Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

Prof. Dr. Michael Gertz

Assignment 2

Graph Theory and Networks in Python

`https://github.com/nilskre/CNA_assignments`

Team Member: Patrick Gï¿½nther, 3660886,
Applied Computer Science
rh269@stud.uni-heidelberg.de

Team Member: Felix Hausberger, 3661293,
Applied Computer Science
eb260@stud.uni-heidelberg.de

Team Member: Nils Krehl, 3664130,
Applied Computer Science
pu268@stud.uni-heidelberg.de

1 Problem 2-1 Erdos-Renyi Network

Consider an Erdos-Renyi network with N = 80 nodes, connected to each other with probability p = 0.05.

1. What is (i) the expected number of links in the graph and (ii) the expected degree of a node?

   (i) The expected number of links in the graph:

   $$\langle L \rangle = p\frac{N(N-1)}{2} = 0.05\frac{80 \cdot (80-1)}{2} = 158 \tag{1}$$

   The expected number of links in the graph is 158.

   (ii) The expected degree of a node:

   $$\langle k \rangle = p(N-1) = 0.05 \cdot (80-1) = 3.95 \tag{2}$$

   The expected degree of a node in the network is 3.95.

2. In which regime is the network?

   $\langle k \rangle$ is 3.95, thus greater than 1 and not in the subcritical regime.

   $\langle k \rangle < ln(N)$ since $3.95 < 4.38$, thus it is not in the connected regime.

   This means that the network is in the **supercritical regime**.

3. What is the probability to find exactly $L = 200$ links in the graph?

   $$p_L = \binom{\frac{N(N-1)}{2}}{L} p^L (1-p)^{(N(N-1)/2)-L} \tag{3}$$

   $$p_{200} = \binom{\frac{80(80-1)}{2}}{200} 0.05^{200} (1-0.05)^{(80(80-1)/2)-200} \approx 1.26e^{-4} \tag{4}$$

   The probability to find exactly 200 links in the graph is arround $1.26e^{-4}$.

4. What is the probability that a node i in the graph has degree $k_i = 5$ (using the binomial distribution)?

   $$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k} \tag{5}$$

Node	Degree
1	3
2	2
3	2
4	1
5	5
6	1
7	1
8	3
9	1
10	1
11	1
12	1

Table 1

$$p_5 = \binom{80-1}{5} 0.05^5 (1-0.05)^{80-1-5} \approx 0.158 \tag{6}$$

The probability of a node i having a degree of 5 is 0.158.

5. Use maximum likelihood estimation to estimate the model parameters (N, p) for the shown graph.

   From Table 1 follows that the average degree of the network is $\langle k \rangle \approx 0.54$.

$$\langle k \rangle = p(N-1) \tag{7}$$

$$p = \frac{\langle k \rangle}{N-1} = \frac{0.54}{12-1} \approx 0.049 \tag{8}$$

   The model parameters are $N = 12$ and $p = 0.049$.

   Alternative:

   $p(G)$ is maximized for

$$p = \frac{m}{n(n-1)} \tag{9}$$

where m is the number of edges and n is the number of vertices in the graph.

$$p = \frac{11}{12(12-1)} \approx 0.083 \tag{10}$$

Following this method, p would be 0.083.