Problem 6-3 Degree Correlation Coefficient

December 13, 2021

1 Lecture: Complex Network Analysis

Prof. Dr. Michael Gertz

Winter Semester 2021/22

1.1 Assignment 6 - Degree Correlations and Assortativity

Students: Felix Hausberger, Nils Krehl, Patrick Günther

2 1. Compute the degree correlation matrix

[1]:

```
import numpy as np
import networkx as nx
```

[2]:

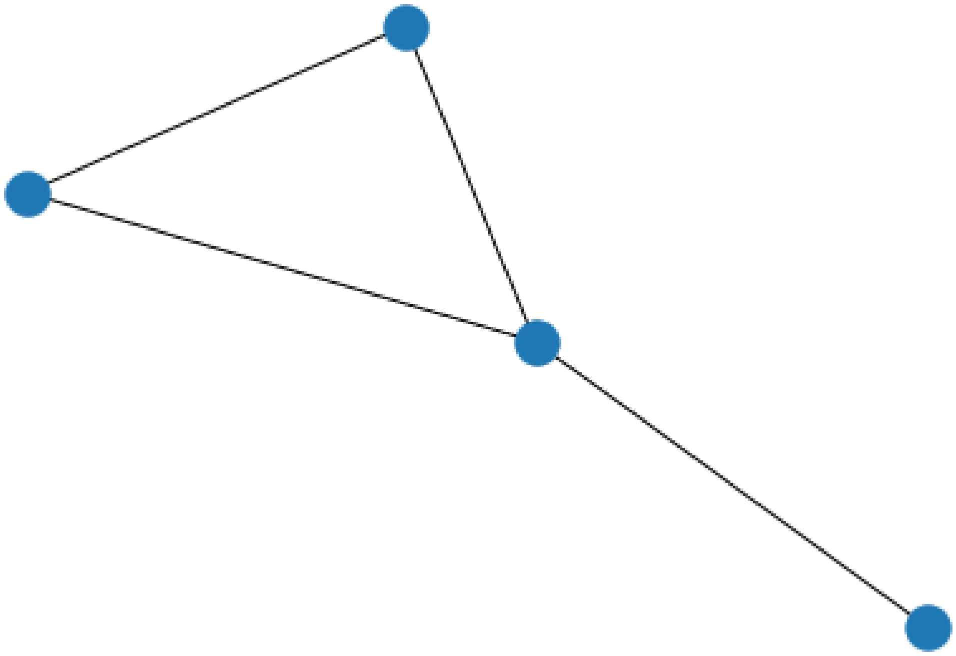
```
A = np.matrix([[0, 1, 0, 1],
               [1, 0, 1, 1],
               [0, 1, 0, 0],
               [1, 1, 0, 0]])
```

[3]:

```
G = nx.convert_matrix.from_numpy_matrix(A)
```

[4]:

```
nx.draw(G)
```



```
[5]: G.degree
```

```
[5]: DegreeView({0: 2, 1: 3, 2: 1, 3: 2})
```

```
[6]: max_degree = max(deg for n, deg in G.degree)
     mapping = {x: x for x in range(max_degree+1)}
     deg_corr_mat = nx.degree_mixing_matrix(G, mapping=mapping)
```

```
[7]: deg_corr_mat
```

```
[7]: array([[0.   , 0.   , 0.   , 0.   ],
            [0.   , 0.   , 0.   , 0.125],
            [0.   , 0.   , 0.25 , 0.25 ],
            [0.   , 0.125, 0.25 , 0.   ]])
```

(the first column/ row is for degree 0. . . that could be cut out, since a node with degree 0 never connects to any other node)

3 2. Compute the probabilities q_k of having a degree k node at the end of a random link

[8]:

```
avg_degree = sum(deg for n, deg in G.degree)/len(G.degree)

q_k = {}
for deg in range(max_degree + 1):
    p_k = [deg for n, deg in G.degree].count(deg)/len(G.degree)
    q_k[deg] = (deg * p_k)/avg_degree
```

[9]:

```
q_k
```

[9]:

```
{0: 0.0, 1: 0.125, 2: 0.5, 3: 0.375}
```

4 3. Compute the degree correlation coefficient r

[10]:

```
sigma_squared = sum([(k**2) * q_k[k] for k in q_k]) - sum([k * q_k[k] for k in␣
 ↪q_k])**2

r = []

for j, row in enumerate(deg_corr_mat):
    for k, e_jk in enumerate(row):
        qk = q_k[k]
        qj = q_k[j]
        r.append((j*k*(e_jk-qj*qk))/sigma_squared)

r = sum(r)
```

[11]:

```
print(f"The degree correlation coefficient of the network is {r}.")
```

```
The degree correlation coefficient of the network is -0.7142857142857144.
```

[12]:

```
# to check our computation, we also use the inbuild function of networkx
nx.algorithms.assortativity.degree_assortativity_coefficient(G)
```

[12]:

```
-0.7142857142857143
```

[ ]:

```
```