Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

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Assignment 5

Growth and Preferential Attachment

`https://github.com/nilskre/CNA_assignments`

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1 Problem 5-1 Configuration Model

1. tbd

2 Problem 5-2 Role of Preferential Attachment

To verify that preferential attachment is a necessary ingredient to build a model of a scale-free network (cf. Slide 5-19), Barabási and Albert also studied alternative versions of their algorithm. In the following, we want to explore how the omission of preferential attachment affects the degree distribution. Note that all tasks can be solved independently! If you skip a task, you may use the corresponding result without a proof. Assume a network is generated as follows: We start with m_0 initial nodes. Links between those nodes are chosen arbitrarily, as long as each node has a link. At each time step, a new node with m links connecting to existing nodes is added. However, in contrast to the model presented in the lecture, the probability that a link of a new node connects to node i does not depend on the degree k_i. We assume a uniform distribution, i.e., all nodes are equally likely and the probabilities are all equal to:

$$\prod = \frac{1}{m_0 + t - 1} \tag{1}$$

1. Show that each node i acquires links according to the following differential equation:

$$\frac{dk_i}{dt} \approx \frac{m}{m_0 + t - 1} \tag{2}$$

Equation (5.2) in Slide 5-8 describes the rate at which existing node i acquires links:

$$\frac{dk_i}{dt} = 1 - (1 - \prod(k_i))^m \approx m \prod(k_i) \tag{3}$$

For $\prod$ we insert equation 1 from above:

$$\frac{dk_i}{dt} = m\frac{1}{m_0 + t - 1} = \frac{m}{m_0 + t - 1} \tag{4}$$

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2. Show that by separating variables and integrating, we obtain the following formula for $k_i(t)$:

$$k_i(t) = m\left[1 + log(\frac{m_0 + t - 1}{m_0 + t_i - 1})\right] \tag{5}$$

Hint: Note that node i joins the network at time t_i with m links, i.e., $k_i(t_i) = m$.

Start with equation 2:

$$\frac{dk_i}{dt} \approx \frac{m}{m_0 + t - 1}$$

Move dt to the other side by multiplying:

$$dk_i \approx \frac{m}{m_0 + t - 1} * dt$$

Integrate:

$$\int_m^{k_i(t)} \frac{1}{k_i}\, dk_i = \int_{t_i}^{t} \frac{m}{m_0 + t - 1}\, dt$$

Build Antiderivatives F ($f(x) = \frac{n}{x}$ => $F(x) = n * log(x)$) and solve integral by $\int_a^b f(x)\, dx = F(b) - F(a)$:

$$log(k_i(t)) - log(m) = (m * log(m_0 + t - 1)) - (m * log(m_0 + t_i - 1))$$

Now extract m on the right side and use log calculation rules $log(u) - log(v) = \frac{log(u)}{log(v)}$

$$log(k_i(t)) - log(m) = m * (\frac{log(m_0 + t - 1)}{log(m_0 + t_i - 1)})$$

Move $log(m)$ to the other side by addition:

$$log(k_i(t)) = m * (\frac{log(m_0 + t - 1)}{log(m_0 + t_i - 1)}) + log(m)$$

Remove log from both sides:

$$k_i(t) = m * (\frac{log(m_0 + t - 1)}{log(m_0 + t_i - 1)}) + m$$

Reformulate:

$$k_i(t) = m * \left[1 + log(\frac{m_0 + t - 1}{m_0 + t_i - 1})\right]$$

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3. Starting from the assertion $k_i(t) < k$, show that nodes have a degree smaller than k exactly if:

$$t_i > 1 - m_0 + (m_0 + t - 1)exp(1 - \frac{k}{m}) \tag{6}$$

Starting from $k_i(t) < k$, insert equation 5 for $k_i(t)$:

$$m * \left[1 + log(\frac{m_0 + t - 1}{m_0 + t_i - 1})\right] < k$$

Divide by m:

$$1 + log(\frac{m_0 + t - 1}{m_0 + t_i - 1}) < \frac{k}{m}$$

Substract $\frac{k}{m}$:

$$1 - \frac{k}{m} + log(\frac{m_0 + t - 1}{m_0 + t_i - 1}) < 0$$

Split log into two parts $(log(\frac{u}{v}) = log(u) - log(v))$:

$$1 - \frac{k}{m} + log(m_0 + t - 1) - log(m_0 + t_i - 1) < 0$$

Use exponential function $(k = log(c) <=> e^k = c)$:

$$e^{1-\frac{k}{m}} + (m_0 + t - 1) - (m_0 + t_i - 1) < 0$$

Move t_i to the other side (first: $+(m_0 + t_i - 1)$ second: $+1 - m_0$):

$$e^{1-\frac{k}{m}} + (m_0 + t - 1) < m_0 + t_i - 1$$

$$1 - m_0 + (m_0 + t - 1) + e^{1-\frac{k}{m}} < t_i$$

Somewhere in our derivation there is still a fault left, because our final expression slightly differs from equation 6 (the exponential term is summed and not multiplied).
