ROBOTICS SYSTEMS

ASSIGNMENT 2

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1. Introduction

1.1 The Robotic Arm

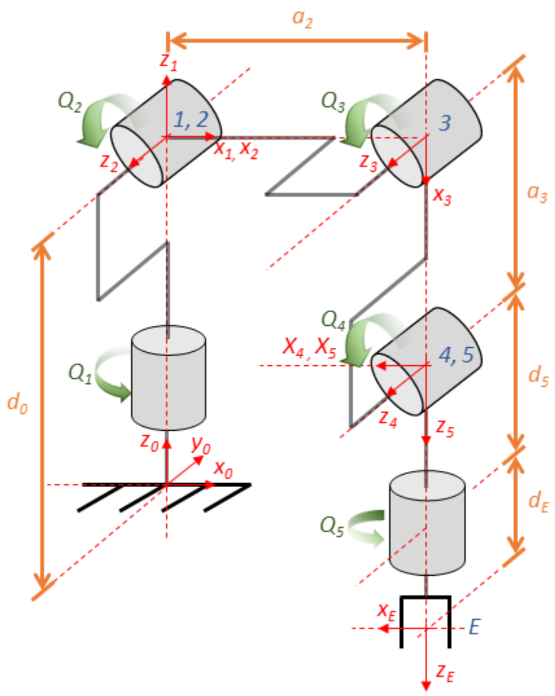

Figure 1: The Robotic Arm in "Zero" Configuration

The following report will be based on the robotic arm depicted in figure 1. All joints have a rotation Q which is positively rotating about the z axis of each of its respective frames. The robotic arm will have the following parameters:

Variables	Length (mm)
d_0	300
a_2	450
a_3	350
d_5	50
d_E	30

1.2 Aim and Purpose

The aim of this report is to investigate the mathematics of the robotic arm such that the Jacobian matrix can be derived. The purpose of deriving and utilizing the Jacobian matrix is to investigate and understand the relationship between the joint space (rotation of each joint) and task space (translational and rotational velocity of the end effector) of a robotic arm.

1.3 Forward Kinematics from Denavit-Hartenberg (DH) Table

From the above diagram, the following DH table can be found:

DH Table

	D_x	R_x	D_z	R_z
i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0°	d_0	$0° + Q_1$
2	0	90°	0	$0° + Q_2$
3	a_2	0°	0	$-90° + Q_3$
4	a_3	0°	0	$-90° + Q_4$
5	0	-90°	0	$0° + Q_5$
E	0	0°	$d_5 + d_E$	0°

Figure 2: DH Table of the Robotic Arm

From the DH table shown in figure 2, the forward kinematic transformation matrix can be calculated as such:

$$
{}^{x}_{y}T = \begin{bmatrix} 1 & 0 & 0 & a_X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_X & -\sin\alpha_X & 0 \\ 0 & \sin\alpha_X & \cos\alpha_X & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_Y \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_X & -\sin\theta_X & 0 & 0 \\ \sin\theta_X & \cos\theta_X & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)
$$

The values of the variables above in equation 1 can be found from the DH table directly, where the matrix ${}^{x}_{y}T$ is the transformation matrix from frame X to frame Y. The equation below demonstrates the transformation matrix from the origin frame to the end-effector frame:

$$
{}^{0}_{E}T = {}^{0}_{1}T * {}^{1}_{2}T * {}^{2}_{3}T * {}^{3}_{4}T * {}^{4}_{5}T * {}^{5}_{E}T \quad (2)
$$

2. Jacobian

2.1 Derivation of Jacobian

2.1.1 Jacobian for Translational Velocity Component

From equation 1 and by inspection of the robotic arm, a list of useful transformation matrices are calculated and shown below:

$$
{}^{0}_{1}T = \begin{bmatrix} \cos Q_1 & -\sin Q_1 & 0 & 0 \\ \sin Q_1 & \cos Q_1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)
$$

$$
{}^{1}_{2}T = \begin{bmatrix} \cos Q_2 & -\sin Q_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin Q_2 & \cos Q_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)
$$

$$
{}^{2}_{3}T = \begin{bmatrix} \cos(Q_3 - 90) & -\sin(Q_3 - 90) & 0 & a_2 \\ \sin(Q_3 - 90) & \cos(Q_3 - 90) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)
$$

$$
{}^{3}_{4}T = \begin{bmatrix} \cos(Q_4 - 90) & -\sin(Q_4 - 90) & 0 & a_3 \\ \sin(Q_4 - 90) & \cos(Q_4 - 90) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)
$$

$$ {}^4_5T = \begin{bmatrix} \cos Q_5 & -\sin Q_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin Q_5 & -\cos Q_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7) $$

$$ {}^5_ET = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_5 + d_E \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8) $$

With respect to figure 1, a new point w is assigned as the "wrist point" at the origin of frames 4 and 5:

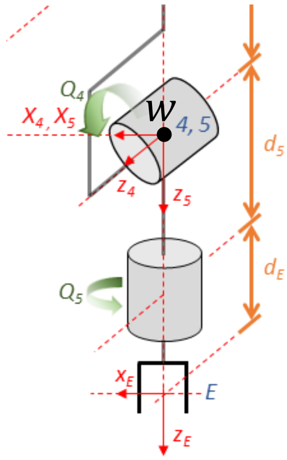

Figure 3: Definition of the "Wrist Point" on the Robotic Arm

To calculate the Jacobian, it is necessary to find the relative position of the "wrist point" from all the origins of each respective frame. This is necessary to find the direction of the translational velocity of the point w, this is further discussed below. For cleanliness, all trigonometric expressions will be expressed as described in equations 9 to 11:

$$ \sin Q_x = Sx, \ \cos Q_x = Cx \quad (9) $$

$$ \sin(Q_x + Q_y) = Sxy, \cos(Q_x + Q_y) = Cxy \quad (10) $$

$$ \sin(Q_2 - Q_1 + Q_3 + Q_4) = S\_1234, \cos(Q_2 - Q_1 + Q_3 + Q_4) = C\_1234 \quad (11) $$

All position matrices below are expressed in frame 0 for simplicity in later calculations (derived from equations 23-17):

$$ {}^0r_{3w} = \begin{bmatrix} C1 * S23 * a_3 \\ S1 * S23 * a_3 \\ -c23 * a_3 \end{bmatrix} \quad (12) $$

$$ {}^0r_{2w} = \begin{bmatrix} C1 * (C2 * a_2 + S23 * a_3) \\ S1 * (C2 * a_2 + S23 * a_3) \\ S2 * a_2 - C23 * a_3 \end{bmatrix} \quad (13) $$

$$ {}^0r_{1w} = \begin{bmatrix} C1 * (C2 * a_2 + S23 * a_3) \\ S1 * (C2 * a_2 + S23 * a_3) \\ S2 * a_2 - C23 * a_3 \end{bmatrix} \quad (14) $$

$$ {}^0r_{wE} = \begin{bmatrix} (d_5 + d_E) * (\frac{S1234}{2} + \frac{S\_1234}{2}) \\ -(d_5 + d_E) * (\frac{C1234}{2} - \frac{C\_1234}{2}) \\ -C234 * (d_5 + d_E) \end{bmatrix} \quad (15) $$

$$ {}^0r_{5E} = \begin{bmatrix} (d_5 + d_E) * (\frac{S1234}{2} + \frac{S\_1234}{2}) \\ -(d_5 + d_E) * (\frac{C1234}{2} - \frac{C\_1234}{2}) \\ -C234 * (d_5 + d_E) \end{bmatrix} \quad (16) $$

$$ {}^0r_{wE} = \begin{bmatrix} (d_5 + d_E) * (\frac{S1234}{2} + \frac{S\_1234}{2}) \\ -(d_5 + d_E) * (\frac{C1234}{2} - \frac{C\_1234}{2}) \\ -C234 * (d_5 + d_E) \end{bmatrix} \quad (17) $$

Where, the position matrices are denoted such that ${}^0r_{xy}$ is the position matrices from point x to point y in frame 0.
To transform any matrices to frame 0, the transformation matrices are calculated from equations 3-8:

$$ {}^0_2T = {}^0_1T * {}^0_2T \quad (18) \qquad {}^0_3T = {}^0_2T * {}^2_3T \quad (19) \qquad {}^0_4T = {}^0_3T * {}^3_4T \quad (20) $$

$${}^{0}_{5}T = {}^{0}_{4}T * {}^{4}_{5}T \quad (21) \qquad\qquad {}^{0}_{E}T = {}^{0}_{5}T * {}^{5}_{E}T \quad (22)$$

Where the position matrices of a particular frame are multiplied with a corresponding transformation matrix to obtain the desired position matrix in frame 0 (as shown in equations 23 to 27), note that $r_i - r_w = r_{iw}$:

$${}^{0}r_{3w} = {}^{0}_{3}T * \begin{bmatrix} a_3 \\ 0 \\ 0 \end{bmatrix} \quad (23) \qquad\qquad {}^{0}r_{2w} = {}^{0}_{2}T * \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} + {}^{0}r_{3w} \quad (24)$$

$${}^{0}r_{1w} = {}^{0}_{1}T * \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + {}^{0}r_{2w} \quad (25) \qquad\qquad {}^{0}r_{5E} = {}^{0}_{5}T * \begin{bmatrix} 0 \\ 0 \\ d_5 + d_E \end{bmatrix} \quad (26)$$

$${}^{0}r_{wE} = {}^{0}_{E}T * \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + {}^{0}r_{5E} \quad (27)$$

As briefly mentioned, the translational velocity of a joint i $\in \{1,2,3,4,5\}$ is shown in equation 28 (v_i), where a cross product is performed between the direction of rotation of each joint $\widehat{z_i}$ and the position vector from the point i to the "wrist point" w (r_{iw}), multiplied by the rotational velocity of the joint i. Where the cross product matrix is the Jacobian for the translational velocity component.

$$v_i = \dot{Q}_i * (\widehat{z_i} \times r_{iw}) = \dot{Q}_{1,2,3,4,5} * [\widehat{z_1} \times r_{1w} \quad \ldots \quad \widehat{z_E} \times r_{Ew}] = \dot{Q}_{1,2,3,4,5} * J_{v,w} \quad (28)$$

Where the $\widehat{z_i}$ axis of each point i are converted to frame 0 for further calculation:

$${}^{0}\widehat{z_1} = {}^{0}_{1}T * {}^{1}\widehat{z_1} = {}^{0}_{1}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (29) \qquad {}^{0}\widehat{z_2} = {}^{0}_{2}T * {}^{2}\widehat{z_2} = {}^{0}_{2}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S1 \\ -C1 \\ 0 \end{bmatrix} \quad (30)$$

$${}^{0}\widehat{z_3} = {}^{0}_{3}T * {}^{3}\widehat{z_3} = {}^{0}_{3}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S1 \\ -C1 \\ 0 \end{bmatrix} \quad (31) \qquad {}^{0}\widehat{z_4} = {}^{0}_{4}T * {}^{4}\widehat{z_4} = {}^{0}_{4}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S1 \\ -C1 \\ 0 \end{bmatrix} \quad (32)$$

$${}^{0}\widehat{z_5} = {}^{0}_{5}T * {}^{5}\widehat{z_5} = {}^{0}_{5}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{S1234}{2} + \frac{S\_1234}{2} \\ \frac{C\_1234}{2} - \frac{C1234}{2} \\ -C234 \end{bmatrix} \quad (33) \qquad {}^{0}\widehat{z_E} = {}^{0}_{E}T * {}^{E}\widehat{z_E} = {}^{0}_{E}T * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{S1234}{2} + \frac{S\_1234}{2} \\ \frac{C\_1234}{2} - \frac{C1234}{2} \\ -C234 \end{bmatrix} \quad (34)$$

By substituting all the equations for r_{iw} from equations 12-14 and all the equations for $\widehat{z_i}$ from equations 29 to 31 into equation 28 for all the joints i $\in \{1,2,3,4,5,E\}$, the following expression is found for the translational velocity of the "wrist point" w:
From equation 28, the Jacobian for the translational velocity component at the wrist point *w*

$$v_w = [\,{}^{0}\widehat{z_1} \times {}^{0}r_{1w} \quad {}^{0}\widehat{z_2} \times {}^{0}r_{2w} \quad {}^{0}\widehat{z_3} \times {}^{0}r_{3w} \quad 0 \quad 0\,] * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ 0 \\ 0 \end{bmatrix} \quad (35)$$

can be derived as shown in equation 36 below, where the 4th and 5th column are zeros, denoting no relationship with the translational velocities of frames 4 and 5.

$$J_{v,w} = \begin{bmatrix} -S1 * (C2 * a_2 + S23 * a_3) & C1 * (C23 * a_3 - S2 * a_2) & C1 * C23 * a_3 & 0 & 0 \\ C1 * (C2 * a_2 + S23 * a_3) & S1 * (C23 * a_3 - S2 * a_2) & S1 * C23 * a_3 & 0 & 0 \\ 0 & C2 * a_2 + S23 * a_3 & S23 * a_3 & 0 & 0 \end{bmatrix} \quad (36)$$

2.1.2 Jacobian for Angular Velocity Component

The angular velocity at the “wrist point” w is calculated through equation 37, where the matrix of $\hat{z}_i$ is the Jacobian for the Angular Velocity component:

$$\omega_w = \begin{bmatrix} {}^{0}\hat{z}_1 & {}^{0}\hat{z}_2 & {}^{0}\hat{z}_3 & {}^{0}\hat{z}_4 & {}^{0}\hat{z}_5 \end{bmatrix} * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} = J_{\omega,w} * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} \qquad (37)$$

Where the equations for $\hat{z}_i$ from equations 29 to 31 are substituted into equation 37 to find the Jacobian for angular velocity component:

$$J_{\omega,w} = \begin{bmatrix} 0 & S1 & S1 & S1 & \frac{S1234}{2} + \frac{S\_1234}{2} \\ 0 & -C1 & -C1 & -C1 & \frac{C\_1234}{2} - \frac{C1234}{2} \\ 1 & 0 & 0 & 0 & -C234 \end{bmatrix} \qquad (38)$$

2.1.3 The Jacobian Matrix at Wrist Point

$$\begin{bmatrix} v_w \\ \omega_w \end{bmatrix} = \begin{bmatrix} J_{v,w} \\ J_{\omega,w} \end{bmatrix} * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} = [J_w] * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} \qquad (39)$$

From equation 39, the overall Jacobian Matrix includes both the translational and angular velocity components at wrist point. The overall Jacobian Matrix J_w is shown below in equation 40:

$$J_w = \begin{bmatrix} -S1*(C2*a_2+S23*a_3) & C1*(C23*a_3-S2*a_2) & C1*C23*a_3 & 0 & 0 \\ C1*(C2*a_2+S23*a_3) & S1*(C23*a_3-S2*a_2) & S1*C23*a_3 & 0 & 0 \\ 0 & C2*a_2+S23*a_3 & S23*a_3 & 0 & 0 \\ 0 & S1 & S1 & S1 & \frac{S1234}{2} + \frac{S\_1234}{2} \\ 0 & -C1 & -C1 & -C1 & \frac{C\_1234}{2} - \frac{C1234}{2} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (40)$$

2.1.3 The Jacobian Matrix at End Effector

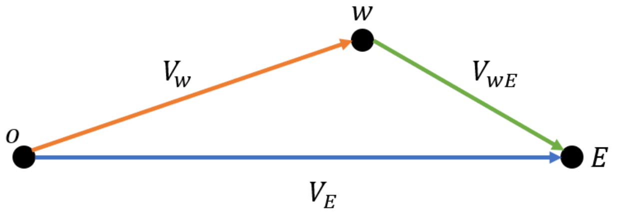

Figure 4: Vector Triangle of Wrist to End Effector

From figure _, it can be seen that the following equation is necessary to find the vector v_E:

$$v_E = v_w + v_{wE} \qquad (41)$$

Where, the translational velocity v_{wE} can be found through the cross product between the angular velocity at point E and the position matrix between point w and E (${}^{0}r_{wE}$):
Furthermore, as the “wrist point” and end effector are aligned (no angular rotation between), the two angular velocities can be defined to be the same:

$$v_{wE} = \omega_E \times \mathrm{r_{wE}} \qquad (42)$$

$$\omega_E \triangleq \omega_w \qquad (43)$$

$$\begin{bmatrix} v_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & Skew(\mathrm{r_{wE}}) & \\ 0 & 0 & 1 & & & \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} v_w \\ \omega_w \end{bmatrix} \quad = (44)$$

Substituting equations 42 and 43 into equation 41 and rearranging, the following matrix equation can be deduced:
Where a skew function is necessary to perform the cross product for $(\omega_w \times \mathrm{r_{wE}})$ in equation 44 (converting dot product to cross product between the two variables).
From the previous section, the matrix $\begin{bmatrix} v_w \\ \omega_w \end{bmatrix}$ can then be substituted by equation 39 into equation 44:

$$\begin{bmatrix} v_E \\ \omega_E \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & Skew(\mathrm{r_{wE}}) & \\ 0 & 0 & 1 & & & \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * [J_w] \right) * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} = J_E * \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} \qquad (44)$$

Where the Jacobian matrix at the end effector J_E is calculated in equation 45 below:

$$J_E = \begin{bmatrix} (d_5 + d_E) * (\frac{C1234}{2} - \frac{C\_1234}{2}) - S1 * (C2 * a_2 + S23 * a_3) & C1 * (C23 * a_3 - S2 * a_2) + C1 * C234 * (d_5 + d_E) & C1 * C23 * a_3 + C1 * C234 * (d_5 + d_E) & 0 & 0 \\ (d_5 + d_E) * \left(\frac{S1234}{2} + \frac{S\_1234}{2}\right) + C1 * (C2 * a_2 + S23 * a_3) & S1 * (C23 * a_3 - S2 * a_2) + S1 * C234 * (d_5 + d_E) & S1 * C23 * a_3 + S1 * C234 * (d_5 + d_E) & 0 & 0 \\ 0 & C2 * a_2 + S23 * a_3 + d_5 * S234 + d_E * S234 & S23 * a_3 + d_5 * S234 + d_E * S234 & 0 & 0 \\ 0 & S1 & S1 & S1 & \frac{S1234}{2} + \frac{S\_1234}{2} \\ 0 & -C1 & -C1 & -C1 & \frac{C\_1234}{2} - \frac{C1234}{2} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(45)

2.2 Justification of Jacobian

2.2.1 “Zero” Position Theoretical Proof

```
>> calculate_Jacobian([0,0,0,0,0])

ans =
```

$$\begin{array}{ll|ccccc} & \textit{Joints}: & 1 & 2 & 3 & 4 & 5 \\ \hline & J_{v_x} & 0, & 0.43, & 0.43, & 0.08, & 0 \\ \textit{Translational Velocity} & J_{v_y} & 0.45, & 0, & 0, & 0, & 0 \\ & J_{v_z} & 0, & 0.45, & 0, & 0, & 0 \\ & J_{\omega_x} & 0, & 0, & 0, & 0, & 0 \\ \textit{Angular Velocity} & J_{\omega_y} & 0, & -1.0, & -1.0, & -1.0, & 0 \\ & J_{\omega_z} & 1.0, & 0, & 0, & 0, & -1.0 \end{array}$$

Figure 5: Jacobian Matrix of End Effector at "Zero" Position

The validation method to check the if the computed Jacobian matrix is correct is by numerically testing the Jacobian matrix at known poses.

As shown in figure 1, the first position used for validation is the initial "zero" position. Here, the input of $Q_{1,2,3,4,5} = 0$ is used where the resultant Jacobian matrix shown in figure 5 is computed. With reference to equations 28 and 37 where the cross product and z-axis is necessary to form the Jacobian matrix, the following calculations from frame 0 were performed to check the values in the Jacobian that were outputted.

By performing a cross product between $z_i \times (r_E - r_i)$, the direction of the velocity can be obtained. For the contribution from joint 1 velocity, the cross product $z_i \times (r_E - r_i)$ is shown below in equation 46:

$$z_1 \times (r_E - r_1) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 \\ 0 \\ -a_3 - d_5 - d_E \end{bmatrix} = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix} \qquad (46)$$

The obtained value for this is the same for the output of the Jacobian above (examining joint one $J_{v,x}, J_{v,y}, J_{v,z}$) where 0.45 is in meters, which is equivalent to $a_2 = 450mm$.

Repeating this process to obtain the other 4 joints:

$$z_2 \times (r_E - r_2) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 \\ 0 \\ -a_3 - d_5 - d_E \end{bmatrix} = \begin{bmatrix} a_3 + d_5 + d_E \\ 0 \\ a_2 \end{bmatrix} \qquad (47)$$

$$z_3 \times (r_E - r_3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -a_3 - d_5 - d_E \end{bmatrix} = \begin{bmatrix} a_3 + d_5 + d_E \\ 0 \\ 0 \end{bmatrix} \qquad (48)$$

$$z_4 \times (r_E - r_4) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -d_5 - d_E \end{bmatrix} = \begin{bmatrix} d_5 + d_E \\ 0 \\ 0 \end{bmatrix} \qquad (49)$$

$$z_5 \times (r_E - r_5) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -a_3 - d_5 - d_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (50)$$

Where the z-axis matrix is found through overlapping each frame's axis to frame 0 axis to obtain the angular velocity component of the Jacobian matrix, this can be seen in figure 6 below:

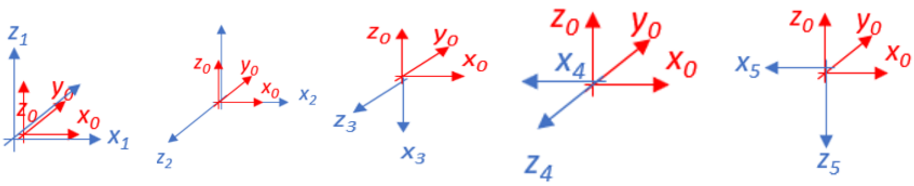

Figure 6: Superposition of Each Respective Frame onto Frame 0

For frame 1, the z-axis is pointing along the positive z direction of frame 0, therefore:

$$J_{\omega_1} = \begin{bmatrix} J_{\omega_{x,1}} \\ J_{\omega_{y,1}} \\ J_{\omega_{z,1}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (51)$$

This is reflected on the computed Jacobian shown in figure 5.

Repeating the above steps, for frames 2, 3 and 4, the z-axis is pointing along the negative y direction of frame 0, therefore:

$$J_{\omega_{2,3,4}} = \begin{bmatrix} J_{\omega_{x,2,3,4}} \\ J_{\omega_{y,2,3,4}} \\ J_{\omega_{z,2,3,4}} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad (52)$$

Finally, for frame 5, the z-axis is pointing along the negative z direction of frame 0, therefore:

$$J_{\omega_5} = \begin{bmatrix} J_{\omega_{x,5}} \\ J_{\omega_{y,5}} \\ J_{\omega_{z,5}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad (53)$$

By inspection, it can be seen that the values obtained from equations 46 to 53 are all present in the computed Jacobian matrix seen in figure 5. Therefore, it can be concluded that the computed Jacobian matrix is theoretically sound.

2.2.2 Verification through Differentiation

Another method of verification is comparing the computed Jacobian (from figure 5) to a Jacobian matrix obtained from differentiation (from the forward kinematics). This process is only valid for comparing between the translational velocity components of the Jacobian. The detailed process of the differentiate method can be encapsulated in the following steps:

1. Use the DH table from above
2. Obtain the position of end-effector symbolically
3. Differentiate the end-effector position vector with respect to time
4. Convert the differentiated term into a state-space matrix, with the variables being the angular velocity of the joints

The state-space matrix is the Jacobian matrix.
As the symbolic solution obtained from the 2 different methods are numerically slightly different, there is no way to compare them symbolically. Therefore, to confirm the similarity of the two solutions, a nested for loop that iterates through values of Q was implemented to numerically solve for both Jacobian matrices.
A script named "*verifying_Jacobian.m*" was written to implement the aforementioned for loop. After computing a list of possible Q values, and output was given and can be seen in figure 7.

```
>> verifying_Jacobian

diff =

[ 0,  1.1e-13, 9.1e-13, -8.5e-14, 0]
[ 0, -2.3e-13,       0,        0, 0]
[ 0,  4.5e-13,       0,        0, 0]

the two results are the same
```

Figure 7: Difference Between Computed Values and Theorised Values

From figure 7, it can be seen that the difference in values are all roughly 0, therefore, proves that the Jacobian computed in figure 5 and the Jacobian derived from the differentiation is virtually the same.