Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

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Assignment 7

Degree Assortativity and Robustness

`https://github.com/nilskre/CNA_assignments`

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1 Problem 7-2 Molloy-Reed Criterion

Consider a configuration model network that has nodes of degree 1, 2, and 3 only, with probabilities p_1, p_2, and p_3, respectively. The degree distribution is given by:

$$p_k = \delta_{k,1}p_1 + \delta_{k,2}p_2 + \delta_{k,3}p_3, \begin{cases} \delta_{k,1} = 3 & \text{if } k = 1 \\ \delta_{k,2} = 2 & \text{if } k = 2 \\ \delta_{k,3} = 2 & \text{if } k = 3 \end{cases}$$

1. Compute the first moment $\langle k \rangle$ and the second moment $\langle k^2 \rangle$ of the degree distribution.
2. Using the Molloy-Reed criterion, show that there is a giant component if and only if $p_1 < 3p_3$.
3. In terms of the structure of the network, discuss the meaning of the condition $p_3 < 3p_1$. Why does the result not depend on p_2?

2 Problem 7-4 Random Failures in Uncorrelated Networks

Compute the critical threshold f_c for each of the following degree distributions, under the assumption that the networks do not exhibit any degree correlation.

1. Poisson distribution, i.e.,
$$p_k = e^{-\mu}\frac{\mu^k}{k!}$$
2. Discrete exponential distribution, i.e.,
$$p_k = (1 - e^{-\lambda})e^{-\lambda k}$$
3. Dirac delta distribution, i.e.,
$$p_k = \delta_{k,k_0} = \begin{cases} 1 & \text{if } k = k_0, \\ 0 & \text{otherwise.} \end{cases}$$

Discuss the consequences of your results for network robustness.

Hint: You may use the first and second moment from the lecture or other literature without a proof