Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

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Assignment 4

Scale-Free Networks

`https://github.com/nilskre/CNA_assignments`

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1 Problem 4-1 Power laws

Consider the degree distribution functions p_k of the following two undirected networks:

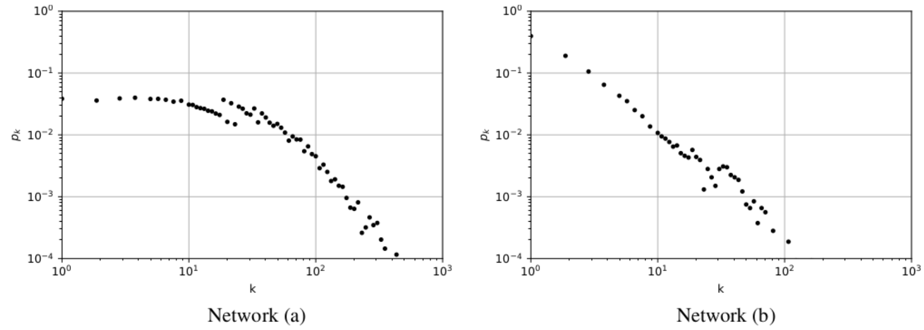

Figure 1: A simple graph with 4 connected triples.

1. One of these networks is approximately scale-free, the other is not. Identify the scale-free network and explain how you came to your conclusion.

   Network b is is scale free. The figure 1 shows the log-log plots of the degree distribution of the two networks a and b. For scale-free networks the degree distribution forms a straight line across the diagonal. This is given for network b.

2. A particular network is believed to have a degree distribution that follows a power law. A random sample of nodes is taken and their degrees are measured. The degrees of the first twenty nodes with degrees 10 or greater are:

   16 17 10 26 13 14 28 45 10 12 12 10 136 16 25 36 12 14 22 10

   For $K_{min} = 10$, estimate the exponent γ of the power law using the estimation method presented in the lecture. Hint: you only have to estimate the exponent (see Slide 4-33)! Calculate the error σ of your estimation using the equation $\sigma = \frac{\gamma-1}{\sqrt{N}}$

   The following solution is based on Slide 4-28 Estimating the degree exponent. From step 1. (Estimate value of the degree exponent corresponding to K_{min}) following formula is used:

$$\gamma = 1 + N\left[\sum_{i=0}^{N}(ln(\frac{k_i}{K_{min}-\frac{1}{2}}))\right]^{-1} \tag{1}$$

Now the values $N = 20$, k_i from the list above and $K_{min} = 10$ are inserted:

$$\gamma = 1 + 20\left[ln(\frac{16}{20-\frac{1}{2}}) + ln(\frac{17}{20-\frac{1}{2}}) + ln(...) + ...\right]^{-1}$$

$$\gamma \approx -14,17$$

Based on N and γ the error σ is calculated:

$$\sigma = \frac{\gamma - 1}{\sqrt{N}} = \frac{-14,17-1}{\sqrt{20}} = -3,39$$