Problem 7-1 Degree Correlations and Assortativity

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1 Lecture: Complex Network Analysis

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1.1 Assignment 7 - Assortativity and Robustness

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2 1. Build graph

[1]:
```
import pandas as pd
import networkx as nx
import numpy as np
import seaborn as sns
import matplotlib.pylab as plt
import scipy
```

[2]:
```
df_blogs = pd.read_csv('assortativity_networks/blogs.txt', sep="\t", header=None)
df_javax = pd.read_csv('assortativity_networks/javax.txt',␣
 ↪delim_whitespace=True, header=None)
df_network_science = pd.read_csv('assortativity_networks/network-science.txt',␣
 ↪sep="\t", header=None)
```

[3]:
```
df_blogs
```

[3]:
```
         0     1
0        1     2
1        1     3
2        1     4
3        1     5
4        1     6
...    ...   ...
33425  975   664
33426  975    67
33427  975  1004
```

```
33428   975  1224
33429  1028   791

[33430 rows x 2 columns]
```

[4]:
```
# since it is an undirected graph, no parallel edges are added
G_blogs = nx.Graph()
G_blogs.add_edges_from(df_blogs.itertuples(index=False))

G_javax = nx.Graph()
G_javax.add_edges_from(df_javax.itertuples(index=False))

G_network_science = nx.Graph()
G_network_science.add_edges_from(df_network_science.itertuples(index=False))

# remove self-loops
G_blogs.remove_edges_from(nx.selfloop_edges(G_blogs))
G_javax.remove_edges_from(nx.selfloop_edges(G_javax))
G_network_science.remove_edges_from(nx.selfloop_edges(G_network_science))
```

[5]:
```
print(f"Number of nodes in blogs is {G_blogs.number_of_nodes()}.")
print(f"Number of edges in blogs is {G_blogs.number_of_edges()}.")
print()
print(f"Number of nodes in javax is {G_javax.number_of_nodes()}.")
print(f"Number of edges in javax is {G_javax.number_of_edges()}.")
print()
print(f"Number of nodes in network-science is {G_network_science.
 ↪number_of_nodes()}.")
print(f"Number of edges in network-science is {G_network_science.
 ↪number_of_edges()}.")
```

```
Number of nodes in blogs is 1224.
Number of edges in blogs is 16715.

Number of nodes in javax is 6120.
Number of edges in javax is 50290.

Number of nodes in network-science is 1461.
Number of edges in network-science is 2742.
```

3 2. Degree correlation matrix

[6]:
```
def calculate_degree_correlation_matrix(G):
    max_degree = max(deg for n, deg in G.degree)
    # create a dict to save the number of degree combinations
    degrees = []
    for i in range(max_degree+1):
```

```
        for j in range(max_degree+1):
            degrees.append((i,j))

    deg_1 = []
    deg_2 = []
    for i in degrees:
        deg_1.append(i[0])
        deg_2.append(i[1])
    d = {'deg_1': deg_1, 'deg_2': deg_2, 'count': 0}
    degree_correlation_df = pd.DataFrame(data=d)

    for u,v,weight in G.edges(data=True):
        degree_correlation_df.loc[degree_correlation_df.eval(f'deg_1 == {G.
↪degree(u)} & deg_2 == {G.degree(v)}'), 'count'] += 1

    deg_corr_mat = np.zeros((max_degree+1, max_degree+1))
    for index, row in degree_correlation_df.iterrows():
        deg_corr_mat[row['deg_1'], row['deg_2']] = row['count']

    deg_corr_mat = deg_corr_mat + deg_corr_mat.T
    deg_corr_mat_prob = deg_corr_mat / np.sum(deg_corr_mat)

    deg_corr_mat_absolute = deg_corr_mat

    return deg_corr_mat_absolute, deg_corr_mat_prob
```

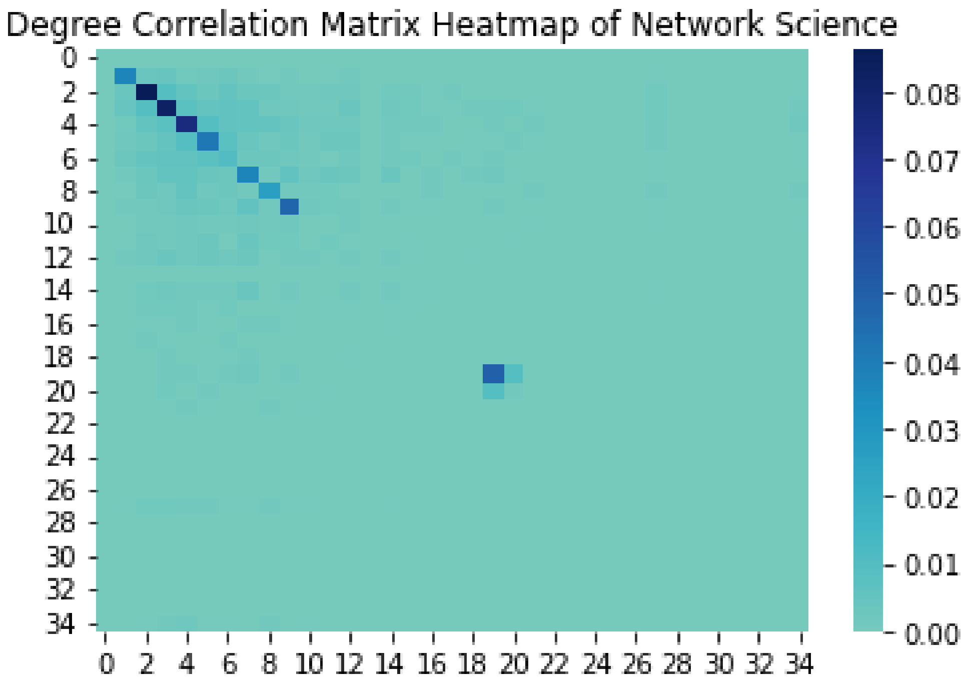
[7]:
```
deg_corr_mat_blogs_absolute, deg_corr_mat_blogs =␣
↪calculate_degree_correlation_matrix(G_blogs)
```

[8]:
```
deg_corr_mat_network_science_absolute, deg_corr_mat_network_science =␣
↪calculate_degree_correlation_matrix(G_network_science)
```

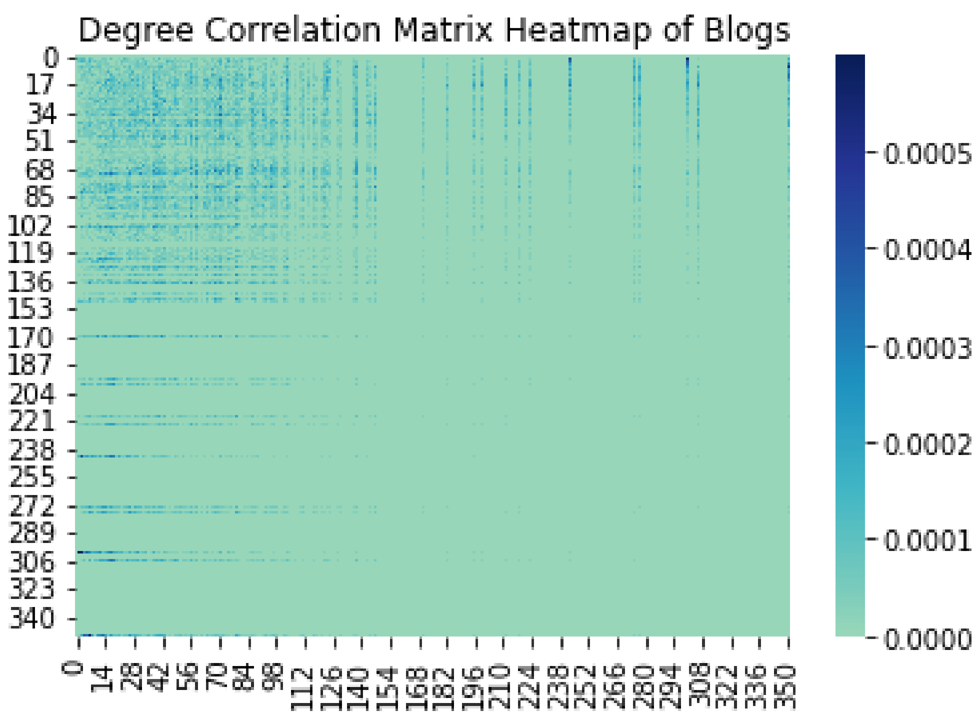
[9]:
```
deg_corr_mat_javax_absolute, deg_corr_mat_javax =␣
↪calculate_degree_correlation_matrix(G_javax)
```

4 3. Heatmap

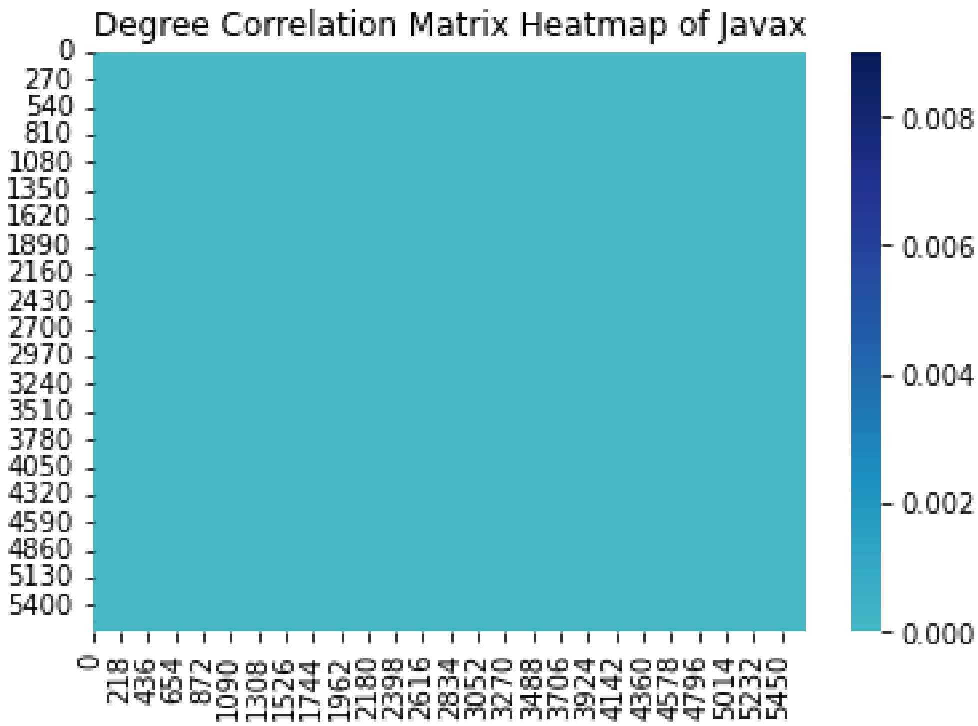
[10]:
```
ax = sns.heatmap(deg_corr_mat_network_science, cmap="YlGnBu", center=0.015)
plt.title("Degree Correlation Matrix Heatmap of Network Science")
plt.show()
```



[11]:
```
ax = sns.heatmap(deg_corr_mat_blogs, cmap="YlGnBu", center=0.00015)
plt.title("Degree Correlation Matrix Heatmap of Blogs")
plt.show()
```



[23]:
```
ax = sns.heatmap(deg_corr_mat_javax, cmap="YlGnBu", center=0.00015)
plt.title("Degree Correlation Matrix Heatmap of Javax")
plt.show()
```



5 4. Nearest neighbor degree

[12]:
```
# calculates nearest neighbor degree for singe nodes
def calculate_k_nn_single_node(G, node):
    neighbors = list(G.neighbors(node))
    return np.sum([G.degree(neighbor) for neighbor in neighbors]) / G.
 ↪degree(node)
```

[13]:
```
# get k_i
degrees_network_science = [G_network_science.degree(node) for node in␣
 ↪G_network_science.nodes]
k_i_network_science = []
for node in list(G_network_science.nodes):
    k_i_network_science.append(calculate_k_nn_single_node(G_network_science,␣
 ↪node))
```

```
degrees_blogs = [G_blogs.degree(node) for node in G_blogs.nodes]
k_i_blogs = []
for node in list(G_blogs.nodes):
    k_i_blogs.append(calculate_k_nn_single_node(G_blogs, node))

k_i_javax = []
degrees_javax = [G_javax.degree(node) for node in G_javax.nodes]
for node in list(G_javax.nodes):
    k_i_javax.append(calculate_k_nn_single_node(G_javax, node))
```

[14]:
```
# calculates nearest neighbor degree for all nodes of degree k
def calculate_k_nn(k, deg_corr_mat_absolute):
    neighbors = deg_corr_mat_absolute[k]
    num_neighbors = np.sum(neighbors)

    return np.sum([k_prime * neighbors[k_prime] / num_neighbors for k_prime in␣
 ↪range(len(neighbors))])
```

[15]:
```
# get k_nn
k_nn_network_science = []
for k in range(len(deg_corr_mat_network_science_absolute[0])):
    k_nn_network_science.append(calculate_k_nn(k,␣
 ↪deg_corr_mat_network_science_absolute))

k_nn_blogs = []
for k in range(len(deg_corr_mat_blogs_absolute[0])):
    k_nn_blogs.append(calculate_k_nn(k, deg_corr_mat_blogs_absolute))

k_nn_javax = []
for k in range(len(deg_corr_mat_javax_absolute[0])):
    k_nn_javax.append(calculate_k_nn(k, deg_corr_mat_javax_absolute))
```

```
/opt/anaconda3/envs/complexnetworkanalysis/lib/python3.7/site-
packages/ipykernel_launcher.py:6: RuntimeWarning: invalid value encountered in
double_scalars
```

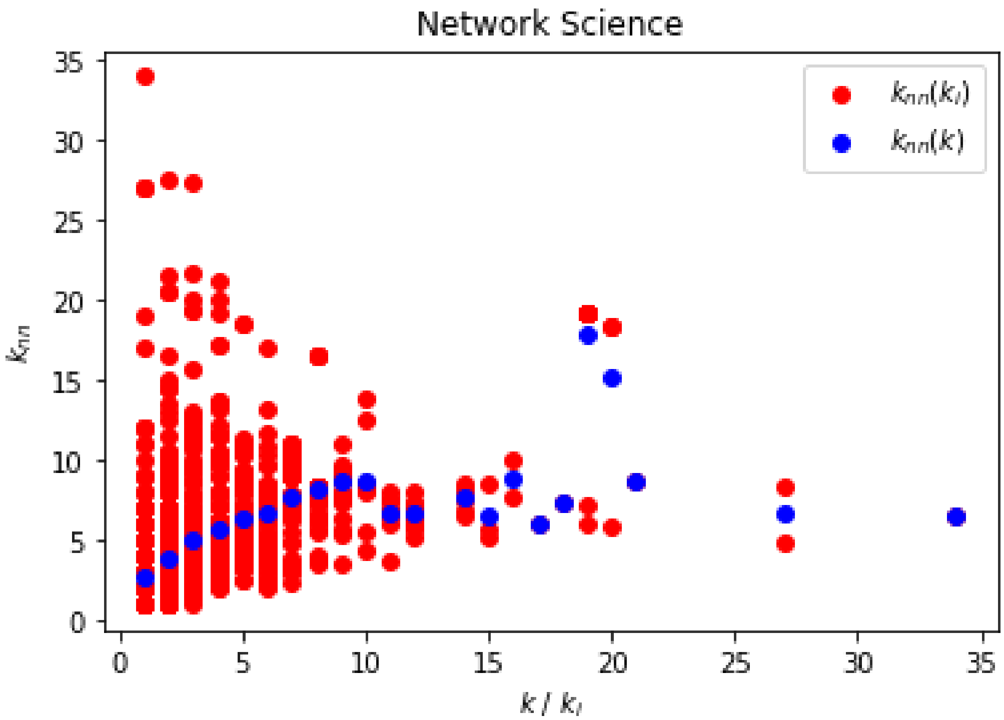
[ ]:
```

```

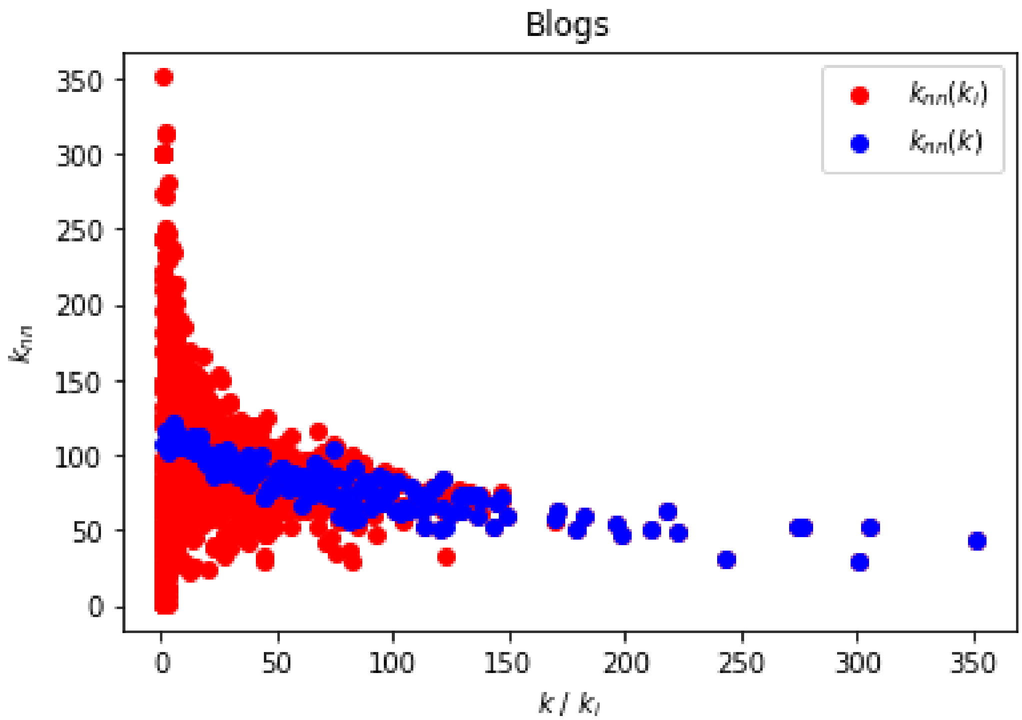
6 Scatter plot

[16]:
```
plt.scatter(degrees_network_science, k_i_network_science, c='red',␣
 ↪label='$k_{nn}(k_i)$')
plt.scatter(range(len(deg_corr_mat_network_science_absolute[0])),␣
 ↪k_nn_network_science, c='blue', label='$k_{nn}(k)$')
```

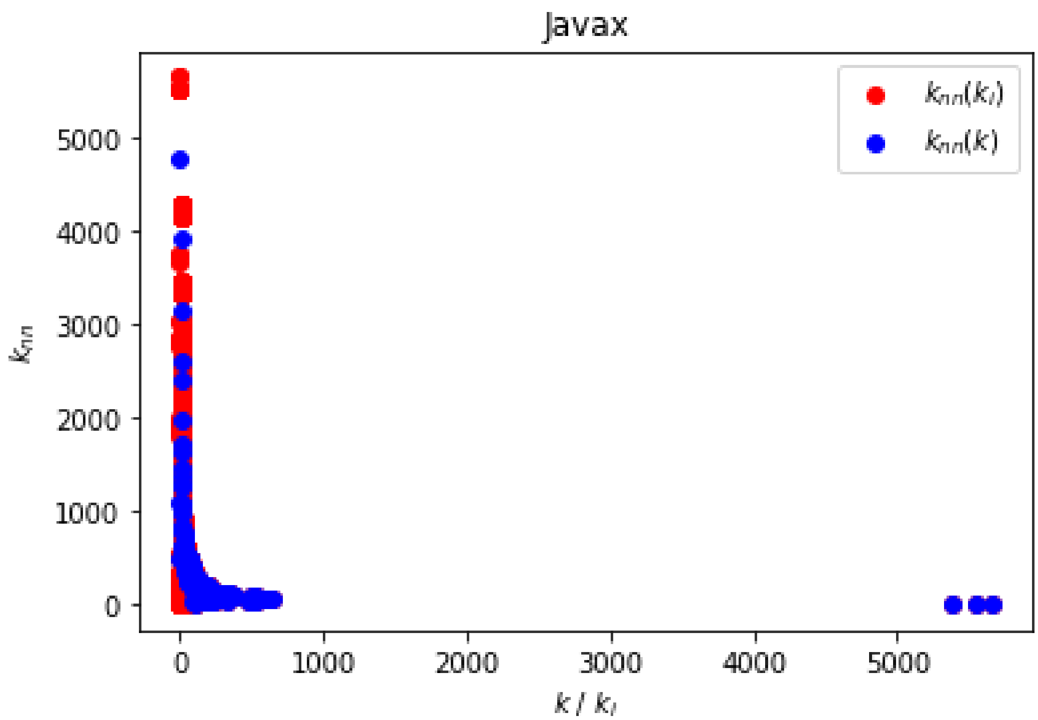
```
plt.title("Network Science")
plt.xlabel('$k$ / $k_i$')
plt.ylabel('$k_{nn}$')
plt.legend()
plt.show()
```



[17]:
```
plt.scatter(degrees_blogs, k_i_blogs, c='red', label='$k_{nn}(k_i)$')
plt.scatter(range(len(deg_corr_mat_blogs_absolute[0])), k_nn_blogs, c='blue',␣
 ↪label='$k_{nn}(k)$')
plt.title("Blogs")
plt.xlabel('$k$ / $k_i$')
plt.ylabel('$k_{nn}$')
plt.legend()
plt.show()
```



[18]:
```
plt.scatter(degrees_javax, k_i_javax, c='red', label='$k_{nn}(k_i)$')
plt.scatter(range(len(deg_corr_mat_javax_absolute[0])), k_nn_javax, c='blue',␣
 ↪label='$k_{nn}(k)$')
plt.title("Javax")
plt.xlabel('$k$ / $k_i$')
plt.ylabel('$k_{nn}$')
plt.legend()
plt.show()
```



7 Degree correlation coefficient

[19]:
```
def compute_degree_correlation_coefficient(G, deg_corr_mat):
    max_degree = max(deg for n, deg in G.degree)

    avg_degree = sum(deg for n, deg in G.degree)/len(G.degree)

    q_k = {}
    for deg in range(max_degree + 1):
        p_k = [deg for n, deg in G.degree].count(deg)/len(G.degree)
        q_k[deg] = (deg * p_k)/avg_degree

    sigma_squared = sum([(k**2) * q_k[k] for k in q_k]) - sum([k * q_k[k] for k␣
 ↪in q_k])**2

    r = []

    for j, row in enumerate(deg_corr_mat):
        for k, e_jk in enumerate(row):
            qk = q_k[k]
            qj = q_k[j]
            r.append((j*k*(e_jk-qj*qk))/sigma_squared)
```

```
    r = sum(r)

    return r
```

[20]:
```
print(f"The degree correlation coefficient with our computation for Network␣
↪Science is r={compute_degree_correlation_coefficient(G_network_science,␣
↪deg_corr_mat_network_science)}")
# to check our computation, we also use the inbuild function of networkx
print(f"The degree correlation coefficient with the inbuild networkx function␣
↪for Network Science is r={nx.algorithms.assortativity.
↪degree_assortativity_coefficient(G_network_science)}")
```

```
The degree correlation coefficient with our computation for Network Science is
r=0.4616224667525837
The degree correlation coefficient with the inbuild networkx function for
Network Science is r=0.4616224667525835
```

[21]:
```
print(f"The degree correlation coefficient with our computation for Blogs is␣
↪r={compute_degree_correlation_coefficient(G_blogs, deg_corr_mat_blogs)}")
# to check our computation, we also use the inbuild function of networkx
print(f"The degree correlation coefficient with the inbuild networkx function␣
↪for Blogs is r={nx.algorithms.assortativity.
↪degree_assortativity_coefficient(G_blogs)}")
```

```
The degree correlation coefficient with our computation for Blogs is
r=-0.2212328638045546
The degree correlation coefficient with the inbuild networkx function for Blogs
is r=-0.22123286380455423
```

[26]:
```
print(f"The degree correlation coefficient with our computation for Javax is␣
↪r={compute_degree_correlation_coefficient(G_javax, deg_corr_mat_javax)}")
# to check our computation, we also use the inbuild function of networkx
print(f"The degree correlation coefficient with the inbuild networkx function␣
↪for Javax is r={nx.algorithms.assortativity.
↪degree_assortativity_coefficient(G_javax)}")
```

```
The degree correlation coefficient with our computation for Javax is
r=-0.2327051928360141
The degree correlation coefficient with the inbuild networkx function for Javax
is r=-0.23270519283601443
```

[28]:
```
# because it took forever: pickle stuff
import pickle
with open('deg_corr_mat_javax_absolute.pkl','wb') as f:
    pickle.dump(deg_corr_mat_javax_absolute, f)

with open('deg_corr_mat_javax.pkl','wb') as f:
```

```python
    pickle.dump(deg_corr_mat_javax, f)
```

[ ]:

```python

```