Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

Prof. Dr. Michael Gertz

Assignment 7
Degree Assortativity and Robustness

https://github.com/nilskre/CNA_assignments

Team Member: Patrick Günther, 3660886,
               Applied Computer Science
               rh269@stud.uni-heidelberg.de

Team Member: Felix Hausberger, 3661293,
               Applied Computer Science
               eb260@stud.uni-heidelberg.de

Team Member: Nils Krehl, 3664130,
               Applied Computer Science
               pu268@stud.uni-heidelberg.de

1 Problem 7-2 Molloy-Reed Criterion

Consider a configuration model network that has nodes of degree 1, 2, and 3 only, with probabilities p_1, p_2, and p_3, respectively. The degree distribution is given by:

$$p_k = \delta_{k,1}p_1 + \delta_{k,2}p_2 + \delta_{k,3}p_3, \begin{cases} \delta_{k,1} = 3 & \text{if } k = 1 \\ \delta_{k,2} = 2 & \text{if } k = 2 \\ \delta_{k,3} = 2 & \text{if } k = 3 \end{cases}$$

1. Compute the first moment $\langle k \rangle$ and the second moment $\langle k^2 \rangle$ of the degree distribution.

   We assume $\delta_{k,k'}$ to be the dirac-delta-function. For the first and second moment is follows:

   $$\langle k \rangle = \sum_{k=1}^{3} k p_k = 3p_1 + 4p_2 + 6p_3$$

   $$\langle k^2 \rangle = \sum_{k=1}^{3} k^2 p_k = 3p_1 + 8p_2 + 18p_3$$

2. Using the Molloy-Reed criterion, show that there is a giant component if and only if $p_1 < 3p_3$.

   The Molloy-Reed criteria propagates a giant component exists in case $\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$. κ can be calculated as:

   $$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} = \frac{3p_1 + 8p_2 + 18p_3}{3p_1 + 4p_2 + 6p_3}$$

   One way for κ being greater than 2 is when all components of the sum in the numerator are at least twice as big as the matching component of the sum in the denominator (with one component being slightly bigger than twice as big for κ being strictly bigger than 2). This means $6p_3 > 3p_1$ and therefore $2p_3 > p_1$. We therefore assume the thesis in the task desciption to be wrong.

3. In terms of the structure of the network, discuss the meaning of the condition $p_3 < 3p_1$. Why does the result not depend on p_2?

The fact that in the calculation of κ the component of p_2 already goes in twice as big in the numerator ($8p_2 > 4p_2$), makes the goal of $\kappa > 2$ only dependent on the scale of p_1 compared to p_3. If $p_3 < 3p_1$ holds, a giant component only exists in case $\frac{1}{3}p_3 < p_1 < 2p_3$. Other than that, we cannot really see what $p_3 < 3p_1$ should lead to.

2 Problem 7-4 Random Failures in Uncorrelated Networks

Compute the critical threshold f_c for each of the following degree distributions, under the assumption that the networks do not exhibit any degree correlation.

1. Poisson distribution, i.e.,

$$p_k = e^{-\mu}\frac{\mu^k}{k!}$$

2. Discrete exponential distribution, i.e.,

$$p_k = (1 - e^{-\lambda})e^{-\lambda k}$$

3. Dirac delta distribution, i.e.,

$$p_k = \delta_{k,k_0} = \begin{cases} 1 & \text{if } k = k_0, \\ 0 & \text{otherwise.} \end{cases}$$

Discuss the consequences of your results for network robustness.

Hint: You may use the first and second moment from the lecture or other literature without a proof