Polocy Gradient

Introduction to Reinforcement Learning

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RL Algorithms Involve

- Optimization
- Delayed consequences
- Exploration
- Generalization
- And statistical and computational efficiency matters

Last time: Generalization and Efficiency

Can use structure and additional knowledge to help constrain and speed reinforcement learning

Policy-Based Reinforcement Learning

- In the last lecture we approximated the value or action-value function using parameters w,

$$V_w(s) \approx V^\pi(s)$$

$$Q_w(s, a) \approx Q^\pi(s, a)$$

- A policy was generated directly from the value function (using ϵ-greedy)
- In this lecture we will directly parametrize the policy, and will typically use θ to show parameterization:

$$\pi_\theta(s, a) = \mathbb{P}\left[a|s; \theta\right]$$

- Goal is to find a policy π with the highest value function V^π
- We will focus again on model-free reinforcement learning

VALUE-BASED AND POLICY-BASED RL

- Value Based
  - learned Value Function
  - Implicit policy (e.g. -greedy)
- Policy Based
  - No Value Function
  - Learned Policy
- Actor-Critic
  - Learned Value Function
  - Learned Policy

Types of Policies to Search Over

- So far have focused on deterministic policies or ϵ-greedy policies
- Now we are thinking about direct policy search in RL, will focus heavily on stochastic policies

Policy Objective Functions

- Goal: given a policy $\pi_\theta(s,a)$ with parameters θ, find best θ
- But how do we measure the quality for a policy π_θ?
- In episodic environments can use policy value at start state $V(s_0,\theta)$
- For simplicity, today will mostly discuss the episodic case, but can easily extend to the continuing / infinite horizon case

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize $V(s_0, \theta)$

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize $V(s_0, \theta)$
- Can use gradient free optimization
  - Hill climbing
  - Simplex / amoeba / Nelder Mead
  - Genetic algorithms
  - Cross-Entropy method (CEM)
  - Covariance Matrix Adaptation (CMA)

Human-in-the-Loop Exoskeleton Optimization (Zhang et al. Science 2017)

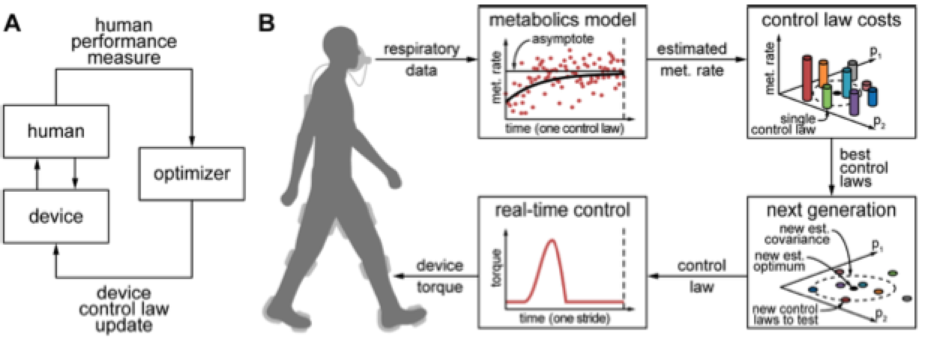

Figure: Zhang et al. Science 2017

Optimization was done using CMA-ES, variation of covariance matrix evaluation

Gradient Free Policy Optimization

Can often work embarrassingly well: ”discovered that evolution strategies (ES), an optimization technique that’s been known for decades, rivals the performance of standard reinforcement learning (RL) techniques on modern RL benchmarks (e.g. Atari/MuJoCo)”
(https://blog.openai.com/evolution-strategies/)

Gradient Free Policy Optimization

- Often a great simple baseline to try
- Benefits
  - Can work with any policy parametrizations, including non-differentiable
  - Frequently very easy to parallelize
- Limitations
  - Often less sample efficient because it ignores temporal structure

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize $V(s_0, \theta)$
- Can use gradient free optimization:
- Greater efficiency often possible using gradient
    - Gradient descent
    - Conjugate gradient
    - Quasi-newton
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure

Policy Gradient

- Define $V(\theta) = V(s_0, \theta)$ to make explicit the dependence of the value on the policy parameters [but don't confuse with value function approximation, where parameterized value function]
- Assume episodic MDPs (easy to extend to related objectives, like average reward)

Policy Gradient

- Define $V^{\pi_\theta} = V(s_0, \theta)$ to make explicit the dependence of the value on the policy parameters
- Assume episodic MDPs
- Policy gradient algorithms search for a local maximum in V (s0,) by ascending the gradient of the policy, w.r.t parameters

$$\Delta\theta = \alpha \nabla_\theta V(s_0, \theta)$$

- Where $\nabla_\theta V(s_0, \theta)$ is the policy gradient

$$\nabla_\theta V(s_0, \theta) = \begin{pmatrix} \frac{\partial V(s_0,\theta)}{\partial \theta_1} \\ \frac{\partial V(s_0,\theta)}{\partial \theta_2} \\ \vdots \\ \frac{\partial V(s_0,\theta)}{\partial \theta_n} \end{pmatrix}$$

- and α is a step-size parameter

Summary of Benefits of Policy-Based RL

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Shortly will see some ideas to help with this last limitation

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Computing the gradient analytically

- We now compute the policy gradient analytically
- Assume policy π_θ is differentiable whenever it is non-zero
- Assume we can calculate gradient $\nabla_\theta \pi_\theta(s, a)$ analytically
- What kinds of policy classes can we do this for?

Differentiable Policy Classes

- Many choices of differentiable policy classes including:
  - Softmax
  - Gaussian
  - Neural networks

Value of a Parameterized Policy

- Now assume policy π_θ is differentiable whenever it is non-zero and we know the gradient $\nabla_\theta \pi_\theta(s, a)$
- Recall policy value is $V(s_0, \theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} R(s_t, a_t); \pi_\theta, s_0\right]$ where the expectation is taken over the states actions visited by π_θ
- We can re-express this in multiple ways
  - $V(s_0, \theta) = \sum_a \pi_\theta(a|s_0) Q(s_0, a, \theta)$

Value of a Parameterized Policy

- Now assume policy π_θ is differentiable whenever it is non-zero and we know the gradient $\nabla_\theta \pi_\theta(s,a)$
- Recall policy value is $V(s_0,\theta) = \sum_a \pi_\theta(a|s_0)Q(s_0,a,\theta)$ where the expectation is taken over the states actions visited by π_θ
- We can re-express this in multiple ways
  - $V(s_0,\theta) = \sum_a \pi_\theta(a|s_0)Q(s_0,a,\theta)$
  - $V(s_0,\theta) = \sum_\tau P(\tau;\theta)R(\tau)$
    - where $\tau = (s_0, a_0, r_0, ..., s_{T1}, a_{T1}, r_{T1}, s_T)$ is a state-action trajectory,
    - $P(\tau;)$ is used to denote the probability over trajectories when executing policy $\pi(\theta)$ starting in state s_0, and
    - $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ the sum of rewards for a trajectory τ

- To start will focus on this latter definition. See Chp 13.1-13.3 of SB for a nice discussion starting with the other definition

Likelihood Ratio Policies

- Denote a state-action trajectory as $\tau = (s_0, a_0, r_0, ..., s_{T1}, a_{T1}, r_{T1}, s_T)$
- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ
- Policy value is

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} R(s_t, a_t); \pi_\theta\right] = \sum_{\tau} P(\tau;\theta)R(\tau)$$

- where $P(\tau;\theta)$ is used to denote the probability over trajectories when executing policy $\pi(\theta)$
- In this new notation, our goal is to find the policy parameters θ:

$$argmax_\theta V(\theta) = argmax_\theta \sum_{\tau} P(\tau;\theta)R(\tau)$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$argmax_\theta V(\theta) = argmax_\theta \sum_\tau P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\begin{aligned}
\nabla_\theta V(\theta) &= \nabla_\theta \sum_\tau P(\tau;\theta)R(\tau) \\
&= \sum_\tau \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau P(\tau;\theta)R(\tau) \frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)} \\
&= \sum_\tau P(\tau;\theta)R(\tau) \nabla_\theta log P(\tau;\theta)
\end{aligned}$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$argmax_\theta V(\theta) = argmax_\theta \sum_\tau P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\nabla_\theta V(\theta) = \sum_\tau P(\tau;\theta)R(\tau)\nabla_\theta logP(\tau;\theta)$$

- Approximate with empirical estimate for m sample trajectories under policy π_θ:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m)\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta logP(\tau^{(i)};\theta)$$

Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample paths under policy :

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta log P(\tau^{(i)}; \theta)$$

$$\begin{aligned}
\nabla_\theta \log P(\tau^{(i)}; \theta) &= \nabla_\theta \log \left[\mu(s_0) \quad \Pi_{t=0}^{T-1} \pi_\theta(a_t|s_t) P(s_{t+1}|s_t, a_t) \right] \\
&= \nabla_\theta \left[\log \mu(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t|s_t) + \log P(s_{t+1}|s_t, a_t) \right] \\
&= \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \quad \text{(score function)}
\end{aligned}$$

Score Function

- A score function is the derivative of the log of a parameterized probability / likelihood
- Example: let $\pi(s, \theta)$ be the probability of state s under parameter
- Then the score function would be
$$\nabla_\theta log \pi(s; \theta)$$
- For many policy classes, it is not hard to compute the score function

Softmax Policy

- Weight actions using linear combination of features $\phi(s,a)^T\theta$
- Probability of action is proportional to exponentiated weight

$$\pi_\theta(s,a) = e^{\phi(s,a)^T\theta} / (\sum_a e^{\phi(s,a)^T\theta})$$

- The score function is

$$\nabla_\theta \log \pi_\theta(s,a) = \phi(s,a) - \mathbb{E}_{\pi_\theta}[\phi(s,\cdot)]$$

Gaussian Policy

- In continuous action spaces, a Gaussian policy is natural
- Mean is a linear combination of state features $\mu(s) = \phi(s)^T\theta$
- Variance may be fixed σ^2, or can also parametrised
- Policy is Gaussian $a \sim N(\mu(s), \sigma^2)$
- The score function is

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}$$

Likelihood Ratio / Score Function Policy Gradient

- Goal is to find the policy parameters :

$$argmax_\theta V(\theta) = argmax_\theta \sum_\tau P(\tau;\theta)R(\tau)$$

- Approximate with empirical estimate for m sample paths under policy π_θ using score function:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m)\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$$

$$= (1/m)\sum_{i=1}^{m} R(\tau^{(i)})\sum_{t=0}^{T-1}\nabla_\theta \log \pi_\theta(a_t^{(i)}|s_t^{(i)})$$

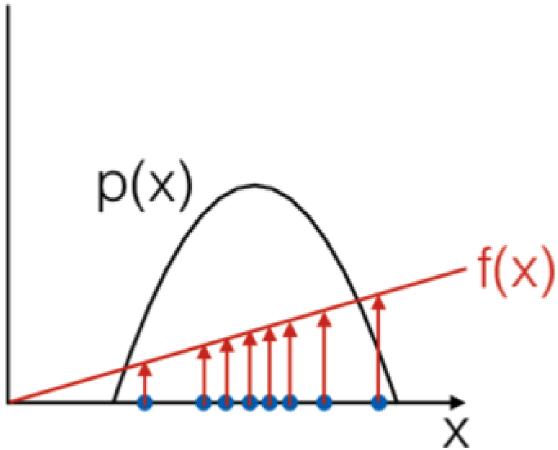
- Do not need to know dynamics model

Score Function Gradient Estimator: Intuition

- Consider generic form of $R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$:
  $\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$
- $f(x)$ measures how good the sample x is.
- Moving in the direction $\hat{g}^i$ pushes up the logprob of the sample, in proportion to how good it is
- Valid even if $f(x)$ is discontinuous, and unknown, or sample space (containing x) is a discrete set

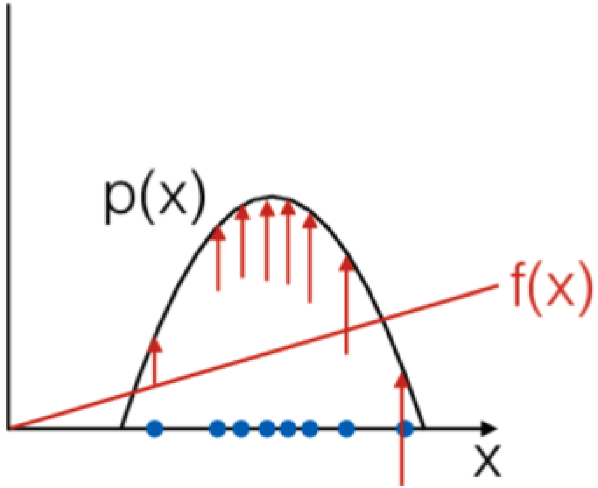
Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$



Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$



Policy Gradient Theorem

The policy gradient theorem generalizes the likelihood ratio approach

Theorem 1

For any differentiable policy $\pi_\theta(s,a)$, for any of the policy objective function $J = J_1$, (episodic reward), J_{avR} (average reward per time step), or $\frac{1}{J_{avV}}$ (average value), the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a) Q^{\pi_\theta}(s,a)\right]$$

Chapter 13.2 in SB has a nice derivation of the policy gradient theorem for episodic tasks and discrete states

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Likelihood Ratio / Score Function Policy Gradient

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline

Policy Gradient: Use Temporal Structure

- Previously:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau\left[\left(\sum_{t=0}^{T-1} r_t\right)\left(\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)\right)\right]$$

- We can repeat the same argument to derive the gradient estimator for a single reward term $r_{t'}$.

$$\nabla_\theta \mathbb{E}[r_{t'}] = \mathbb{E}\left[r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t)\right]$$

- To see this, recall $V(s_0,\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} R(s_t,a_t); \pi_\theta, s_0\right]$ expectation is taken over the states actions visited by π_θ

Policy Gradient: Use Temporal Structure

- Previously:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\left(\sum_{t=0}^{T-1} r_t \right) \left(\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \right) \right]$$

- We can repeat the same argument to derive the gradient estimator for a single reward term $r_{t'}$.

$$\nabla_\theta \mathbb{E}[r_{t'}] = \mathbb{E} \left[r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

- Summing this formula over t, we obtain

$$V(\theta) = \nabla_\theta \mathbb{E}[R] = \mathbb{E} \left[\sum_{t'=0}^{T-1} r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

Policy Gradient: Use Temporal Structure

Recall for a particular trajectory $\tau^{(i)}$, $\sum_{t'=t}^{T-1} r_{t'}^{(i)}$ is the return $G_t^{(i)}$

$$\nabla_\theta \mathbb{E}[R] \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t, s_t) G_t^{(i)}$$

MONTE-CARLO POLICY GRADIENT (REINFORCE)

Leverages likelihood ratio / score function and temporal structure

$$\Delta\theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t$$

REINFORCE:
Initialize policy parameters θ arbitrarily
for each episode $\{s_1, a_1, r_2, \cdots, s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**
  for $t = 1$ to $T - 1$ **do**
    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t$
  endfor
endfor
return θ

Likelihood Ratio / Score Function Policy Gradient

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline
  - Alternatives to using Monte Carlo returns $R(\tau^{(i)})$ as targets

DESIRED PROPERTIES OF A POLICY GRADIENT RL ALGORITHM

Goal: Converge as quickly as possible to a local optima

- Incurring reward / cost as execute policy, so want to minimize number of iterations / time steps until reach a good policy

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Policy Gradient: Introduce Baseline

- Reduce variance by introducing a baseline $b(s)$

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta) \left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t) \right) \right]$$

- For any choice of b, gradient estimator is unbiased.
- Near optimal choice is the expected return,

$$b(s_t) \approx \mathbb{E}\left[r_t + r_{t+1} + \ldots + r_{T1}\right]$$

- Interpretation: increase logprob of action a_t proportionally to how much returns $\sum_{t'=t}^{T-1} r_{t'}$ are better than expected

Baseline b(s) Does Not Introduce Bias–Derivation

$$
\begin{aligned}
&\mathbb{E}_{\tau}[\nabla_\theta \log \pi(a_t|s_t;\theta) b(s_t)] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t;\theta) b(s_t)]\right] \text{ (break up expectation)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t;\theta)]\right] \text{ (pull baseline term out)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\mathbb{E}_{a_t}[\nabla_\theta \log \pi(a_t|s_t;\theta)]] \text{ (remove irrelevant variables)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \pi_\theta(a_t|s_t)\frac{\nabla_\theta \pi(a_t|s_t;\theta)}{\pi_\theta(a_t|s_t)}\right] \text{ (likelihood ratio)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \nabla_\theta \pi(a_t|s_t;\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\nabla_\theta \sum_a \pi(a_t|s_t;\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\nabla_\theta 1] \\
&= \mathbb{E}_{s_0:t,a_0:(t-1)}[b(s_t)\cdot 0] = 0
\end{aligned}
$$

"Vanilla" Policy Gradient Algorithm

Initialize policy parameter θ, baseline b

for iteration$=1, 2, \cdots$ **do**

  Collect a set of trajectories by executing the current policy

  At each timestep t in each trajectory τ^i, compute

    Advantage estimate $\hat{A}_{it}^{n}$

  Update the policy, using a policy gradient estimate $\hat{g}$,

    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_{it}^{n}$.

    (**Plug $\hat{g}$ into SGD or ADAM**)

endfor

Other Choices for Baseline?

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \cdots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep t in each trajectory τ^i, compute
    Return $G_t^i = \sum_{t'=t}^{T-1} r_{t'}^i$, and
    Advantage estimate $\hat{A}_t^i = G_t^i - b(s_t)$.
  Re-fit the baseline, by minimizing $\sum_i \sum_t ||b(s_t) - G_t^i||^2$,
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
    (Plug $\hat{g}$ into SGD or ADAM)
endfor

Choosing the Baseline: Value Functions

- Recall Q-function / state-action-value function:

$$Q^{\pi}(s, a) = \mathbb{E}_{\pi}[r_0 + \gamma r_1 + \gamma^2 r_2 ... | s_0 = s, a_0 = a]$$

- State-value function can serve as a great baseline

$$V^{\pi}(s) = \mathbb{E}_{\pi}[r_0 + \gamma r_1 + \gamma^2 r_2 ... | s_0 = s]$$
$$= \mathbb{E}_{a \sim \pi}[Q^{\pi}(s, a)]$$

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Likelihood Ratio / Score Function Policy Gradient

- Policy gradient:

$$\nabla_\pi \mathbb{E}[R] \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t, s_t)(G_t^{(i)} - b(s_t))$$

- Fixes that improve simplest estimator
  - Temporal structure (shown in above equation)
  - Baseline (shown in above equation)
  - Alternatives to using Monte Carlo returns G_t^i as estimate of expected discounted sum of returns for the policy parameterized by θ?

Choosing the Target

- G_t^i is an estimation of the value function at st from a single roll out
- Unbiased but high variance
- Reduce variance by introducing bias using bootstrapping and function approximation
  - Just like in we saw for TD vs MC, and value function approximation

Actor-critic Methods

- Estimate of V/Q is done by a critic
- Actor-critic methods maintain an explicit representation of policy and the value function, and update both
- A3C (Mnih et al. ICML 2016) is a very popular actor-critic method

Policy Gradient Formulas with Value Functions

- Recall:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta)\left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t)\right)\right]$$

$$\nabla_\theta \mathbb{E}_\tau[R] \approx \mathbb{E}_\tau\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta)\left(Q(s_t,a_t;w) - b(s_t)\right)\right]$$

- Letting the baseline be an estimate of the value V , we can represent the gradient in terms of the state-action advantage function

$$\nabla_\theta \mathbb{E}_\tau[R] \approx \mathbb{E}_\tau\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta)\hat{A}^\pi(s_t,a_t)\right]$$

- where the advantage function $A^\pi(s_t,a_t) = Q^\pi(s,a) - V^\pi(s)$

Choosing the Target: N-step estimators

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} R_t^i \nabla_\theta log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Note that critic can select any blend between TD and MC estimators for the target to substitute for the true state-action value function.

$$\hat{R}_t^{(1)} = r_t + \gamma V(s_{t+1})$$

$$\hat{R}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2})$$

$$...$$

$$\hat{R}_t^{(inf)} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ...$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$

$$\hat{A}_t^{(inf)} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... - V(s_t)$$