Problem 7-3 Xalvi-Brunet and Sokolov Algorithm

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Lecture: Complex Network Analysis

Prof. Dr. Michael Gertz

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Assignment 7 - Assortativity and Robustness

Problem 7-3: Xalvi-Brunet and Sokolov Algorithm

Students: Felix Hausberger, Nils Krehl, Patrick Günther

[1]:
```
import pandas as pd
import networkx as nx
import numpy as np
import seaborn as sns
import matplotlib.pylab as plt
import scipy
```

1. Xalvi-Brunet and Sokolov algorithm

[2]:
```
def xalvi_brunet_sokolov_algorithm(graph, num_iterations, assortative):
    network = graph.copy()
    for i in range(num_iterations):
        links = np.array(list(network.edges))
        degrees = network.degree()
        # choose two random links
        choosen_indices = np.random.choice(range(len(links)), 2, replace=False)
        choosen_links = links[choosen_indices]

        # get corresponding nodes and their node degrees
        corresponding_nodes = choosen_links.flatten()
        corresponding_node_degrees = np.array([degrees[x] for x in␣
 ↪corresponding_nodes])

        # sort the nodes by their degrees in descending order
        index_array = np.argsort(corresponding_node_degrees)[::-1]
```

```python
        ordered_nodes = corresponding_nodes[index_array]
        ordered_nodes_degrees = corresponding_node_degrees[index_array]

        # remove the selected links
        network.remove_edge(choosen_links[0][0], choosen_links[0][1])
        network.remove_edge(choosen_links[1][0], choosen_links[1][1])

        # rewiring
        if assortative == True:
            network.add_edge(ordered_nodes[0], ordered_nodes[1])
            network.add_edge(ordered_nodes[2], ordered_nodes[3])
        else:
            network.add_edge(ordered_nodes[0], ordered_nodes[3])
            network.add_edge(ordered_nodes[1], ordered_nodes[2])

    return network
```

2. Create networks with Xalvi-Brunet and Sokolov algorithm

[3]:
```python
df_neutral_network = pd.read_csv('neutral_network.txt', delim_whitespace=True,␣
↪header=None)
```

[4]:
```python
G_neutral_network = nx.Graph()
G_neutral_network.add_edges_from(df_neutral_network.itertuples(index=False))
```

[5]:
```python
G_assortative = xalvi_brunet_sokolov_algorithm(G_neutral_network, 5000, True)
G_disassortative = xalvi_brunet_sokolov_algorithm(G_neutral_network, 5000, False)
```

[6]:
```python
print("Degree Correlation Coefficient")
print("r = 0: neutral network; r < 0: disassortative network; r > 0: assortative␣
↪network \n")
print("Neutral network Degree Correlation Coefficient: {}".format(nx.
↪degree_pearson_correlation_coefficient(G_neutral_network)))
print("Assortative network Degree Correlation Coefficient: {}".format(nx.
↪degree_pearson_correlation_coefficient(G_assortative)))
print("Disassortative network Degree Correlation Coefficient: {}".format(nx.
↪degree_pearson_correlation_coefficient(G_disassortative)))
```

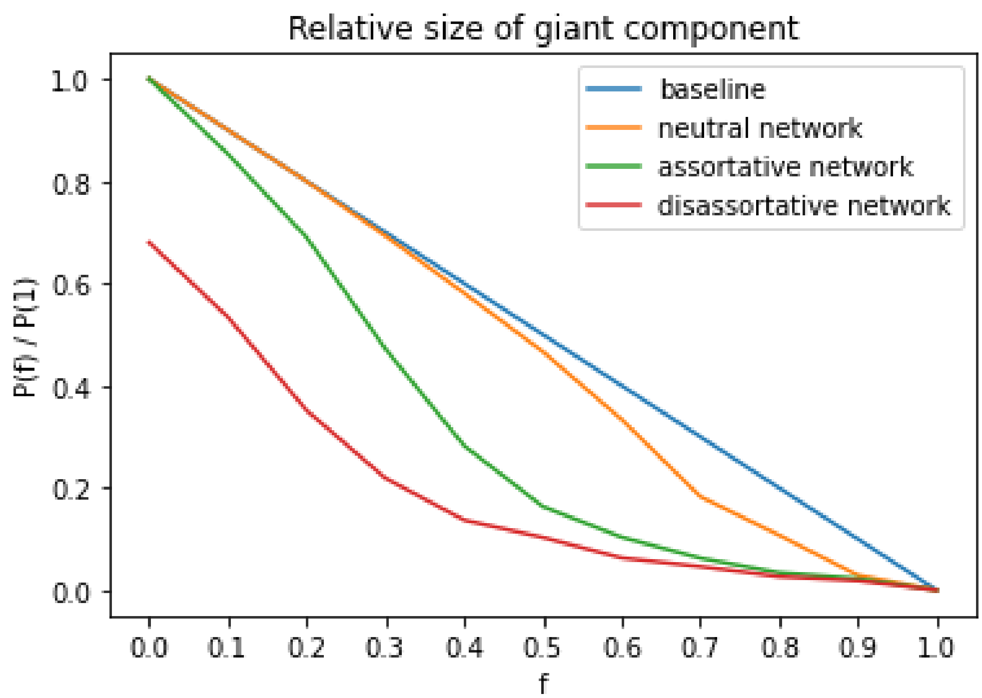
```
Degree Correlation Coefficient
r = 0: neutral network; r < 0: disassortative network; r > 0: assortative
network

Neutral network Degree Correlation Coefficient: -0.009246262701730106
Assortative network Degree Correlation Coefficient: 0.9037799701732246
Disassortative network Degree Correlation Coefficient: -0.6223530885853903
```

3. Plot giant component size

[69]:
```
def get_giant_component_size(network):
    if network.number_of_nodes() > 0:
        giant_component = max(nx.connected_components(network), key=len)
        giant_component_size = len(giant_component)
        return giant_component_size
    else:
        return 0


def get_relative_size_of_giant_component(graph, num_samples=20):
    network = graph.copy()
    number_nodes = network.number_of_nodes()
    f = []

    relative_size_of_giant_component = []
    for f_value in np.arange(0,1.1,0.1):
        giant_component_size = []
        for sample in range(num_samples):
            minimized_network = network.copy()
            number_to_be_removed = int(f_value * number_nodes)

            random_sample = np.random.choice(minimized_network.nodes(),␣
 ↪number_to_be_removed, replace=False)
            minimized_network.remove_nodes_from(random_sample)

            giant_component_size.
 ↪append(get_giant_component_size(minimized_network))
        relative_size_of_giant_component.append(np.mean(np.
 ↪array(giant_component_size)))

    return np.array(relative_size_of_giant_component) / 100

neutral_relative_size_of_giant_component =␣
 ↪get_relative_size_of_giant_component(G_neutral_network)
assortative_relative_size_of_giant_component =␣
 ↪get_relative_size_of_giant_component(G_assortative)
disassortative_relative_size_of_giant_component =␣
 ↪get_relative_size_of_giant_component(G_disassortative)
```

[71]:
```
plt.plot(np.arange(10,-1,-1) / 10, label="baseline")
plt.plot(neutral_relative_size_of_giant_component, label="neutral network")
plt.plot(assortative_relative_size_of_giant_component, label="assortative␣
 ↪network")
```

```
plt.plot(disassortative_relative_size_of_giant_component, label="disassortative␣
↪network")
plt.xticks(ticks=range(11), labels=(np.arange(0,11,1) / 10))
plt.legend()
plt.title("Relative size of giant component")
plt.xlabel("f")
plt.ylabel("P(f) / P(1)")
plt.show()
```



4. Discussion

Discuss the results from the previous task: Which network is the most robust against random failures? Explain why this is the case.

- The plot above shows, that with increasing f (increased number of removed nodes), the size of the giant component decreases slowest in the neutral network. That is why the most robust network against random failues is the neutral network. In a neutral network nodes are linked randomly and consequently the density of links is around the average degree.
- In assortative networks hubs tend to link to each other and small-degree nodes tend to connect to small degree nodes.
- In disassortative networks hubs avoid each other. Small-degree nodes tend to connect to hubs, and hubs tend to connect to small-degree nodes (this is called hub-and-spoke character). When removing hubs the network is quickly divided into parts. This explains the rapid reduction of the giant component size in disassortative networks.