

$$\begin{pmatrix} 1 & -10 & 5 & -5 \\ 2 & -14 & 7 & -7 \\ 1 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{-2R_1} \begin{pmatrix} 1 & -10 & 5 & -5 \\ 0 & 6 & -3 & -3 \\ 0 & -10 & 5 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & -3 & -3 \\ 0 & -10 & 5 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Nullspace: } \begin{pmatrix} 0 & x_2 & x_3 & x_4 \end{pmatrix} \begin{cases} x_1 = 0 \\ -2x_2 + x_3 - x_4 = 0 \end{cases} \quad \begin{cases} x_1 = 0 \\ x_2 = -\frac{x_3}{2} + \frac{x_4}{2} \end{cases}$$

$$n - r = 4 - 2 = 2$$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= -\frac{1}{2}x_3 + \frac{1}{2}x_4 \end{aligned}$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Пусто } x = 0, x_4 = 2$$

(2)

$$x_2 = 1$$

$$\sqrt{1} \quad x_2 = 2 \quad x_1 = 0$$

$$x_2 = -1$$

$$x_2 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ 0 \end{pmatrix}$$

О.С.Р.

$$X = C_1 X_1 + C_2 X_2 = C_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 2 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ -1 \\ 2 \\ 0 \end{pmatrix}$$

$$\vec{A}(\vec{x}) = [\vec{x}, \vec{a}]$$

$$1) A(u+v) = A(u) + A(v)$$

$$2) A(\lambda \cdot u) = \lambda \cdot A(u)$$

$$1) A(\vec{x} + \vec{y}) = [\vec{x} + \vec{y}, \vec{a}] = [\vec{x}, \vec{a}] + [\vec{y}, \vec{a}] = A(\vec{x}) + A(\vec{y})$$

$$2) A(\lambda x) = [\lambda \vec{x}, \vec{a}] = \lambda \cdot [\vec{x}, \vec{a}] = \lambda \cdot A(\vec{x})$$

$$\vec{A}(\vec{x}) = 2\vec{x}$$

$$1) \vec{A}(\vec{x} + \vec{y}) = 2 \cdot \vec{x} + 2 \cdot \vec{y} = \vec{A}(\vec{x}) + \vec{A}(\vec{y})$$

$$2) \vec{A}(x \cdot \lambda) = 2x \cdot \lambda = \lambda \cdot \vec{A}(\vec{x})$$

$$\vec{a} = 4\vec{i} + 1\vec{k}$$

$$\vec{x} = x_1 \cdot \vec{i} + x_2 \cdot \vec{j} + x_3 \cdot \vec{k}, \quad \vec{k} \rightarrow 1.6$$

$$\vec{A}(\vec{x}) = [\vec{x}, \vec{a}] = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & x_2 + (x_3 - x_2) & 1 - kx_2 \end{vmatrix}$$

$$A = \begin{pmatrix} x_1 & 4 & -1 & 0 \\ x_2 & 1 & 0 & -1 \\ x_3 & 1 & 0 & 0 \end{pmatrix}$$

$$N_g \vec{A}(p(t)) = p(t) - 2 \cdot p(t)$$

$$P_2 \rightarrow (t^2, t, 1)$$

$$P_1 \rightarrow (t^3, t^2, t, 1)$$

$$\frac{13}{1}$$

$$P(t) = At^2 + Bt + C$$

$$A(P(t)) = (At^2 + Bt + C) - 2(At^2 + Bt + C) =$$

$$= (2At + B) + (2At^2 + Bt + 2C) =$$

$$= (\underbrace{A+B} \cdot 2)t + 2At^2 + 2C + B - \underline{\underline{2At^2 + (2A+2B)t + B + 2C}}$$

$$\bar{A} = \begin{matrix} A \\ B \\ C \end{matrix} \begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 & 0 \end{pmatrix} A$$

$$\begin{pmatrix} 0 & \underline{2} & \underline{1} \\ 0 & \underline{0} & \underline{2} \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \end{pmatrix} =$$

$$\begin{cases} \underline{2A + 2B = 0} \\ 2B + C = \\ 2C = 0 \end{cases}$$

$$\begin{cases} A = 0 \\ B = 0 \\ C = 0 \end{cases}$$

$$p(t) = At^2 + Bt + C$$

$$p(t) = 2t^2 + t$$

$$\underline{\text{Ker} A(p/t)} = \{ \underline{p(t)} = 0 \}$$

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \end{pmatrix} = A \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + B \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + C \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\text{Im } \bar{A} = \{ p(t) = \underline{(2A + 2B)t^2 + (2B + C)t - 2C} \}$$

$$\text{Im } \bar{A} = P_2$$

$$IV \quad G = \begin{pmatrix} \textcircled{g_{11}} & g_{12} \\ g_{21} & \textcircled{g_{22}} \end{pmatrix} = \begin{pmatrix} |\vec{e}_1|^2 & \vec{e}_1 \cdot \vec{e}_2 \\ \vec{e}_1 \cdot \vec{e}_2 & |\vec{e}_2|^2 \end{pmatrix}$$

$$\begin{pmatrix} \textcircled{2} & \textcircled{1} \\ 1 & 3 \end{pmatrix}$$

$$|\vec{e}_1| = \sqrt{2}$$

$$|\vec{e}_2| = \sqrt{3}$$

$$\vec{e}_1 \cdot \vec{e}_2 = 1$$

$$\vec{x} = 1\vec{e}_1 - 1\vec{e}_2$$

$$\vec{y} = 1\vec{e}_1 + 1\vec{e}_2$$

$$\vec{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\vec{y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\cos \angle(\vec{x}, \vec{y}) = \frac{\textcircled{(\vec{x}, \vec{y})}}{\textcircled{|\vec{x}|} \cdot \textcircled{|\vec{y}|}}$$

$$\textcircled{|\vec{x}|^2} = \vec{x} \cdot \vec{x}$$

$$\vec{x} = \begin{pmatrix} 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= (1 - 2) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= 3$$

$$|\vec{x}| = \sqrt{3}$$

$$\vec{y} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 7$$

$$|\vec{y}| = \sqrt{17}$$

$$(\vec{x}, \vec{y}) = \vec{x}^T \cdot G \vec{y} = (1 - 2) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} =$$

$$= -1$$

$$\cos \angle(\vec{x}, \vec{y}) = \frac{-1}{\sqrt{3} \cdot \sqrt{17}} = -\frac{1}{\sqrt{51}}$$

$$\angle(\vec{x}, \vec{y}) = \arccos \left(\frac{1}{\sqrt{21}} \right) = \pi - \arccos \frac{1}{\sqrt{2}}$$