Assignment 1
NLP 201: Natural Language Processing I

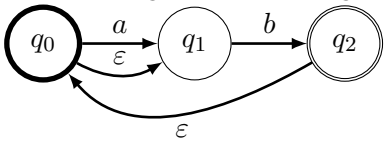
Out: October 13, 2020
Due: October 27, 2020 11:59pm

All assignments are to be completed individually. You may discuss with the TAs and the instructor, but you may not receive help from anyone else.

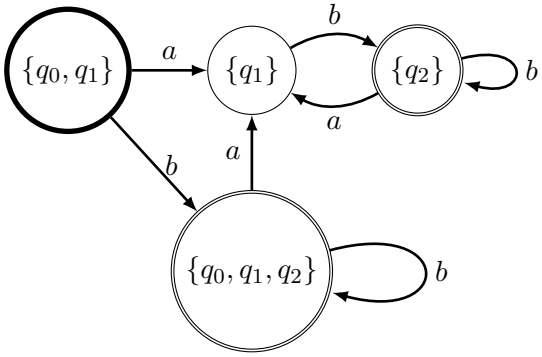
Instructions. Create a LaTeX-typeset write-up of your solutions and submit it to Canvas. Some free tools that might help: TexStudio (Windows), MacTex (Mac), TeX Live (cross-platform), and Overleaf (online). For the problems that require that you give FSAs, we encourage you to create them using drawing tools such as OmniGraffle (Mac) or yEd (online or cross-platform), but we will accept hand-drawn figures for full credit. Make sure initial and final states are clearly identified and explicitly list all symbols on each arc (i.e., no wildcards). We ask that you do not use any automated tools to *solve* these problems as you will not have access to such tools for similar problems on the exams.

Problem 1 [15 points]

For the following NFA containing ε-moves,



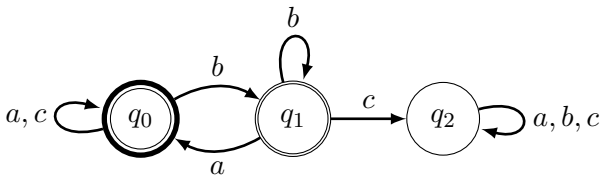
give an equivalent DFA.



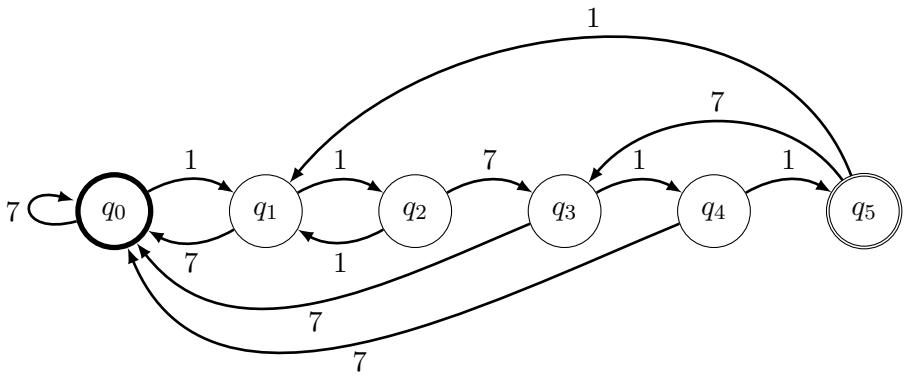
Problem 2 [15 points]

Give DFAs accepting the following languages.

1. **[5 points]** The set of strings over the alphabet $\{a, b, c\}$ in which the substring bc never occurs.



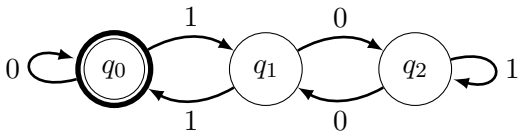
2. **[5 points]** The set of strings over the alphabet $\{1, 7\}$ that end in 11711.



3. **[5 points]** The set of strings over the alphabet $\{0, 1\}$ which are divisible by three when interpreted as a binary number (ignoring leading zeroes).

   For example $00000_b = 0$, which is divisible by 3, so 00000 should be accepted. $001001_b = 9$ and thus should be accepted also. $101_b = 5$ is not divisible by 3 and thus should be rejected.

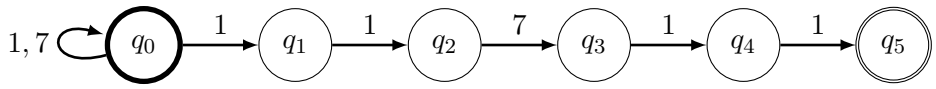
   Note: ε should be interpreted as 0 and thus should be accepted.



Problem 3 [10 points]

Give a *non-deterministic* FSA (possibly with ε-moves) accepting the following language.

1. The set of strings over $\{1, 7\}$ ending in 11711. How does your NFA differ from the DFA you constructed in Problem 2.2?

   Because the ending of the string is the only part of any consequence, we ignore the first n characters of the string until parsing 11711 at the end. Since the FSA is non-deterministic, we only transition from q_0 to q_1 at this final substring. This stands in contrast to the deterministic FSA above, which must revert to previous states if the target substring is interrupted.

Problem 4 [15 points]

Give regular expressions for each of the following languages:

1. **[7 points]** The set of strings over $\{a, b, c\}$ in which the substring bc never occurs.

   ```
   ((a|c)*|(a|c)*bb*a(a|c)*)
   ```

2. **[8 points]** The language described in Problem 2.3: The set of strings over $\{0, 1\}$ which are divisible by three when interpreted as a binary number (ignoring leading zeroes).

   For example $00000_b = 0$, which is divisible by 3, so 00000 should be accepted. $001001_b = 9$, and thus should be accepted also. $101_b = 5$, is not divisible by 3, and thus should be rejected.

   Note: ε should be interpreted as 0, and thus should be accepted.

   ```
   0*(1(01*0)*1)*0*
   ```

Problem 5 [20 points]

We may define **generalized regular expressions** (GREs) as follows:

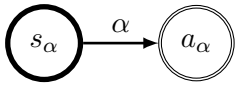
1. $\emptyset$ is a GRE denoting the empty language;
2. ε is a GRE denoting the language $\{\varepsilon\}$;
3. for each $\sigma \in \Sigma$, σ is a GRE denoting the language $\{\sigma\}$;
4. if α and β are GREs, denoting the languages A and B, respectively, then
   - $(\alpha \mid \beta)$ is a GRE denoting $A \cup B$;
   - $(\alpha\beta)$ is a GRE denoting $A.B$;
   - α^* is a GRE denoting A^*;
   - **[new]** $(\alpha \wedge \beta)$ is a GRE denoting $A \cap B$; and
   - **[new]** $\neg\alpha$ is a GRE denoting $\overline{A}$.

Prove that the languages denoted by GREs are regular.

Note: you may use refer to the definitions and proofs about REs we provided in class and only deal with the two new clauses in the generalized definition.

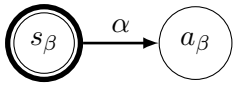
Proof. To prove $A \cap B$ is regular, we will first prove (without loss of generality) that $\overline{A}$ is regular.
Consider the following FSA, which accepts the language A:

M_α :



We then create a new FSA by swapping any accept states:

$M_{\neg\alpha}$:



This is a valid FSA, which accepts only the language of $\overline{A}$ (meaning $\overline{A}$ is regular).

Now we consider the language of $A \cap B$. Using De Morgan's Law we can show that $\overline{\overline{A} \cup \overline{B}} = A \cap B$. Because regular languages are known to be closed under union and complement, we can see that $A \cap B$ is also regular.

□

Problem 6 [25 points]

For each of the following statements, answer whether the claim is **true** or **false**, and give a *short* (one- to two-sentence) explanation if true or counterexample if false.

1. **[5 points]** Let $L_1 \subset L_2$. If L_1 is not regular, then L_2 must also be not regular.

   False. Consider the languages $L_1 = \{0^n 1^n \mid n \in \mathbb{N}\}$ and $L_2 = \{0^* 1^*\}$. Then $L_1 \subset L_2$, but L_1 is irregular, but L_2 is regular.

2. **[5 points]** $L = L_1 \cap L_2$. If L_1 and L are regular languages, then L_2 must also be a regular language.

   False. Consider $L_1 = \{\varepsilon\}$ (regular) and $L_2 = \{0^n 1^n\}$ (irregular). Then $L = L_1 \cap L_2 = \{\varepsilon\}$ (regular).

3. **[5 points]** $L = L_1 \cup L_2$. If L_1 and L are regular languages, then L_2 must also be a regular language.

   False. Let $L_1 = \{0^n 1^m\}$ (regular) and $L_2 = \{0^n 1^n\}$ (irregular) for $n, m \in \mathbb{N}$ and $n < m$ (that is, $L_1 \subset L_2$). Then $L = L_1 \cup L_2 = \{0^n 1^m\}$, which is still regular.

4. **[5 points]** $L = \bigcap_{i=1}^{\infty} L_i$. If all of the L_i are regular languages, then L is also a regular language.

   False. Let $L_i^p = \{0^n \mid n \leq i \text{ is prime}\}$ and $L_i^+ = \{0^n \mid n > i\}$. Define $L_i := L_i^p \cup L_i^+$. We see that L_i^p is regular because it has finitely many prime-length words; L_i^+ is also clearly regular. As a result, L_i is regular, being the union of two regular languages. However, we have $L = \bigcap_{i=1}^{\infty} L_i = \{0^p \mid p \text{ is prime}\}$, which is irregular by the pumping lemma.

5. **[5 points]** Let α, β and γ be regular expressions. If $L(\beta \mid \alpha\gamma) \subseteq L(\gamma)$, then $L(\alpha^*\beta) \subseteq L(\gamma)$.

   True/False.