Available online at www.sciencedirect.com

SCIENCE DIRECT®

Pattern Recognition Letters 25 (2004) 865–874

Pattern Recognition Letters

www.elsevier.com/locate/patrec

Threshold selection using fuzzy set theory

Tamalika Chaira *, A.K. Ray

Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur 721302, India

Received 8 September 2003; received in revised form 21 January 2004
Available online 3 March 2004

Abstract

This paper introduces an image thresholding method using four types of fuzzy thresholding methods taking Gamma membership into account for determining the membership values of the pixels of an image. The effectiveness of this method is illustrated by using a set of images having various types of histograms. A comparative study on images has also been done.

The experimental results have demonstrated good performance in unilevel, bilevel and trilevel thresholding.

Keywords: Fuzzy divergence; Indices of fuzziness; Fuzzy compactness; Membership function; Gamma distribution; Thresholding

1. Introduction

The problem of gray level thresholding plays a key role in image processing and recognition. Thresholding is a process of partitioning a digital image into mutually exclusive and distinctive regions.

In order to extract an object from background in a multilevel gray scale image, it is necessary to compute an appropriate threshold. It is widely accepted that the selection of correct threshold will lead to good extraction of objects. At the same time selecting a threshold is not an easy problem.

Segmentation of multimodal image involves the task of assigning each pixel of an image into several regions including the background region. This process becomes complex when the image quality is not good i.e. the image possesses noise or imprecision, represented as fuzziness. The nature of this ambiguity (fuzziness) in the image therefore arises from the uncertainty present. When the regions of an image are ill defined, it is appropriate to avoid crisp segmentation. The segments in that case may be viewed as fuzzy subsets of the image.

A number of excellent investigations on various thresholding techniques have been reported in the literature. Kapur et al. (1985) and Li and Lee (1992) used the concept of entropy while Brink and Pendcock (1996) and Abutaleb (1989) used two-dimensional entropy threshold an image. Otsu (1979) suggested the threshold detection by maximizing the class separability, which was based on within class, between class and total variance of

* Corresponding author. Address: C/O B.B. Chaira, A-7 Aurobinda Nagar, Judges Court Road, Midnapore (W), West Bengal 721101, India.

E-mail addresses: tamalika@lycos.com, tamalika@mailcity.com (T. Chaira), akray@ece.iitkgp.ernet.in (A.K. Ray).

0167-8655/$ - see front matter
doi:10.1016/j.patrec.2004.01.018

gray levels. Whatmough (1991) used the exponential hull method, which is a variation of convex hull for concavity analysis. Kittler and Illingworth (1985) minimized the classification error probability based on the condition that a mixture of Gaussian densities governs the histograms.

Several researchers have investigated fuzzy based thresholding techniques. Pal and Rosenfeld (1988) optimized the fuzzy compactness using the Zadeh's S-function for the membership evaluation for image thresholding. Huang and Wang (1995) used Shannon and Yager's measure for fuzzy thresholding. Ramar et al. (2000) used the neural network for selecting the best threshold using various fuzzy measures. Fuzzy homogeneity vectors and fuzzy co-occurrence matrix was reported by Cheng and Chen (1997) for image thresholding.

In this paper the image has been thresholded using several methods viz. fuzzy divergence proposed by Chaira and Ray (2003), linear and quadratic indices of fuzziness, fuzzy compactness and fuzzy similarity. Gamma membership function proposed by Chaira and Ray (2003) has been used for finding the membership values of the pixels of the image. The minimization of the indices of fuzziness yields the appropriate value of the threshold. In a likewise fashion, the maximization of two other measures, viz, fuzzy compactness and fuzzy similarity between an image thresholded by an ideal value and the same image thresholded by any arbitrary value (practically thresholded image) yields the appropriate threshold.

2. Fuzziness and the membership function

Let A be an image of size $M \times M$ having L gray levels where $\mu_A(a_{ij})$, $0 \leqslant \mu_A(a_{ij}) \leqslant 1$, is the membership value of (i,j)th pixel in A.

Mathematically the fuzzy image may be represented as $A = \{a_{ij}, \mu_A(a_{ij})\}, \forall a_{ij} \in A$.

Let the count (f) denote the number of occurrences of the gray level f in the image. Given certain threshold value t, which separates the object and the background, the average gray level of the background region and object region respectively is given by the relation:

$$\mu_0 = \frac{\sum_{f=0}^{t} f \cdot \mathrm{count}(f)}{\sum_{f=0}^{t} \mathrm{count}(f)} \tag{1}$$

$$\mu_1 = \frac{\sum_{f=t+1}^{L-1} f \cdot \mathrm{count}(f)}{\sum_{f=t+1}^{L-1} \mathrm{count}(f)} \tag{2}$$

The membership value of a pixel in a certain region depends on its affinity to the region to which it belongs. The membership values of the pixels have been determined using Gamma distribution, which is another way of representing the membership function as has been described below.

2.1. Gamma membership function

The general formula for the probability density function of the Gamma distribution is

$$f(x) = \frac{\left(\frac{(x-\mu)}{\beta}\right)^{\gamma-1} \exp\left(-\frac{(x-\mu)}{\beta}\right)}{\Gamma(\gamma)}, \quad x \geqslant \mu, \ \gamma, \beta > 0 \tag{3}$$

where γ is the shape parameter, μ is the location parameter, β is the scale parameter and Γ is the Gamma function.

When $\mu \neq 0$, $\beta = 1$ and $\gamma = 1$, the Gamma distribution in Eq. (3) takes the form

$$f(x) = \exp[-(x-\mu)] \quad \text{as } \Gamma(1) = 1 \tag{4}$$

The membership function $\mu_A(a_{ij})$ of the object region may be computed from Eq. (4) by choosing $\mu = \mu_1$, the average gray level of the object region, and is given by

$$\mu_A(a_{ij}) = \exp(-c \cdot |a_{ij} - \mu_1|) \quad \text{if } a_{ij} > t, \text{ for object region} \tag{5}$$

In a likewise manner, the membership function $\mu_A(a_{ij})$ of the background region may be computed from Eq. (4) by choosing $\mu = \mu_0$, the average gray level of the background region. This is given by

$$\mu_A(a_{ij}) = \exp(-c \cdot |a_{ij} - \mu_0|) \quad \text{if } a_{ij} \leqslant t, \text{ for background region} \tag{6}$$

where t is any chosen threshold as stated above.

It may be pointed out that in the membership function, the constant 'c' has been taken to ensure membership value of the gray level feasible in the range $[0,1]$. Here 'c' may been chosen as $c = 1/(f_{\max} - f_{\min})$, where $f_{\min}$ and $f_{\max}$ are the minimum and maximum gray level in the image respectively. The absolute value of the distance between the mean of the region to which a pixel belongs and the gray level of that pixel has been considered for computing the membership function.

For trilevel thresholding i.e. where there are three regions in the image, two threshold values t_1 and t_2 have been selected such that $0 \leqslant t_1 < t_2 \leqslant L-1$, where L is the maximum gray level of the image. Extending the concept of bilevel thresholding, the membership function, in case of trilevel thresholding will take the form

$$\begin{aligned}\mu_A(a_{ij}) &= \exp(-c_1 \cdot |a_{ij} - \mu_0|) \quad \text{if} \quad a_{ij} \leqslant t_1 \\ &= \exp(-c_1 \cdot |a_{ij} - \mu_1|) \quad \text{if } t_1 < a_{ij} \leqslant t_2 \\ &= \exp(-c_1 \cdot |a_{ij} - \mu_2|) \quad \text{if } a_{ij} > t_2\end{aligned} \tag{7}$$

where μ_0, μ_1 and μ_2 are the average gray levels for the three regions separated by the thresholds t_1 and t_2. and the constant 'c_1' is like 'c' in Eq. (7).

3. Types of fuzzy threshold methods

Here the proposed fuzzy methods that have been used for finding the thresholds by optimization, are now discussed below.

3.1. Fuzzy divergence

Fan and Xie (1999) proposed fuzzy divergence from fuzzy exponential entropy by using a single row vector. The authors (2003) extended the concept of Fan and Xie to an image which may be represented by a matrix for image thresholding.

In an image of size $M \times M$ with L distinct gray levels, the fuzzy divergence between images A and B is given by

$$\begin{aligned}D(A,B) = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1}\big[2 - (1 - \mu_A(a_{ij}) \\ + \mu_B(b_{ij})) \cdot e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} - (1 - \mu_B(b_{ij}) \\ + \mu_A(a_{ij})) \cdot e^{\mu_B(b_{ij}) - \mu_A(a_{ij})}\big]\end{aligned} \tag{8}$$

$i,j = 0,1,2\ldots,(M-1)$ and $\mu_A(a_{ij})$ and $\mu_B(b_{ij})$ are the membership value of the pixel where a_{ij} and b_{ij} are the (i,j)th pixel of the image A and B.

3.2. Index of fuzziness

The index of fuzziness has been defined as

$$I(A) = (2/n^k)d(A,\overline{A})$$

where $d(A,\overline{A})$ denotes the distance between A and its nearest ordinary set $\overline{A}$.

k denotes the type and distance used, e.g. for $k=1$ for Hamming distance, $k=0.5$ for Euclidean distance.

The linear index of fuzziness is defined as

$$\begin{aligned}\text{L.I.} &= (2/n)\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}|\mu_A(a_{ij}) - \mu_{\overline{A}}(a_{ij})| \\ &= (2/n)\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}\min(\mu_A(a_{ij}),(1-\mu_A(a_{ij})))\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mu_{\overline{A}}(a_{ij}) &= 0 \quad \text{if } \mu_A(a_{ij}) \leqslant 0.5 \\ &= 1 \quad \text{if } \mu_A(a_{ij}) > 0.5\end{aligned}$$

The quadratic index of fuzziness is defined as

$$\begin{aligned}\text{Q.I.} &= (2/\sqrt{n})\left(\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}(\mu_A(a_{ij}) - \mu_{\overline{A}}(a_{ij}))^2\right)^{1/2} \\ &= (2/\sqrt{n})\left(\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}(\min(\mu_A(a_{ij}),(1-\mu_A(a_{ij}))))^2\right)^{1/2}\end{aligned} \tag{10}$$

In the present work the method of threshold selection has been used using the indices of fuzziness, taking into account the Gamma membership function as stated above in Section 2.

3.3. Fuzzy similarity measure

In this work a new type similarity measure for fuzzy sets has been used for thresholding the image where the images are considered as fuzzy image. Wang (1997) proposed a similarity measure between two fuzzy sets A and B given by the relation.

$$S_1(A,B) = (1/n) \cdot \sum_{i=0}^{N-1} \frac{\min(\mu_A(a_i), \mu_B(b_i))}{\max(\mu_A(a_i), \mu_B(b_i))}$$

Here the concept of Wang has been extended to images for image thresholding using Gamma membership function. The similarity between two images, A and B, may be given as

$$S_1(A,B) = (1/n) \cdot \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} \frac{\min(\mu_A(a_{ij}), \mu_B(b_{ij}))}{\max(\mu_A(a_{ij}), \mu_B(b_{ij}))},$$
$$n = M \times M \tag{11}$$

where $\mu_A(a_{ij})$ and $\mu_B(b_{ij})$ are the membership values of the (i,j)th pixels i.e. a_{ij} and b_{ij} of the images A and B respectively. The image B is the ideally segmented image that has been described in the Section 4.1 below. If the image A is almost towards the ideally segmented image, the similarity value is maximum and the threshold value corresponding to the maximum value, is the optimum threshold.

3.4. Geometry of fuzzy sets

Rosenfeld (1984) and Pal and Rosenfeld (1988) used the concepts of digital picture geometry i.e. fuzzy compactness to threshold an image.

A fuzzy image subset is characterized by μ_A (a matrix) or simply μ (for simplicity) whose elements are the membership values of the pixels of the image A. If μ is a piecewise constant as in a digital image, then the area of μ i.e. $a(\mu)$ is the weighted sum of the areas of the regions on which μ has constant non-zero values.

μ_{ij} is the (i,j)th element of the membership matrix μ formed from the membership values of the pixel a_{ij} i.e. $\mu_A(a_{ij})$ of image A. Here image name A has been omitted for simplicity.

$$a(\mu) = \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_{ij} \tag{12}$$

For a piecewise constant, perimeter of μ has been defined as

$$P(\mu) = \sum_{i=1}^{M} \sum_{j=1}^{M-1} |\mu_{ij} - \mu_{i,j+1}| + \sum_{j=1}^{M} \sum_{i=1}^{M-1} |\mu_{ij} - \mu_{i+1,j}| \tag{13}$$

The compactness of μ has defined as

$$\mathrm{Comp}(\mu) = a(\mu)/p^2(\mu) \tag{14}$$

Here we have used maximizing the fuzzy compactness of fuzzy image subset for threshold selection using Gamma membership function, derived from Gamma distribution described in Section 2.

4. Method of computation and discussion

Here the computation algorithm for threshold selection has been described for all the four fuzzy measures. The reason for selecting the threshold corresponding to the minimum or the maximum value depending on the measures used has also been discussed.

4.1. Using fuzzy compactness

Threshold region has been chosen from the searching strategy as explained in the Section 5. For each threshold, the membership values of all the pixels in the image have been found out using the above procedures described in Section 2 and also the compactness of μ has been found out from Eq. (14).

While using the compactness from Eq. (14), the value is very large for ideally thresholded image where the membership values of the pixels are all 1 (as area is large and perimeter is very less). *Ideally segmented image* is that image which is exactly thresholded so the pixels, which are in the object or the background region, are totally in their regions. For ideally thresholded image, all the object pixels should contribute more to the object regions i.e. the membership values of all the pixels to the object class μ_{ij}^{obj} should be 1. Similarly the membership values of the pixels in the background region μ_{ij}^{back} to the background class is equal to 1. Thus for any threshold, the more the pixel membership values are towards 1, the better is the segmentation. This ensures the belongingness of object pixels to the object region and background pixels to the background region and consequently the fuzzy area becomes large and fuzzy perimeter is less for digital image (piecewise constant). This implies that the maximization of compactness is the criterion of threshold. The resulting thresholded image becomes equivalent to the ideally thresholded image. In this way for each threshold, compactness has been found out. Maximum compactness value has been selected and the corresponding gray level is the optimum thresholds.

4.2. Using indices of fuzziness

Threshold region has been chosen from the searching strategy.

While using the index of fuzziness for segmentation from (Eqs. (9) and (10)) for each threshold, the distance between the membership value of the pixels of an image and its nearest ordinary set has been found out. Nearest ordinary set is either 0 or 1 depending on the membership values less than or greater than 0.5. Here while using the Gamma membership function, another constant term of $\ln(0.5) = -0.6931$ has been used to control the membership values in the range $0.5 \leqslant \mu_A(i,j) \leqslant 1$ or $[0.5, 1]$. So the membership function from Eqs. (5) and (6) has been remodified as

$$\begin{aligned}\mu_A(a_{ij}) &= \exp(-0.6931 \cdot c \cdot |a_{ij} - \mu_0|) \\ &\qquad \text{if } a_{ij}, \quad \text{for background} \\ &= \exp(-0.6931 \cdot c \cdot |a_{ij} - \mu_1|) \\ &\qquad \text{if } a_{ij} > t, \quad \text{for object}\end{aligned} \tag{15}$$

where t is the any chosen threshold.

In this Gamma membership function, the membership values are all in the range $[0.5, 1]$. So the nearest ordinary set is 1 while computing the indices of fuzziness. For all threshold gray level in the searching region, the index of fuzziness has been found out using the above Eqs. (9) and (10). Then the minimum value among all the indices of fuzziness has been selected and the corresponding gray level has been selected for threshold. For any threshold, when the membership value is more towards 1, then the distance from the nearest set i.e. 1 is less, which implies that the index of fuzziness is minimum. Thus selection of the minimum index of fuzziness value means that the separation between the calculated thresholded image and the ideally thresholded image is less.

4.3. Using fuzzy similarity

While using the fuzzy similarity measure, membership values have been found out using the above two procedure. Similarity here means that the thresholded image is towards the ideally segmented image. Now substituting $\mu_B(b_{ij}) = 1$ in Eq. (11) (as for an ideally thresholded image, the membership values are all 1 where the object pixels lie in the object region and the background pixels lie in the background region), the measure has been modified as:

$$\begin{aligned}S_1(A,B) &= (1/n) \cdot \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} \frac{\min(\mu_A(a_{ij}), 1)}{\max(\mu_A(a_{ij}), 1)} \\ &= (1/n) \cdot \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} \mu_A(a_{ij})\end{aligned} \tag{16}$$

For each threshold the similarity values have been found out. When the similarity value of a segmented image is more towards the ideally segmented image for a particular threshold, the segmentation may be considered as extremely good. So the optimum threshold is the threshold value at which the fuzzy similarity measure is maximum. This implies that the thresholded image is almost towards the ideally thresholded image.

5. Experimental results and discussion

Four sets of the thresholded images obtained using each of the four measures discussed in the Section 3 have been presented here.

Again the threshold value selected is either minimum or the maximum value depending on the measure we are dealing with. While using divergence, linear index and quadratic index, the minimum value from the plot have been selected and while using fuzzy compactness, fuzzy similarity measure maximum value from the plots have been selected. To accommodate all the measures in the same graph, all the fuzzy measure values have been appropriately scaled i.e. fuzzy divergence values are divided by 1000, fuzzy similarity values are divided by 10 and fuzzy compactness values are multiplied by 10. As the plot of fuzzy similarity is not so clear when all the measures are plotted together, so a plot of fuzzy similarity vs. gray level is shown separately (when not divided by 10).

Our results have been compared with the Huang's method on Shannon's entropy, where he used inverse membership function and also fuzzy divergence method by the authors. The time complexity depends on the searching process.

In a multimodal histograms, a selection strategy searches threshold values in the region between the two consecutive peaks of the histogram as in Fig. 1, while in unimodal thresholding linear search has been used from minimum to maximum value of the gray level.

As we have restricted the searching length so the time required is less than that of Huang's method whereas in Huang's method the searching length is from minimum gray level to maximum gray level.

The images in 'tif' format on which the thresholding have been performed are:

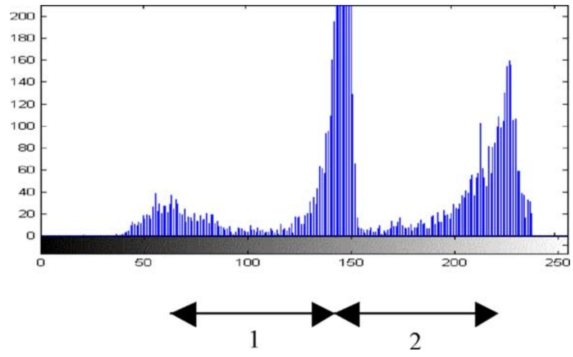


Fig. 1. "1 and 2" is the region of searching.

(i) Fig. 2(a) shows the '*Rice*' image of size 128×128. The corresponding thresholded images using fuzzy divergence, linear index, quadratic index, fuzzy similarity and fuzzy compactness are presented in Fig. 2(b)–(f). From the results, it has been observed that the threshold results, using fuzzy compactness (Fig. 2(f)) is not so better. It is somewhat more segmented. Huang's method (Fig. 2(g)) is almost the same as that of other fuzzy measures Fig. 2(b)–(e), which are better. The threshold values are displayed in Table 1, which are obtained from Graph 1.

(ii) '*Eight*' is a image of size 200×200. Its histogram is bilevel. It has two prominent peaks. From the thresholded images, it has been observed that results using all the measures are almost similar giving good results when com-

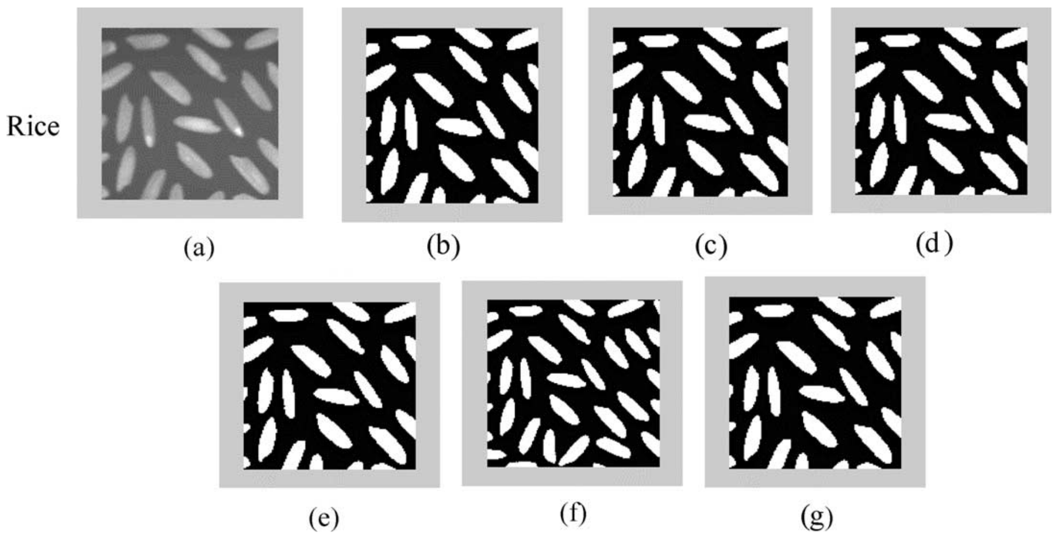
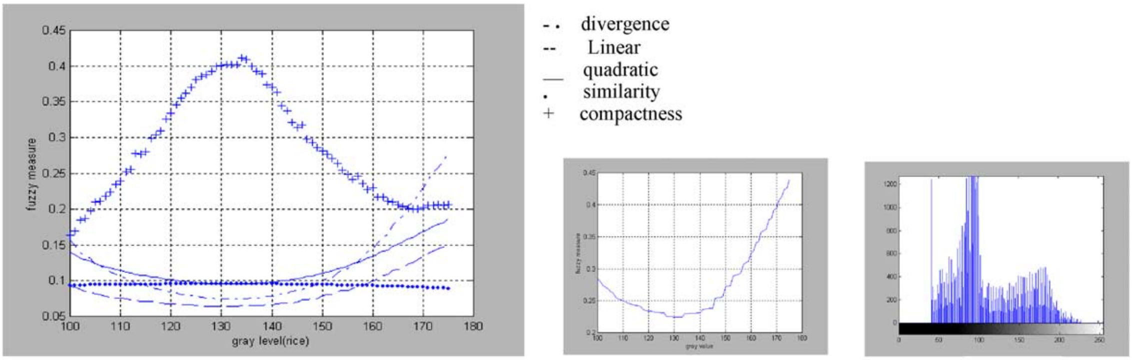


Fig. 2. (a) Input image 'rice', (b) thresholded using fuzzy divergence, (c) thresholded using quadratic index, (d) thresholded using linear index, (e) thresholded using fuzzy similarity, (f) thresholded using fuzzy compactness and (g) thresholded using Huang's method.

Table 1
Comparison table for various images using Gamma membership function

Image	Divergence	Linear index	Quadratic index	Fuzzy similarity	Fuzzy compactness	Huang's method
Rice	131, 132	131, 132	131, 132	131–132	134	131, 132
Eight	164, 165	172	164, 165	172	166–167	196
Shot 1	106, 180	108, 184	106, 180	106, 184	110, 180	104, 186
Rice (uni)	179	179	177–178	179	174	175



(a) (b) (c)

Graph 1. (a) Plots of gray level/fuzzy threshold methods on the 'rice' image, (b) plot using Huang method and (c) histogram of the image 'rice'.

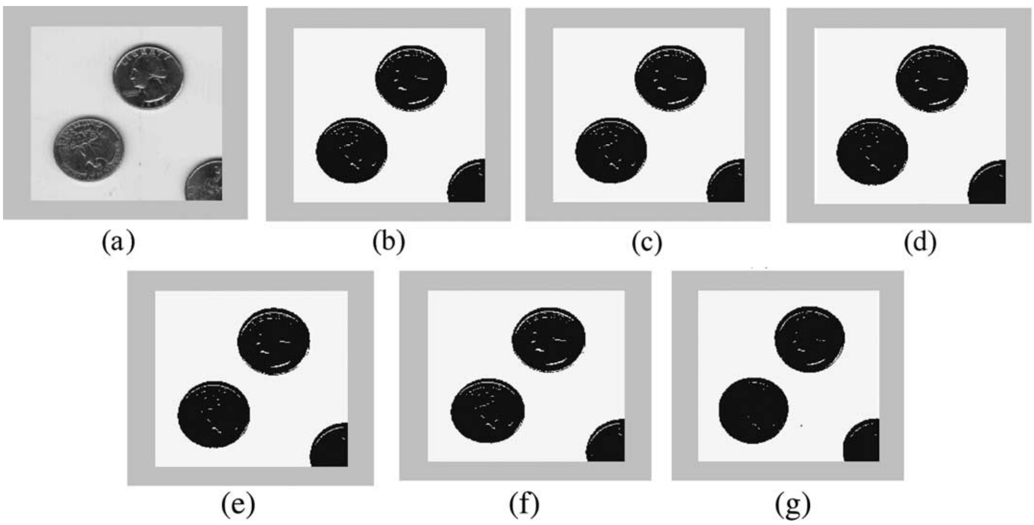
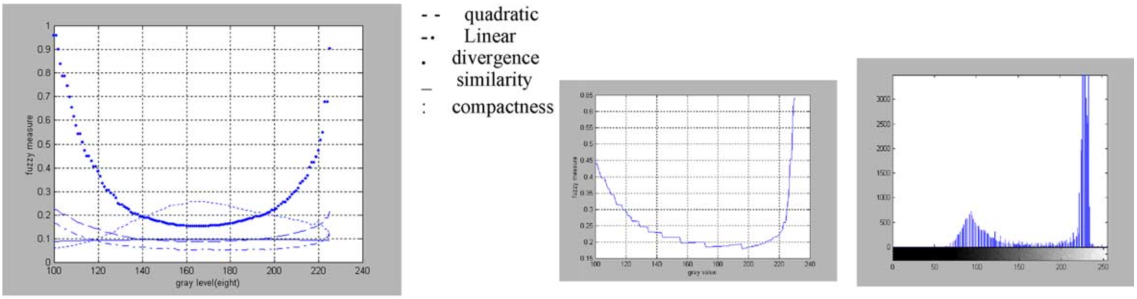


Fig. 3. (a) Input image 'eight', (b) thresholded using fuzzy divergence, (c) thresholded using quadratic index, (d) thresholded using linear index, (e) thresholded using fuzzy similarity, (f) thresholded using fuzzy compactness and (g) thresholded using Huang's method.

pared to Huang's method where the inside details are not in picture (Fig. 3(g)). The thresholded values are displayed in the Table 1, which are obtained from Graph 2.

(iii) Fig. 4 shows the threshold results of image of '*shot*'. It is an example of trilevel thresholding. It has three peaks and two valleys. The stream of circular shots in the image is embedded in another circular homogeneous region, while the background forms the third region in the



(a) (b) (c)

Graph 2. (a): Plots of gray level/fuzzy threshold methods on 'eight' image, (b) plot using Huang's method and (c) histogram of the image 'eight'.

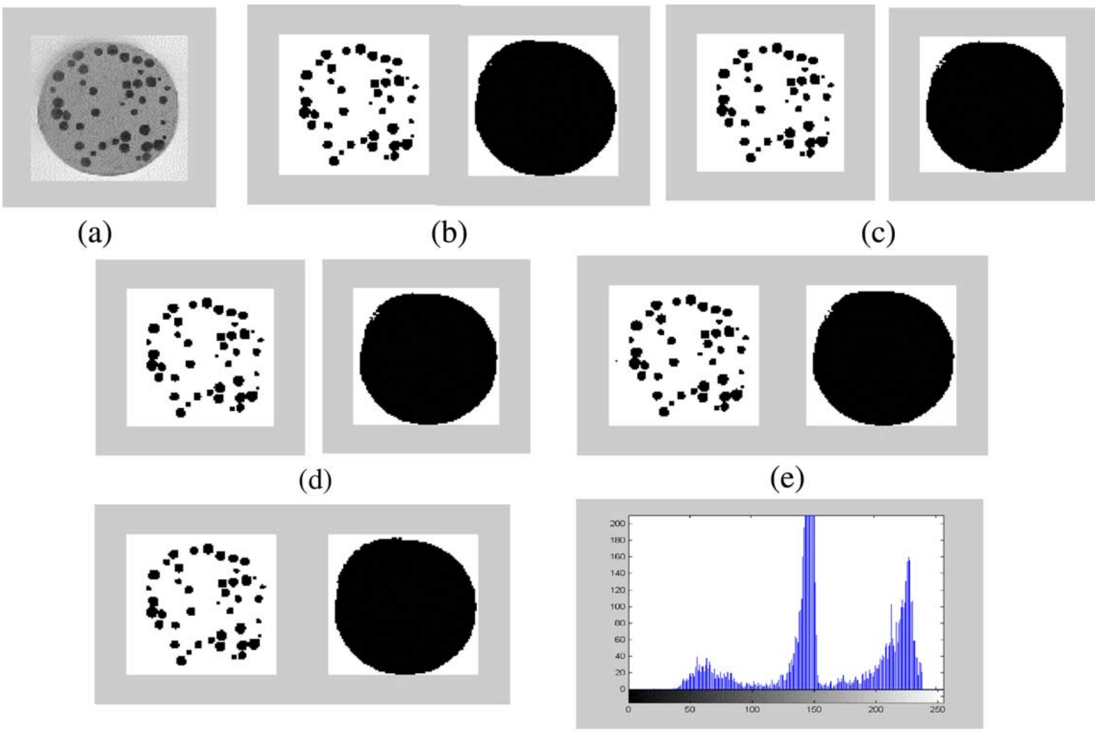


Fig. 4. (a) Input image 'shot', (b) thresholded using fuzzy divergence and quadratic index, (c) thresholded using linear index, (d) thresholded at fuzzy similarity, (e) thresholded using fuzzy compactness, (f) thresholded using Huang's method and (g) histogram of the input image.

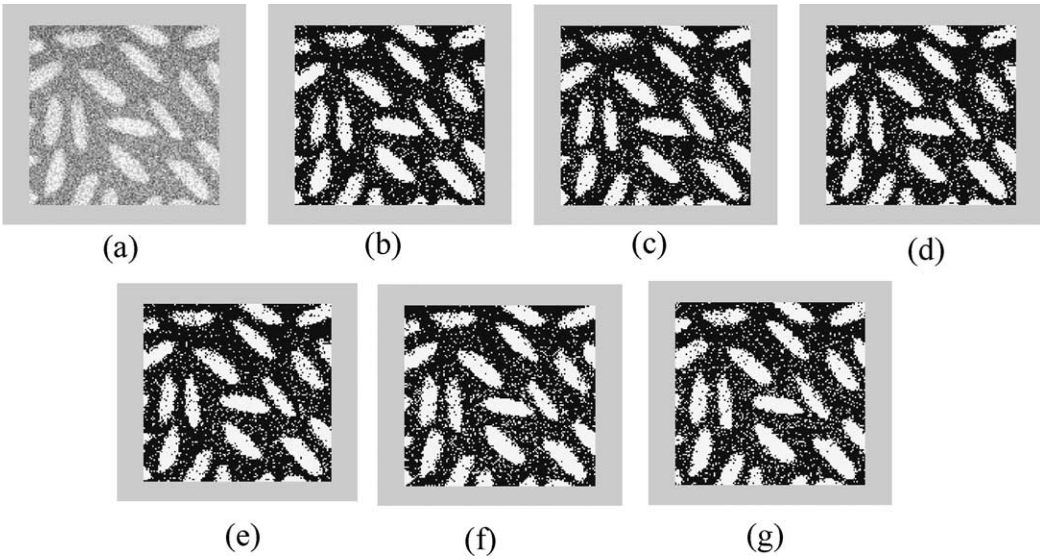
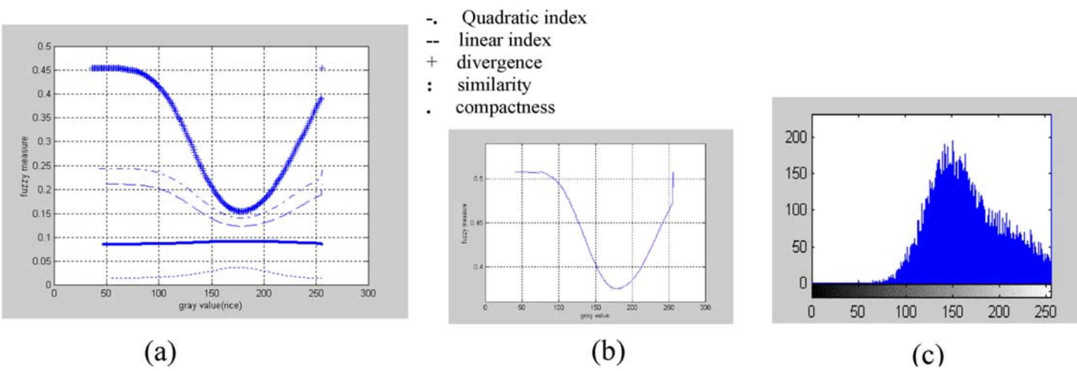


Fig. 5. (a) Input image 'rice' (unimodal), (b) thresholded using fuzzy divergence, (c) thresholded at quadratic index, (d) thresholded using linear index, (e) thresholded at fuzzy similarity, (f) thresholded using fuzzy compactness and (g) thresholded using Huang's method.



Graph 3. (a) Plots for gray level/fuzzy threshold methods on 'rice' mixed with noise image, (b) plot using Huang's method and (c) histogram of the image 'rice'.

image. The minor differences in the threshold results are not properly visible and the thresholds values are displayed in Table 1.

Experiment has also been performed on unimodal histogram. The unimodal histogram has been obtained by mixing Gaussian noise to the image 'rice' (Fig. 5). The threshold values are displayed in Table 1, which are obtained from Graph 3.

6. Conclusion

Based on the concept of fuzzy sets and the type of the membership function used i.e. the Gamma membership function, a new thresholding method has been proposed here. It utilizes the fuzzy methods of an input image to identify the appropriate threshold value. From the plots of various measures, the threshold values have been selected according to the measure used. The optimal threshold values obtained by various measures on the same image need not be unique. While using the indices of fuzziness, membership values are all greater than 0.5, so more the membership values, the less is the value of indices of fuzziness. Using fuzzy similarity, the more the thresholded image is towards the ideally segmented image, the more is the similarity. For fuzzy compactness, when the area becomes large and perimeter is less, i.e., more compactness, the thresholded image is almost towards the ideally thresholded image implying maximum compactness. Thus gray level corresponding to the maximum compactness is the optimum threshold. And while using fuzzy divergence, the more the thresholded image is towards the ideally thresholded image, the less is the divergence.

The proposed method, which is based on minimizing/maximizing the measure of fuzziness of an image, has demonstrated satisfactory performance in unimodal, bimodal and trimodal thresholding. The use of fuzzy range can help to locate effectively the deep valley in the histogram.

References

Abutaleb, A.S., 1989. Automatic thresholding of gray level pictures using two-dimensional entropy. Comput. Vision Graphics Image Process. 47, 22–32.

Brink, A.D., Pendcock, N.E., 1996. Minimum cross-entropy threshold selection. Pattern Recognition 29, 179–188.

Chaira, T., Ray, A.K., 2003. Segmentation using fuzzy divergence. Pattern Recognition Lett. 12 (24), 1837–1844.

Cheng, H.D., Chen, H.H., 1997. Image segmentation using fuzzy homogeneity criterion. Information Science 98, 237–262.

Fan, J., Xie, W., 1999. Distance measure and induced fuzzy entropy. Fuzzy Sets System 104, 305–314.

Huang, L.K., Wang, M.J., 1995. Image thresholding by minimizing the measure of fuzziness. Pattern Recognition 28 (1), 41–51.

Kapur, J.N., Sahoo, P.K., Wong, A.K.C., 1985. A new method of gray level picture thresholding using the entropy of the histogram. Comput. Vision, Graphics Image Process. 29, 273–285.

Kittler, J., Illingworth, J., 1985. On threshold selection using clustering criteria. IEEE Trans. System Man Cybernet. SMC-15, 652–655.

Li, E.H., Lee, C.K., 1992. Minimum cross-entropy thresholding. Pattern Recognition, 617–625.

Otsu, N., 1979. A Threshold selection method from gray level histograms. IEEE Trans. System Man Cybernet. 9, 62–66.

Pal, S.K., Rosenfeld, A., 1988. Image enhancement and thresholding by optimization of fuzzy compactness. Pattern Recognition Lett. 7, 77–86.

Ramar, K., Arumugam, S., Sivanandam, S.N., Ganesan, L., Manimegalai, D., 2000. Quantitative fuzzy measures for threshold selection. Pattern Recognition Lett. 21 (1), 1–7.

Rosenfeld, A., 1984. The fuzzy geometry of image subsets. Pattern Recognition Lett. 2, 311–317.

Wang, W.J., 1997. New similarity measures on fuzzy sets and fuzzy elements. Fuzzy Sets Systems 85, 305–309.

Whatmough, R.J., 1991. Automatic threshold selection from a histogram using the exponential hull. Graphical Models Image Process. 53, 592–600.