Heidelberg University
Institute of Computer Science
Database Systems Research Group

Lecture: Complex Network Analysis

Prof. Dr. Michael Gertz

Assignment 1

Graph Theory and Networks in Python

`https://github.com/nilskre/CNA_assignments`

Team Member: Patrick Günther, 3660886,
Applied Computer Science
rh269@stud.uni-heidelberg.de

Team Member: Felix Hausberger, 3661293,
Applied Computer Science
eb260@stud.uni-heidelberg.de

Team Member: Nils Krehl, 3664130,
Applied Computer Science
pu268@stud.uni-heidelberg.de

1 Problem 3-2 Parametrized Random Networks

1. Determine the average degree. We use the equation $\langle k \rangle = p(N-1)$:

   - $a = 0.5$, $z = 1$

     $$\langle k \rangle = \lim_{N \to \infty} \frac{0.5}{N}(N-1) = \lim_{N \to \infty} 0.5 - \frac{0.5}{N} = 0.5 \tag{1}$$

     $\langle k \rangle = 0.5$ and $\langle k \rangle < 1$„ which means there is **no GC**.
   - $a = 2$, $z = 1$

     $$\langle k \rangle = \lim_{N \to \infty} \frac{2}{N}(N-1) = \lim_{N \to \infty} 2 - \frac{2}{N} = 2 \tag{2}$$

     $\langle k \rangle = 0.5$ and $\langle k \rangle > 1$„ which means there is a **GC**.
   - $a > 0$, $z = 2$

     $$\langle k \rangle = \lim_{N \to \infty} \frac{a}{N^2}(N-1) = \lim_{N \to \infty} \frac{a}{N} - \frac{a}{N^2} = 0 \tag{3}$$

     $\langle k \rangle = 0$ and $\langle k \rangle < 1$„ which means there is **no GC**.
   - $a > 0$, $z = 0.5$

     $$\langle k \rangle = \lim_{N \to \infty} \frac{a}{\sqrt{N}}(N-1) = \lim_{N \to \infty} a\sqrt{N} - \frac{a}{\sqrt{N}} = \infty \tag{4}$$

     $\langle k \rangle = \infty$ and $\langle k \rangle > 1$„ which means there is a **GC**.

2. Determine the average degree. Again with $\langle k \rangle = p(N-1)$:

   $$\langle k \rangle = \lim_{N \to \infty} \frac{a}{N^z}(N-1) = \lim_{N \to \infty} aN^{1-z} - \frac{a}{N^z} \tag{5}$$

   This means:

   $$\langle k \rangle = \begin{cases} \infty, & \text{if } z < 1 \\ 0, & \text{if } z \geq 1 \end{cases}$$

3. Determine the conditions on a and z for which these random networks are critical, again in the limit $N \to \infty$.

   Networks are critical, if $\langle k \rangle = 1$

   We have this equation from the previous subtask:

   $$\langle k \rangle = \lim_{N \to \infty} aN^{1-z} - \frac{a}{N^z} \tag{6}$$

   Since here $1 - z$ must be 0 and aN^0 must be 1, the conditions are: z must be 1 and a must be 1.