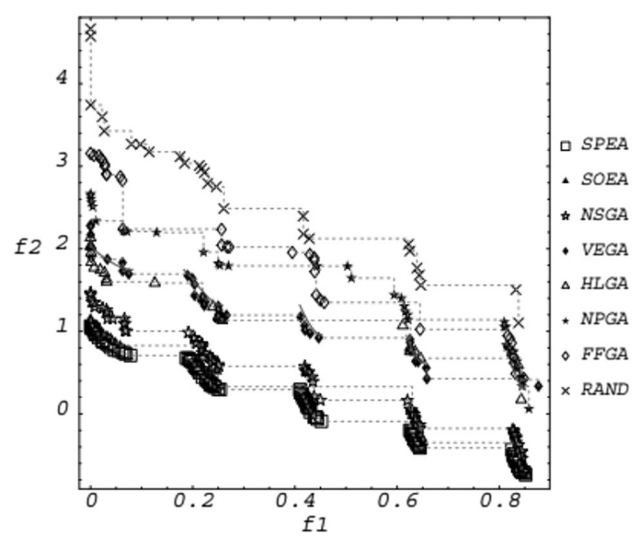


Fig. 5 Performance of the main evolutionary algorithms in the ZDT3 function [adapted from Zitzler et al. [193]]

It is possible to observe a huge variety of stochastic algorithms found in the literature. For Yang [179], there are no good or bad algorithms, but one more appropriate for a given optimization problem. It is a difficult task for a single algorithm to face any type of MOP with good balance of exploration, exploitation, convergence, coverage and low computational cost at the same time. This fact suggests that there is always an opening for the development of new meta-heuristics.

3 Literature Review

Multi-objective optimization has been improving and developed for some decades as it faces several problems that may be encountered. The number of applications has been growing in Mechanical Engineering and the following are those that are considered most relevant or recent, divided into the subareas where multi-objective optimization is present: design optimization, manufacturing and structural health monitoring.

3.1 Multi-objective Problems in Design Optimization

Undoubtedly one of the greatest applications in mechanical engineering is in determining the geometry of structures to obtain the best possible performances. Generally related to weight, stiffness, aerodynamics and/or modal parameters. The more objectives that must be met at the same time, the more complex MOP becomes. The most relevant applications are cited in the text and the objective functions, decision variables and algorithm employed are in Table 1.

Grachi et al. [62] found a design for a meta-material that found the viscoelastic with the highest vibration attenuation in a low-frequency range. Meta-materials are composites that are artificially engineered to have unconventional mechanical properties that stem from their microstructural geometry rather than from their chemical composition.

Schlieter and Dlugosz [155] optimize the design of an airfoil by proposing a new optimization methodology and obtaining many optimal Pareto solutions for a large number of decision variables. Li et al. [97] proposed a new method for optimizing the design of wind turbine blades that surpassed traditional methods, since in addition to optimizing structural strength and stiffness of the blade also considers the noise and power generation efficiency. Wang et al. [173, 175] also studied multi-objective wind turbine blade design proposing a novel gradient-based multi-objective evolution algorithm based on both uniform decomposition and differential evolution (MODE/D), which according to the author, performed better than NSGA-II mainly when the number of objectives is increased.

Fan et al. [47] designed a new mixed flow flan improving a currently standard model. The author increased the efficiency by 11.71% and the pressure raised by 50.15% using Orthogonal Method (OM). The purpose of this method is to test the influence of a particular factor over the whole outcome, with the view of obtaining optimal configuration in terms of the performance levels [187]. Zhang et al. [186, 188, 191] also applied multi-objective optimization using OM in design of Axial Flow Pump where the new design's head and efficiency increased by 17.8% and 4.26%, whilst the shaft power and the pressure pulsation coefficient reduced by 1.22% and 11%, respectively. Wang and Huo [177] used the same approach to improve the hydraulic performance of a centrifugal pump. Karimi et al. [84] proposed a design optimization approach for floating offshore wind turbine support structures, where the author found the locus of platform cost minima and wind performance maxima for a given environmental condition and sea state spectrum modifying the geometry of the structure (multi-objective constrained problem).

Moleiro et al. [117] studied metal-ceramic functionally graded (FG) plates, which are composed of a main functionally graded material layer and may include metal and/or ceramic faces under thermo-mechanical loadings. Other authors who also studied this type of structure with a pit in multi-objective optimization were: Ashjari and Khoshravan [14] and Franco Correia [54, 55]. Both considered constraints on the problem.

Multi-objective optimization is widely found in the design of collision structures. Asanjarani et al. [13] presented a crashworthiness optimization of the tapered thin-walled square tube with indentations using one and multi-objective approach, where the author found an optimized collision geometry. Zhang et al. [186] developed a hybrid multi-objective optimization approach for absorbing structures in train collisions that brought good results using multi-objective artificial bee colony. Peng et al. [132] also studied structures for collision using NSGA-II with an approach using finite element methods and design of experiments (DOE).

Ebrahimi-Nejad et al. [44] tried to found the best design of a sports car suspension system using simplified quarter-car models and TOPSIS. Other authors who studied suspension designs using a multi-objective approach were: (i) Zhang et al. [188] performed the multi-objective suspension system optimization for an in-wheel-motor driven electric vehicle, (ii) Zhang and Wang [189] conducted a parametric study to optimize a half-vehicle suspension system model, (iii) Fossati et al. [51] used NSGA-II and numerical computational studies comprising the multi-objective optimization of a full-vehicle suspension, (iv) Jiang and Wang [79, 80] used TOPSIS to optimize the suspension system of a truck and also to optimize handling stability and ride comfort.

Table 1 Multi-objective optimization system in design problems

Authors	Objective functions	Decision variables	Optimization method	Structure
Panagant et al. [130]	Structural mass and Compliance	Several variables related to the size and number of trusses	Fourteen meta-heuristics	Truss
Panagant et al. [129]	Strucutural mass, welding cost and compliance	Topology, shape and size	Fourteen meta-heuristics	Floor-flame
Grachi et al.[62]	Metamaterial inertia and Bragg scattering effect	Layer thickness and number of layers	MO based on GA	Metamaterial
Schlieter and Dlugosz [155]	Equivalent stress, displacement, frequency, mass	24 design variables (geometry)	MO based on DE	Airfoil
Moleiro et al. [117]	Mass, displacement and Tsai-Hill failure criteria	Thickness, power-low distribution, the thickness of metal	Direct MultiSearch (DMS)	FG blades
Ashjari, Khoshravan [14]	Mass and deflection	FG core volume fractions and thickness of the face sheets	NSGA II	FG blades
Franco Correia et al. [54, 55]	Mass, cost, natural frequency	Index of the power-law of volume fractions, thickness of FGM layer and face sheets	DMS	FG blades
Li et al. [97]	Structural strength, stiffness, noise reduction and aerodynamic performance	Chord length and twist angle (for each cross section)	MOPSO and finite volume method	Wind turbine blade
Wang et al. [173, 175]	Energy production, blade mass, root thrust, cost	30 related to design	MODE/D	Wind tubine blade
Fan et al. [47]	Efficiency and Pressure	Hub angle of impeller (and wrap) and diffuser	Orthogonal method	Mixed flow FAN
Zhang et al. [191]	Head, efficiency, shaft power and pressure pulsation	Number of blades, blade setting angle, hub ratio, distance between the blade and the guide vane	Orthogonal method	Axial flow pump
Wang and Huo [177]	Indexes head, efficiency, shaft power and pump net positive suction head	Impeller outlet width, blade inlet angle, blade outlet angle and cape angle	Orthogonal method	Centrifugal pump
Karimi et al. [84]	Cost model and wind turbine performance metric	Nine geometric variables of multi-body platform	NSGA-II	Offshore support
Asanjarani et al. [13]	Specific energy absorption and ratio between average and maximum crushing forces	Cross section, thickness, taper angle, number and radius of indentations	RSM, NSGA-II and desirability function	Tapered thin-walled square tube
Zhang et al. [186]	Capability of absorbing impact and energy	Side length and wall thickness of hexagonal tube	MOABC	Train collision piece
Ebrahimi-Nejad et al. [44]	unsprung and sprung mass accelerations, displacement and suspension travel	Stiffness and damping	TOPSIS	Sports car suspension
Zhang et al. [187]	Sensitive of the front double pivot and the rear double wishbone suspensions	Eighteen parameters related to stiffness and damping coefficients	NSGA-II	Suspension system
Fossati et al. [51]	Three objective functions related to comfort and safety	Six parameters being stiffness and damping coefficients of each suspension	NSGA-II	Full-vehicle suspension
Rao et al. [142, 144]	Total surface area, total annual cost, total pressure drop and effectiveness	Seven design variables related to geometry	Jaya algorithm	Plate-fin heat exchangers

Table 1 (continued)

Authors	Objective functions	Decision variables	Optimization method	Structure
Bahadormanesh et al. [16]	Thermal efficiency and size parameter	Different organic working fluids, mass flow rate, evaporator temperature, maximum pressure	Multi-objective firefly algorithm	Radial expanders
Vo-duy et al. [170]	Weight, natural frequency	Volume fractions, thickness and fiber orientation angles	NSGA-II	Beam structure
Ghasemi [59]	Mass/cost, buckling	Cylinder thickness, radius and lenght	NSGA-II	Composite cylindrical
Arian Nik et al. [12]	Rigidity, buckling	Fiber orientation	NSGA-II	Composite plate
Lee et al. [92]	Weight, deformation	Stacking sequence, thickness, material	MOGA	Composite plate
Omkar et al. [126]	Weight, cost	Fiber orientation	VEPSO	Composite plate
Kalantari et al. [83]	Weight, cost	Thickness, fiber and resin	NSGA-II	Composite plate
Ikeya et al. [77]	Mass, compliance	Volume fraction, thickness	GA	Composite plate

Panagant et al. [130] conducted a research using 14 types of meta-heuristics in 8 types of classical trusses subject to bound and stress constraints and compared the results of each one, concluding that the algorithm proposed by them was the one that had the best performance, the Success History-based Adaptive Multi-objective Differential Evolution (SHAMODE-WO). Panagant et al. [129] also used 14 meta-heuristics, this time in an automotive floor-frame. The author concluded that the meta-heuristic proposed in his work was one of the best algorithms, the Real-code Population-Based Incremental Learning hybridized with Adaptive Differential evolution (RPBILADE).

In the thermal area some designs can be found. Rao et al. [142, 144] applied single and multi-objective optimization in design of plate-fin heat Exchangers, which the design involves a number of geometric and physical parameters with high complexity. The general approaches are based and trial and few studies before him used multi-objective optimization. One of these was Ahmadi et al. [5] who used NSGA-II. However, Rao compared this study to his and concluded that Jaya had a better result. Bahadormanesh et al. [16] applied multi-objective optimization in improvement design of radial expanders of Organic Rankine Cycles (ORC) using firefly algorithm, where it was possible to preselect better parameters for the construction of these rotors. Wang et al. [174] also applied multi-objective optimization in ORC, but aiming to improve aspects thermodynamics and economics using NSGA-II.

Works in the field of composite materials can also be found. Vo-duy et al. [170] optimized a beam structure made by laminated composite using finite element method. Ghasemi and Hajmohammad [59] applied multi-objective optimization in design of laminated composite cylindrical shell under external hydrostatic pressure for minimum cost and maximum buckling pressure. Other actors who also worked with the optimization of carbon fiber structures using a multi-objective approach were: Arian Nik et al. [12], Lee et al. [92], Omkar et al. [126], Kalantari et al. [83], Ikeya et al. [77] and Diniz et al. [41].

3.2 Multi-objective Problems in Manufacturing

Processing engineering is an extensive area in Mechanical Engineering, the largest areas of which are Welding, Machining and Molding.

3.2.1 Welding

Welding is one of the most important areas of engineering and there are currently many types of processes. Some of these processes can have a large number of decision variables controlling the process, with high complexity of correlation or analytical solutions. Still, you can have numerous objectives to be optimized simultaneously. Some of them are to increase productivity, decrease costs, reduce the thermally affected zone, increase the reinforcement, decrease (or increase) the hardness, decrease the emission of toxic gases, decrease the consumption of electric energy, increase the impact strength, increase the tensile strength, increase elongation in the weld area, reduce noise pollution, among others. As seen, the use of Meta-heuristics has collaborated a lot to deal with this extensive area. The most relevant applications are cited in the text and the objective functions, decision variables and algorithm are in Table 2.

Ahmad et al. [4] studied Submerge Arc Welding (SAW) looking for optimal parameters to achieve productivity and weld quality. SAW is a versatile welding process widely used in fabrication and manufacturing of marine and pressure vessels, pipelines and offshore structures. Other authors who also studied the SAW process aiming at multi-objective optimization, generally as weld quality, strength, hardness and/or productivity, were: Choudhary et al. [29], Ahire et al. [3], Sailender et al. [153], Silva et al. [154], Rao et al. [142, 144], Al Dawood et al. [6], Yifei et al. [182] (welding robot parameters), Torres et al. [168], among others.

Sowrirajan et al. [162] applied multi-objective optimization to find the optimum clad layer dimensions in pressure vessels using stainless steel that maximize clad height and clad width and minimize depth of penetration in a process FCAW (Flux Cored Arc Welding). Paula et al. [131] and Almeida et al. [11] also studied the optimization of the parameters of this process.

Shao et al. [161] studied the optimization of gas metal arc welding (GMAW) parameters and sequences for low-carbon steel (Q345D) T-joint using FEM and DOE concluding that the welding residual deformation and stress always have opposite behavior and are very influence by the process parameters. Lorza et al. (2018) also studied the optimization of welded joints in GMAW using FEM, but another algorithm, the NSGA II. Like the previous author, this work approached a methodology to reduce the error between the FEM and a real case. Both seek the selection of optimal process parameters.

Another welding process that many authors approached to optimize their parameters was the friction stir welding (FSW). Gupta et al. ([60] and [61]) studied this process for joining different alloys in two works with different approaches, but the same materials. In both studies, the optimal parameters found were the same. Shanjeevi et al. [160] studied for AISI 304 l austenitic stainless steel and copper points. Wakchaure et al. [172] for Alloy 6082 using Taguchi-GRA and artificial neural network (ANN) and Senthil et al. [156] for AA6063-T6 pipes using Analysis of Variance (ANOVA) and RSM.

Saha and Mondal [151] studied the optimization of manual metal arc welding (MMAW) process parameters for nanostructured hardfacing material using hybrid approach with Taguchi, TOPSIS and PCA (Principal Component Analysis) identifying the optimal process parameters. The study of the optimization of welding parameters by a multi-objective approach was found less frequently in other processes because they are not so common or are recent: (i) Laser-magnetic hybrid welding (LMW) by Yang et al. [180], (ii) hot wire laser welding (HLW) by Yang et al. [181] (iii) Hybrid laser-arc welding (HLAW) by Gao et al. [57], (vi) laser welding process (LW) by Jiang et al. [81] using FEM, Kriging (a meta-model) and NSGA-II, (v) Micro resistance spot welding (MRSW) by Chen et al. [27] and (vi) Laser beam machining by Belinato et al. [18].

Another type of multi-objective problem related to welding is the local scheduling. Lu et al. [104] studied an approach to welding shop scheduling that according to the authors, should simultaneously consider economic, environmental and social impacts. In this way, the authors proposed a multi-objective approach using a novel hybrid multi-objective grey wolf optimizer (HMOGWO) for makespan (total sum time of each process), energy consumption and noise pollution (ignores in previous studies). The same author in Lu et al. [105] applied the same algorithm for dynamic scheduling in a real-world welding industry. In both cases the authors concluded that the algorithm used outperforms known EA's.

3.2.2 Machining

Another major area of process engineering is machining. As with welding, machining has a wide range of processes like milling, turning, drilling or cutting and each can have a great number of decision variables that control the processes. Some of the conflicting objectives to be optimized simultaneously are minimizing roughness, minimizing cost, minimizing cutting force, increasing productivity, increasing material removal, decreasing process variability, reducing residual stress, among others. The most relevant applications are cited in the text and the objective functions, decision variables and algorithm employed are in Table 3.

Rao et al. [141] applied multi-objective optimization using Non-dominated sorting Teaching–Learning Based algorithm (NSTLBO) in three machining process (turning, wire-eletric-discharge machining and laser cutting) and two micro-machining process (ion beam micro-milling and micro wire-electric-discharge machining) looking for the best process parameters. The author compared this algorithm with NSGA-II and others algorithms.

Lin et al. [98] studied machining parameters in multi-pass turning operations for low carbon manufacturing considering reducing machine cost, energy consumption

Table 2 Multi-objective optimization system in welding process

Authors	Objective functions	Decision variables	Optimization method	Process
Choudhary et al. [29]	Bead width, reinforcement and penetration	Voltage, feed, speed, nozzle to plate distance, flux condition and plate distance	GA, JAYA Algorithm and desirability function	SAW
Ahmad et al.[4]	UTS, Hardness, deposition rate, reinforcement height and bead width	Current, voltage, speed and heat input	Taguchi-desirability function	SAW
Ahire et al. [3]	Welding strength, welding deposition rate	Current, speed, root gap and electrode angle	Response Surface and GA	SAW
Sailender et al. [153]	UTS and Hardness	Voltage, feed, speed and nozzle to plate distance	Taguchi	SAW
Silva et al. [154]	Dilution, reinforcement and bead width radio	Voltage, feed and nozzle to plate distance	ANOVA	SAW
Rao et al. [142, 144]	Bead width, weld reinforcement, weld penetration, tensile strength and weld hardness	Current, speed and feed	JAYA, GA, PSO and Imperialist Competitive Algorithm	SAW
Al Dawood et al. [6]	UTS and hardness	Current, voltage, speed	Taguchi-fuzzy interference system	SAW
Yifei et al. [182]	Productivity and cost	Welding path	GA, Particle Swarm optimization	SAW
Torres et al. [168]	Joint dimensions and dilution	Voltage, speed, wire feed rate, contact distance	Generalized reduced gradient (GRG)	SAW
Rivas et al. [149]	Carbon dioxide emissions, slag, wastes, electric power, material, labor and energy cost,	Current, voltage, welding speed	NSGA II, MOEA/D, MOPSO, SPEA II and PESA II	SAW
Sowrirajan et al. [162]	Clad height, clad width and depth of penetration	Open circuit voltage, wire feed rate, welding speed, distance and electrode angle	RSM and NSGA-II	FCAW
Shao et al. [161]	Welding stress and deformation	Current, voltage, speed, sequence and direction	DOE and MOPSO	GMAW
Lorza et al. [103]	Temperature field and angular distortion	Current and voltage	NSGA II	GMAW
Gupta et al. [60]	Tensile strength, average hardness and average grain size at weld nugget zone	Rotational speed, welding speed, shoulder and pin diameter	Grey relational analysis coupled with PCA and Taguchi	FSW
Gupta et al. [61]	Tensile strength, micro-hardness and grain size	Idem Gupta et al. [60]	Artificial Intelligence and NSGA II	FSW
Shanjeevi et al. [160]	Tensile strength, metal loss and weld time	Friction and upset pressure, rotational speed	TOPSIS and Taguchi	FSW
Wakchaure et al. [172]	Tensile strength and impact strength	Tool rotation speed, welding speed and tilt angle	ANN, GRA and Taguchi	FSW
Senthil et al. [156]	Yield, tensile strength and elongation	Rotational and weld speed	ANOVA and RSM	FSW
Saha and Mondal [151]	Weld bead width, reinforcement and bead hardness	Current, voltage and welding speed	TOPSIS-PCA	MMAW
Yang et al. [180]	Macro-weld profile, microstructure and hardness	Magnetic flux density, laser power, welding speed	NSGA II and Taguchi	LMW
Yang et al. [181]	Welding depth and reinforcement and strength	Laser power, welding speed, hot-wire current	Meta-models and NSGA II	HLW
Chen et al. [27]	Tensile-shear, weld nugget size, failure energy	Ramp time, welding time, current and force	NSGA-II	MRSW
Gao et al. [57]	Depth of penetration, bead width and bead reinforcement	Laser power, current, distance between laser and arc and travelling speed	NSGA II and Taguchi	HLAW
Jiang et al. [81]	Bead width and depth of penetration	Laser power and position and welding speed	Kriging and NSGA-II	LW
Lu et al. [104]	Makespan, total penalty of machine load and instability	Permutation of tasks, actual quantify of the welders and the starting time	HMOGWO	–

Table 2 (continued)

Authors	Objective functions	Decision variables	Optimization method	Process
Lu et al. [105]	Makespan, noise pollution, energy consumption	Actual, start and finish processing time	HMOGWO	–

Table 3 Multi-objective optimization system in machining process

Authors	Objective functions	Decision variables	Optimization method	Process
Rao et al. [141]	Tool flank wear and surface roughness	Cutting speed, feed and depth of cut	NSTLBO and NSGA II	5 different machining process -txt
Lin et al. [98]	Carbon emissions, operation time and cost	Velocity, feed rate, dept of cut and spindle speed of machine tools	MOTLBO	Turning
Sahu and Andhare [152]	Roughness and force of cutting	Speed of cutting, feed rate and depth of cut	TLBO, JAYA, GA and RSM	Turning
Sivaiah and Chakradhar [158]	roughness, flank wear and remove rate	Speed of cutting, feed rate and depth of cut	Taguchi and TOPSIS	Cryogenic turning
Mia et al. [112]	Cutting force, specific energy, temperature, surface roughness, material removal	Cutting speed, feed rate and number of jets	Gray-Taguchi	Cryogenic turning
Gaudêncio et al. [58]	Roughness and MMSE	Cutting speed, cutting feed, machining depth	RSM, GRG and NBI	Turning
Almeida et al. [10]	Mean and deviation of roughness	Cutting speed, feed and depth of cut	RSM	
Park et al. [135]	Cutting energy and energy efficiency	Cutting speed, feed rate, nose radius, edge radius, rake and relief angles	RSM, NSGA-II and TOPSIS	Turning
Warsi et al. [178]	Cutting energy, material removal and roughness	Cutting speed, feed and depth of cut	Gray-RSM	Turning
Qu et al. [139]	Cutting force, roughness, milling efficiency	Spindle speed, feed per tooth, axial depth of cut	NSGA-II	Milling
Montalvo-Urquizo et al. [119]	Deformation, stress, shape error and tool wear	Cutting velocity and axial cutting depth	Simulation-based multi-objective optimization	Milling
Niu et al. [122]	Residual stress, material removal rate, roughness and surface hardness	Milling speed, feed per tooth, width of cut, depth of cut and amplitude ultrasonic	NSGA II	Milling
Prakash et al. [138]	Surface roughness and micro-hardness	Peak current, pulse duration, duty cycle and silicon powder concentration	Taguchi, RSM and NSGA-II	PMEDM
Tripathy and Tripathy [169]	Material removal rate, tool wear rate, electrode wear ratio and surface roughness	Powder concentration, peak current, pulse on time, duty cycle and gap voltage	ANOVA, GRA and TOPSIS	PMEDM
Abidi et al. [1]	Material removal rate, roughness and tool wear rate	Capacitance, electrode material and discharge	MOGA II	EDM
Prakash et al. [137]	Material removal rate and surface roughness	Peak current, pulse-on, duty cycle and powder concentration	MOPSO	Mixed-EDM
Dumbhare et al. [43]	Surface roughness and kerf taper angle	Abrasive flow rate, standoff distance and transverse speed	ANOVA and RSM	AWJM
Rao et al. [142, 144]	Surface roughness and kerf taper angle	Transverse speed, pressure, stand-off distance, tilt angle, surface speed and abrasive flow rate	MO-Jaya and PROMETHEE	AWJM

and environmental impacts using a multi-objective teaching–learning-based optimization algorithm (MOTLBO) in dry and wet cut. Sahu & Andhare [152] applied multi-objective optimization to improve the machinability of Titanium alloy in cryogenic turning process using Teaching–Learning Based Optimization (TLBO), JAYA, GA and RSM. Mia et al. [112] also studied cryogenic turning of a Titanium alloy. Sivaiah and Chakradhar [158] improved the cryogenic turning process during machining of hardened stainless steel.

Gaudêncio et al. [58] proposed a model for machining quality in the AISI 12L14 steel turning process using fuzzy multivariate mean square error, that is, the objectives are decrease roughness to minimize the roughness and its own variability (Multivariate mean square error—MMSE). Almeida et al. [10] used this same steel and mean and standard deviation roughness objectives to optimize parameters in turning. Park et al. [135] studied turning process of hardened material aiming to resolve environmental issues reducing the consumed energy and improving energy efficiency. The energy decreased 16% and the efficiency could be improved 11% compared to the non-optimized system. Warsi et al. [178] studied a sustainable turning of Al 6061 T6 where the proposed parameters resulted in reduction of specific cutting energy by 5% and improvement of 33% in material removal rate while surface roughness remained unaffected.

Another machining process is the milling. Qu et al. [139] applied multi-objective optimization to select optimum parameters in milling thin-walled plates. Montalvo-Urquizo et al. [119] also studied milling creating a simulation model with FEM that the accuracy of the method compares very well with experimental data.

Other welding processes can be found in the literature that have gained more space and multi-objective approaches, but even less common, such as: (i) powder mixed electric discharge (PMEDM)—studied by Prakash et al. [138](first in this process to use NSGA-II) and Tripathy and Tripathy [169], (ii) electrical discharge machine—studied by Abidi et al. [1] and Prakash et al. [137] and (iii) Abrasive Water jet Machine (AWJM)—studied by Dumbhare et al. [43] and Rao et al. [142, 144].

3.2.3 Molding

Molding is another area in which multi-objective optimization is not yet fully explored. It also has a significant number of decision variables that control the process and many objectives to be optimizing simultaneously. Among the analyzed processes, this process had the least number of studies. Most were in the injection process and aimed at reducing the warpage, minimizing the cycle time or reducing costs, materials or energy used. The most relevant applications are cited in the text and the objective functions, decision variables and algorithm employed are in Table 4.

Li et al. [96] proposed an approach to optimize the fiber-reinforced composite injection molding process. Kitayama et al. [87] proposed another methodology to multi-objective optimization of injection parameters for short cycle time and warpage reduction using conformal cooling channel. The same author in Kitayama et al. [88] used the same methodology to found the optimal parameters in plastic injection molding for minimizing warpage and cycle time. Zhang et al. [192] studied the optimization of the injection molding process parameters for diesel engine oil cooler. Okabe et al.

Table 4 Multi-objective optimization system in molding process

Authors	Objective functions	Decision variables	Optimization method	Process
Li et al. [96]	Warpage, volumetric shrinkage and residual stress	Fiber content, fiber aspect ratio, melt temperature, injection pressure and cooling time	Taguchi, RSM and NSGA-II	Injection (fiber-reinforce composite)
Kitayama et al. [87]	Cycle time and warpage	Melt temperature, injection time, packing pressure, packing time, cooling time, cooling temperature	Sequential approximate optimization and radial basis function	Injection (plastic)
Kitayama et al. [88]	Packing pressure profile and warpage	Packing pressure profile, melt temperature, injection time, cooling temperature of coolant and cooling time	Sequential approximate optimization and radial basis function	Injection (plastic)
Zhang et al. [192]	Warpage and clamping force	Gate open time, molding temperature, melt temperature, injection time, packing pressure and time, cooling time	Neural network and MOPSO	Injection (plastic)
Li et al. [94]	Stent expansion in a stenotic artery and molding quality	Length, thickness and outer diameter	Kriging Surrogate	Microinjection
Okabe et al. [124]	Fill time, dry spot, weld line and wasted resin	Injection points	MOGA	Resin-transfer

[124] proposed a new multi-objective optimization approach for resin transfer molding process using FEM, MOGA, self-organizing map and scatter plot matrix.

Li et al. [94] applied multi-objective optimization in a biodegradable polymer stent structure and stent micro-injection molding process using FEM, DOE and Kriging surrogate method. The author improved the expansion performance and the microinjection molding process of the biresorbable polymeric stent with tiny struts.

3.3 Multi-objective Problems in Structural Health Monitoring

Structural Health Monitoring (SHM) is a structural inspection methodology that allows an early diagnosis of structural health through non-destructive techniques, algorithms and the use of sensors in real time. The application of multi-objective optimization can be in the numerical analysis of the collected data and also in the optimization of the reading of the structure through, for example, positioning of the sensors [66]. The most relevant applications are cited in the text and the objective functions, decision variables and algorithm employed are in Table 5.

The study of the positioning of the sensors aims cost reduction by minimizing the number of sensors by improving the efficiency of data reading. In the literature some cases can be found considering multi-objective optimization. In highlight are: (i) Ferentinos and Tsiligiridis [48] (wireless sensors subject to the connectivity and spatial density constraints), (ii) Gomes et al. [66] (composite plates), (iii) Lin et al. [99] (truss structure—3D), (iv) Zhou et al. [185] (wireless sensors) and (v) Li et al. [95, 96] also studied composite plates and used wavelet decomposition.

With positioned sensors, the numerical approach to the treatment of these data considering inverse methods can have a multi-objective approach. It is important to note that these data can be acquired directly in the structure and/or through finite element (FE) model updating procedure. As can be seen in Alexandrino et al. [66], where the authors proposed a robust method of identifying ellipses and circular holes in composite plates. The ellipse having one of its rays much larger than the other could be considered in the authors' work as a crack.

Figure 6 shows a summary of a standard methodology applied in SHM when using a multi-objective approach. First, the structure to be analyzed is defined. Then, a study of the strategy to be adopted in relation to the sensors is made. Finally, an iterative process takes place using an optimization algorithm, usually a meta-heuristic, until some convergence criterion is reached (number of iterations or stipulated difference).

Perera and Ruiz [133] successfully applied a methodology to large structures such as bridges through a two-step methodology. The first stage is to detect the occurrence of damage and the location of damaged zones by FE model updating using damage functions [164]. The use of damage functions avoids adjusting the possible damage values of all the elements separately. This results in a reduced number of parameters to determine which contributes to avoiding optimization numerical problems and makes its application to large-scale structures easier. In the second stage, refined location of damage and estimation of severity, a standard FE model updating technology with independent adjustment of the design variables is applied but, in this case, the number of variables is much reduced because only the elements belonging to the zones identified previously as damaged are now assumed to be damaged.

Kim and Park [85] made a very interesting study of identifying a crack on a rectangular plate where they addressed the multi-objective problem in two ways: comparing the two objectives into one using the method of weighting sum method and without composing them, leaving them free and on analysis of the dominance relationship. They reported that the proposed multi-objective EA was more efficient than the single objective EA.

Some authors have analyzed three-dimensional steel structures: (i) Cha and Buyukozturk [26] showed a methodology that could be used effectively for detecting minor local damage, although real-world validation using experimental data is still needed, (ii) Alkayem et al. [8] studied 3D steel structures through two algorithms applying them in their mono and multi-objective versions and concluded that in this case PSO and MOPSO had more accurate results and less computational cost, (iii) Wang et al. [176] compared FE model updating using NSGA-II, differential evolution for multi-objectives (DEMO) and multi-objective particle swarm optimization (MOPSO) and noted that DEMO outperformed NSGA-II and MOPSO for all damage patterns and (iv) Alkayem et al. [9] used a methodology that uses at the same time two meta-heuristics and the TOPSIS getting good results even with bad conditions or with incomplete date.

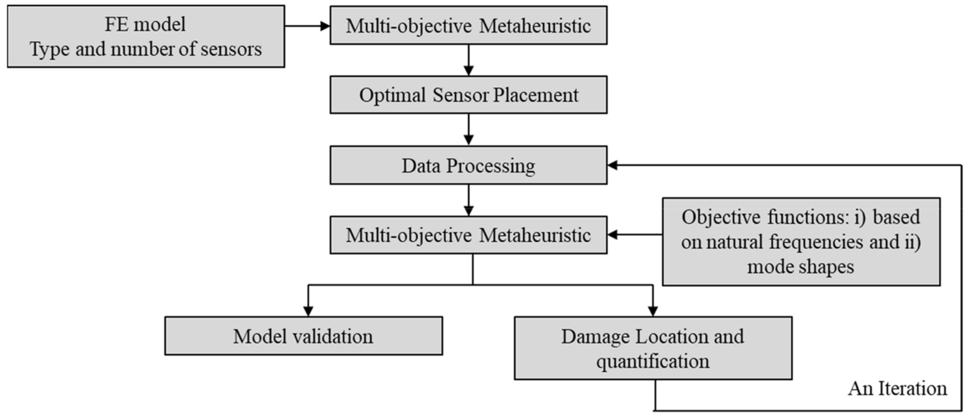
Slipping away from the application of multi-objective optimization in the positioning of sensors or in the identification of damage itself, a relevant article was found on the planning application of the SHM methodology: Kim and Frangopol [86] proposed a probabilistic optimum SHM planning based on the probabilistic fatigue damage assessment which aimed to determine the most important parameters and their weights in order to have the best possible planning of methodology.

According to Alkayem et al. [8], some of the studies in Table 5 have effectively demonstrated the advantage of multi-objective functions rather than conversion into single-objective functions using the weighted sum method. This is because when using a combination between a

Table 5 Multi-objective optimization system in structural health monitoring

Authors	Objective functions	Decision variables	Optimization method	Object
Ferentinos and Tsiligiridis [48]	Total energy related to parameters and sensing points uniformity	Uniformity of sensors points, number of sensors, operational energy consumption battery energy consumption	GA (mono-objective) with weighted sum method	Composite plates
Gomes et al. [66]	Fisher information matrix and difference module of mode shapes	Location and number of sensors	NSGA	Composite plates
Lin et al. [99]	Response covariance sensitivity and correlation analysis	Location and number of sensors	NSGA-II	Truss structure
Zhou et al. [185]	Information effectiveness (mode shapes) and network performance (standard deviation of total energy)	Location and number of sensors	MO discrete firefly algorithm based on neighboring information (MDFA/NS) and NSGA II	Composite plates
Li et al. [95, 96]	Number of sensors and vibration response	Location and number of sensors	NSGA-II and Wavelet decomposition	Composite plates
Alexandrino et al. [66]	Differences in the mean stress and variance	x, y, r (circular hole) x, y, a, b, θ (elliptical hole)	NSGA II, neural networking and fuzzy decision making	Composite plates
Kim and Park [85]	Difference module of the natural frequencies and Modal Assurance Criterion (MAC)—related to mode shapes	Element position and stiffness reduction factor in this element	GA with sum method and MOGA	Composite plates
Perera and Ruiz [133]	Modal frequencies and mode shapes	Element location and damage index	SPGA	Truss structure
Cha and Buyukozturk [26]	Modal Strain Energy and mode shapes	Structural element and Young´s modulus reduction	Implicit Redundant Representation GA and NSGA-II	Truss structure
Wang et al. [176]	Natural Frequencies and accumulative MAC	Truss element and reduction ration	MOPSO, NSGA-II and MODE	Truss structure
Alkayem et al. [8]	Difference module of the natural frequencies and MAC	Truss element and Young's modulus reduction	GA, MOGA, PSO and MOPSO	Truss structure
Alkayem et al.[9]	Modal strain energy and mode shape	Truss element and Young's modulus reduction	MOPSO and Lévy flights	Truss structure
Kim and Frangopol [86]	Expected damage detection delay, expected maintenance delay, damage detection time-based reliability index, expected total life extension and expected life cycle cost	Uncertainties of fatigue damage, maintenance delay, damage detection delay, effects if maintenance actions, service life and costs, maintenance and structural failure	Multi-objective probabilistic optimization process (MOPOP)	–

Fig. 6 Framework for SHM in a multi-objective approach



single-objective optimization algorithm (such EA) and the weighting sum method to solve multiple objectives, the outcome is a sub-set of total Pareto optimal solutions, while a powerful multi-objective meta-heuristic algorithm can generate the whole Pareto optimal solutions or at least the majority of them.

3.4 General Discussion

All recent or relevant publications found for multi-objective optimization in mechanical engineering in this work are in Tables 1, 2, 3, 4 and 5, where it contains the decision variables, the objective functions, the algorithms and the structure or process to be optimized employed. In total, 90 researches were detailed from the point of view of the multi-objective approach.

Of these works, 23 different algorithms were used and since in some works more than one is used, it totaled 102 applications. NSGA-II was the most used, appearing in 32 researches, just behind came MOPSO with 11 appearances, MOGA with 5, Jaya with 5, MODE with 3 and Ortoghonal Method with 3. The other algorithms found in the Table appeared less than twice and this includes the only gradient-based method in the list: the GRG. Note that of the 23 algorithms found, 18 appeared less than twice.

A widely used methodology was that of RSM, where the authors performed analysis using DOE or Taguchi and found polynomial regression curves that were optimized using desirability function or other algorithms. In all, 18 studies followed this methodology. Of those who used and detailed the decision-making technique adopted, TOPSIS was the most present, being in 6 of the works found.

4 Conclusion

Multi-objective optimization is an area that has been highly developed in the last decades and several methodologies, algorithms and decision techniques have been created. However, the amount of information in the literature can make the choice of a methodology to address a problem confusing and this work aimed to show through a systematic and detailed review of the literature what are the most used algorithms, techniques, decision variables and objective functions employed in ninety different research papers in mechanical engineering.

It can be seen that classic optimization methods, such as gradient-based methods, that had their importance in the past lost space for new algorithms that emerged with the advancement of computing, better able to deal with a greater number of variables, multiple objectives and nonlinearities. Meta-heuristics are the most suitable to deal with multi-objective optimization problems, being evolutionary algorithms the most used in the literature, mainly the NSGA-II. Next comes a swarm algorithm, MOPSO. Even the evolutionary and swarm algorithms are being challenged by new and more powerful meta-heuristics with more appropriate routines for specific problems, such as Jaya algorithm, Multi-objective Grey Wolf Optimizer, Success History-based Adaptive Multi-objective Differential Evolution, Real-code Population-Based Incremental Learning hybridized with Adaptive Differential evolution or Non-dominated sorting Teaching–Learning Based algorithm. Since most of these algorithms use the Pareto dominance relationship to find non-dominated solutions, the overwhelming majority of recent problems use a *posteriori* decision-making technique, where the decision maker tries to come up with all possible optimal solutions to the problem before choosing the best one. A widely used a posteriori decision-making technique is TOPSIS.

Even so, it was possible to verify that these powerful tools of multi-objective optimization are still little used in mechanical engineering and who made their use, obtained great improvement in the object in which they work.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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