Advanced Econometrics Lecture 02

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Today's agenda

1. Quick intro with examples in R
   - The fixed-effects estimator
   - The random-effects estimator
   - The Hausman test - fixed vs random effects
2. Extension of the basic model
3. Exercise from last year
4. The basic models' restrictions
5. Dancing with the Stars
6. A little bit of theory
7. Previous years exams

Final grades

Final grade

a) 20% homeworks
b) 40% project
c) 40% exam

Lab class grade

a) 20% homeworks
b) 80% project

You have to:

a) Pass the project
b) Pass the exam

Grade

- $< 50\%$ - 2
- $[50\%, 60\%)$ - 3
- $[60\%, 70\%)$ - 3.5
- $[70\%, 80\%)$ - 4
- $[80\%, 90\%)$ - 4.5
- $[90\%, 100\%]$ - 5

What does panel data look like?

farm	year	prod	area	labor	fert
1	1990	7.87	2.5	160	207.5
1	1991	7.18	2.5	138	295.5
1	1992	8.92	2.5	140	362.5
1	1993	7.31	2.5	127	338
1	1994	7.54	2.5	145	337.5
2	1990	10.35	3.8	184	303.5
2	1991	10.21	3.8	151	206
2	1992	13.29	3.8	185	374.5
2	1993	18.58	3.8	262	421
2	1994	17.07	3.8	174	595.7
3	1990	9.98	3.4	170	252
3	1991	10.44	4.3	148	444
3	1992	10.4	4.3	198	283.5
3	1993	8.92	4.3	189	287
3	1994	7.36	4.3	214	294
⋮	⋮	⋮	⋮	⋮	⋮

Models for panel data – Greene p. 343

- Data sets that combine time series and cross section are common in economics.
- For example, the published statistics for the OECD contain numerous series of economic aggregates observed yearly for many countries.
- Recently constructed **longitudinal data sets** contain observations on thousands of individuals or families, each observed at several points in time.
- Other empirical studies have examined time-series data on sets of firms, states, countries, or industries simultaneously. These data sets provide rich sources of information about the economy.
- The analysis of panel data allows the model builder to learn about economic processes while accounting for both heterogeneity across individuals, firms, countries, and so on and for dynamic effects that are not visible in cross sections.

The One-way Error Component Regression Model

$$y_{it} = x_{it}\beta + u_i + \varepsilon_{it} \tag{1}$$

- Subscript i runs through all entities while subscript t defines time the entity was observed.
- i.e. y_{it} denotes i-th observation observed in period t.
- ε_{it} is a random disturbance
- u_i's are entities' individual characteristics that are unobserved and time-invariant.
- In a model for wage a part of individual effect may be attributed to intelligence.

The Fixed Effects Model – Baltagi p. 12

- In this case, the u_i are assumed to be fixed parameters to be estimated and the remainder disturbances stochastic with ε_{it} independent and identically distributed $IID(0, \sigma_\varepsilon^2)$.
- The $\mathbb{X}_{it}$ are assumed independent of the ε_{it} for all i and t.
- The fixed effects model is an appropriate specification if we are focusing on a specific set of N firms, say, IBM, GE, Westinghouse, etc. and our inference is restricted to the behaviour of these sets of firms.
- Alternatively, it could be a set of N OECD countries, or N American states.
- Inference in this case is conditional on the particular N firms, countries or states that are observed.

The Fixed Effects Model – An Example

The example below uses 1982-1988 state-level data for 48 U.S. states on traffic fatality rate (deaths per 100,000).
We model the highway fatality rates as a function of several commonfactors:

- `beertax` – the tax on a case of beer;
- `spircons` – a measure of spirits consumption;
- `unrate` – the state unemployment rate;
- `perincK` – state per capita income, in thousands.

Source: Ch.F. Baum, *Introduction to Modern Econometrics Using Stata*, p. 222.

The Fixed Effects Model – An Example

```
Call:
plm(formula = fatal ~ beertax + spircons + unrate + perincK,
    data = traffic, model = "within", index = c("state", "year"))

Balanced Panel: n = 48, T = 7, N = 336
Coefficients:
           Estimate Std. Error t-value  Pr(>|t|)
beertax  -0.4840728  0.1625106 -2.9787  0.003145 **
spircons  0.8169651  0.0792118 10.3137 < 2.2e-16 ***
unrate   -0.0290499  0.0090274 -3.2180  0.001441 **
perincK   0.1047103  0.0205986  5.0834 6.738e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    10.785
Residual Sum of Squares: 6.9816
R-Squared:      0.35265
Adj. R-Squared: 0.2364
F-statistic: 38.6774 on 4 and 284 DF, p-value: < 2.22e-16
```

The Fixed Effects Model – An Example

```
> fixef(fixed)
        AL          AR          AZ          CA          CO
        CT          DE          FL          GA          IA
 1.26922654  0.92694626  0.12313743 -1.20967986 -1.08978969
 -2.24823877 -1.35259271 -0.21482860  0.84989551 -0.24285625
...
        TX          UT          VA          VT          WA
        WI          WV          WY
 0.15212238  0.50289787 -0.62267775 -0.57507925 -0.90698507
 -1.14703336  1.05500578  0.55087312
> ols<-lm(fatal~beertax+spircons+unrate+perincK, data=traffic)
> pFtest(fixed, ols)

F test for individual effects

data:  fatal ~ beertax + spircons + unrate + perincK
F = 59.768, df1 = 47, df2 = 284, p-value < 2.2e-16
alternative hypothesis: significant effects
```

The Random Effects Model – Baltagi p. 14-15

- There are too many parameters in the fixed effects model and the loss of degrees of freedom can be avoided if the u_i can be assumed random. In this case $u_i \sim IID(0, \sigma_u^2)$, $\varepsilon_{it} \sim IID(0, \sigma_\varepsilon^2)$ and the u_i are independent of the ε_{it}.
- In addition, the $\mathbb{X}_{it}$ are independent of the u_i and ε_{it} , for all i and t.
- The random effects model is an appropriate specification if we are drawing N individuals randomly from a large population. This is usually the case for household panel studies.
- Care is taken in the design of the panel to make it "representative" of the population we are trying to make inferences about. In this case, N is usually large and a fixed effects model would lead to an enormous loss of degrees of freedom. The individual effect is characterised as random and inference pertains to the population from which this sample was randomly drawn.

The Random Effects Model – An Example

The file *rice.csv* contains 352 observations on 44 rice farmers in the Tarlac region of the Philippines for the 8 years 1990-1997. Variables in the data set are tonnes of freshly threshed rice (*PROD*), hectares planted (*AREA*), person-days of hired and family labour (*LABOR*), and kilograms of fertiliser (*FERT*).

```
------------------------------------------------------------
variable name   variable label
------------------------------------------------------------
firm            firm number = 1 to 44
year            year = 1990 to 1997
prod            Rice production (tonnes)
area            Area planted to rice (hectares)
labor           Hired + family labor (person days)
fert            Fertilizer applied (kilograms)
------------------------------------------------------------

  1.  Data source: These data were used by O'Donnell, C.J. and W.E. Griffiths
  (2006), Estimating State-Contingent Production Frontiers ,
      American Journal of Agricultural Economics, 88(1), 249-266.
```

Source: Principles of Econometrics, R. Carter Hill, William E. Griffiths and Guay C. Lim

The Random Effects Model – An Example

```
Call:
plm(formula = prod ~ area + labor + fert, data = rice,
    model = "random", index = c("firm", "year"))

Balanced Panel: n = 44, T = 8, N = 352
Coefficients:
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept) -0.0608034  0.2557774 -0.2377  0.812238
area         1.5598888  0.2220582  7.0247 1.135e-11 ***
labor        0.0225103  0.0040135  5.6087 4.160e-08 ***
fert         0.0043355  0.0013386  3.2387  0.001316 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    5396.6
Residual Sum of Squares: 1363.3
R-Squared:      0.74737
Adj. R-Squared: 0.74519
F-statistic: 343.166 on 3 and 348 DF, p-value: < 2.22e-16
```

The Random Effects Model – An Example

```
Effects:
                  var std.dev share
idiosyncratic 3.7121  1.9267 0.894
individual    0.4393  0.6628 0.106
theta: 0.2833
```

`pooltest` tests the hypothesis that the same coefficients apply to each individual. It is a standard F test, based on the comparison of a model obtained for the full sample and a model based on the estimation of an equation for each individual.

```
> pooltest(prod~area+labor+fert, data=rice,
           index=c("firm", "year"), model="random")

F statistic

data:  prod ~ area + labor + fert
F = 1.1834, df1 = 172, df2 = 176, p-value = 0.1336
alternative hypothesis: unstability
```

Hausman's Specification Test – Baltagi p. 66

- A critical assumption in the error component regression model is that

$$E(\alpha_{it}|X_{it}) = 0 \tag{2}$$

- This is important given that the disturbances contain individual invariant effects (the u_i) which are unobserved and may be correlated with the X_{it}.
- Hausman (1978) suggests comparing β_{GLS} and β_{Within}, both of which are consistent under the null hypothesis $H_0: \; E(u_{it}|X_{it}) = 0$, but which will have different probability limits if H_0 is not true.
- H_0 : $\mathit{Cov}(u_i, x_i) = 0 \Rightarrow$ random effects
- H_1 : $\mathit{Cov}(u_i, x_i) \neq 0 \Rightarrow$ fixed effects

	RE estimator	**FE estimator**
H_0 : $\mathit{Cov}(u_i, x_i) = 0$	consistent, efficient	consistent, inefficient
H_1 : $\mathit{Cov}(u_i, x_i) \neq 0$	inconsistent	consistent

Hausman's test

```
> phtest(fixed, random)

Hausman Test

data:  prod ~ area + labor + fert
chisq = 26.277, df = 3, p-value = 8.346e-06
alternative hypothesis: one model is inconsistent
```

Quick intro

- This is the end of quick intro!
- Up to now – "*applied econometrics*"
- Let's extend the basic models

The One-way Error Component Regression Model

$$y_{it} = x_{it}\beta + u_i + \varepsilon_{it} \tag{3}$$

- Subscript i runs through all entities while subscript t defines time the entity was observed.
- i.e. y_{it} denotes i-th observation observed in period t.
- ε_{it} is a random disturbance
- u_i's are entities' individual characteristics that are unobserved and time-invariant.
- In a model for wage a part of individual effect may be attributed to intelligence.

Individual effects – examples

- Another example is given by Hajivassiliou (1987) who studies the external debt repayments problem using a panel of 79 developing countries observed over the period 1970–82. These countries differ in terms of their colonial history, financial institutions, religious affiliations and political regimes. All of these country-specific variables affect the attitudes that these countries have with regards to borrowing and defaulting and the way they are treated by the lenders.
- Deaton (1995) gives another example from agricultural economics. This pertains to the question of whether small farms are more productive than large farms. OLS regressions of yield per hectare on inputs such as land, labor, fertilizer, farmer's education, etc. usually find that the sign of the estimate of the land coefficient is negative. These results imply that smaller farms are more productive. [...] Deaton (1995) offers an alternative explanation. This regression suffers from the omission of unobserved heterogeneity, in this case "land quality", and this omitted variable is systematically correlated with the explanatory variable (farm size).

The Two-way Error Component Regression Model

$$y_{it} = \alpha + x_{it}\beta + u_i + \lambda_t + \varepsilon_{it} \tag{4}$$

- u_i denotes the unobservable individual effect,
- λ_t denotes the unobservable time effect,
- ε_{it} is the remainder stochastic disturbance term.

Source: Badi H. Baltagi, *Econometric Analysis of Panel Data*, 2005, p. 33.

The Two-way Error Component Regression Model

$$y_{it} = \alpha + x_{it}\beta + u_i + \lambda_t + \varepsilon_{it} \tag{4}$$

- u_i denotes the unobservable individual effect,
- λ_t denotes the unobservable time effect,
- ε_{it} is the remainder stochastic disturbance term.
- Note that λ_t is individual-invariant and it accounts for any time-specific effect that is not included in the regression.
- For example, it could account for
  - strike year effects that disrupt production,
  - oil embargo effects that disrupt the supply of oil and affect its price,
  - Surgeon General reports on the ill-effects of smoking, or government laws restricting smoking in public places, all of which could affect consumption behavior.

Source: Badi H. Baltagi, *Econometric Analysis of Panel Data*, 2005, p. 33.

Balanced and unbalanced panels – Greene p. 348

- A "panel" data set will consist of n sets of observations on individuals to be denoted $i = 1, \ldots, n$.
- If each individual in the data set is observed the same number of times, usually denoted T, the data set is a **balanced panel**.
- An **unbalanced panel** data set is one in which individuals may be observed different numbers of times.
- A **fixed panel** is one in which the same set of individuals is observed for the duration of the study.
- A rotating panel is one in which the cast of individuals changes from one period to the next.
- For example, Gonzalez and Maloney (1999) examined self-employment decisions in Mexico using the National Urban Employment Survey. This is a quarterly data set drawn from 1987 to 1993 in which individuals are interviewed five times. Each quarter, one-fifth of the individuals is rotated out of the data set.

Well-behaved panel data – Greene p. 348

The asymptotic properties of the estimators in the classical regression model were established in Section 4.4 under the following assumptions:

A.1. Linearity: $y_i = x_{i1}\beta_1 + x_{i2}\beta_2 + \ldots + x_{iK}\beta_K + \varepsilon_i$

A.2. Full rank: The $n \times K$ sample data matrix, $\mathbb{X}$ has full column rank.

A.3. Exogeneity of the independent variables:
$\mathbb{E}[\varepsilon_i | x_{j1}, x_{j2}, \ldots, x_{jK}] = 0; \quad i, j = 1 \ldots, n$

A.4. Homoscedasticity and nonautocorrelation.

A.5. Data generating mechanism-independent observations.

Why should we use panel data? Benefits and limitations

Hsiao (2003) and Klevmarken (1989) list several benefits from using panel data. These include the following.

1. **Controlling for individual heterogeneity.** Panel data suggests that individuals, firms, states or countries are heterogeneous. Time-series and cross-section studies not controlling this heterogeneity run the risk of obtaining biased results.
2. Panel data give more informative data, more variability, less collinearity among the variables, more degrees of freedom and more efficiency.
3. Panel data are better able to study the dynamics of adjustment.
4. Panel data are better able to identify and measure effects that are simply not detectable in pure cross-section or pure time-series data
5. Panel data models allow us to construct and test more complicated behavioral models than purely cross-section or time-series data.

Source: Econometric analysis of panel data / Badi H. Baltagi. — 3rd ed, p. 4-6.

Econometric Analysis of Panel Data

1. Pooled model – OLS, GLS
2. Between estimator
3. The One-way Error Component Regression Model
   - the fixed effects model
   - the random effects model
4. The Two-way Error Component Regression Model
   - the fixed effects model
   - the random effects model
5. Instrumental variable models for panel data
   - Hausman-Taylor estimator
   - IV estimator
6. Dynamic panel-data models
   - Anderson-Hsiao estimator
   - Arrellano-Bond estimator
7. Other methods

1. wide panels – $N \to \infty$, T is small
   - We are here! This lecture is devoted to this class of models.
   - Only static models.
2. long panels – N is small, $T \to \infty$
3. wide and long panels – $N \to \infty$, $T \to \infty$
4. short and narrow panels – N and T are small

Pooled regression and panel data

Pooled models assume that regressors are exogenous and simply write the error as u_{it} rather than using the decomposition $\alpha_i + \varepsilon_{it}$. Then

$$y_{it} = \alpha + \boldsymbol{x}_{it}^{'}\boldsymbol{\beta} + u_{it} \tag{5}$$

What are disadvantages of applying OLS estimator to panel data set?

Fixed vs Random

- Having discussed the fixed effects and the random effects models and the assumptions underlying them, the reader is left with the daunting question, which one to choose?
- This is not as easy a choice as it might seem. In fact, the fixed versus random effects issue has generated a hot debate in the biometrics and statistics literature which has spilled over into the panel data econometrics literature.
- we will study a specification test proposed by Hausman (1978) which is based on the difference between the fixed and random effects estimators.
- Unfortunately, applied researchers have interpreted a rejection as an adoption of the fixed effects model and nonrejection as an adoption of the random effects model.
- For the applied researcher, performing fixed effects and random effects and the associated Hausman test reported in standard packages like Stata, LIMDEP, TSP, etc., the message is clear: Do not stop here. Test the restrictions implied by the fixed effects model derived by Chamberlain (1984) and check whether a Hausman and Taylor (1981) specification.

Hausman's Specification Test – Baltagi p. 66

- A critical assumption in the error component regression model is that

$$E(\alpha_{it}|X_{it}) = 0 \tag{6}$$

- This is important given that the disturbances contain individual invariant effects (the u_i) which are unobserved and may be correlated with the X_{it}.
- Hausman (1978) suggests comparing β_{GLS} and β_{Within}, both of which are consistent under the null hypothesis $H_0: \; E(u_{it}|X_{it}) = 0$, but which will have different probability limits if H_0 is not true.
- H_0 : $Cov(u_i, x_i) = 0 \Rightarrow$ random effects
- H_1 : $Cov(u_i, x_i) \neq 0 \Rightarrow$ fixed effects

	RE estimator	**FE estimator**
$H_0 : Cov(u_i, x_i) = 0$	consistent, efficient	consistent, inefficient
$H_1 : Cov(u_i, x_i) \neq 0$	inconsistent	consistent

Hausman's test

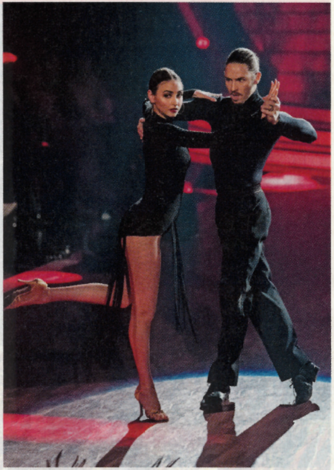
```
> phtest(fixed, random)

Hausman Test

data:  prod ~ area + labor + fert
chisq = 26.277, df = 3, p-value = 8.346e-06
alternative hypothesis: one model is inconsistent
```

Exercise – Last year homework

Baltagi i Levin (1986) oszacowali dynamiczny model dla popytu na papierosy na danych panelowych dla 46 stanów USA. Zaktualizowane dane dotyczyły okresu 1963-1992. Rozważmy niedynamiczną wersję tego modelu postaci:

$$ln\ C_{it} = \alpha + \beta_1 ln\ P_{it} + \beta_2 ln\ Y_{it} + \beta_3 ln\ Pn_{it} + \mu_i + \lambda_t + \varepsilon_{it}, \quad (7)$$

The Economist March 17th 2018



Psychology

The last shall be first

Paper

Check for updates

Research Article

Psychological Science
1–12

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Do Evaluations Rise With Experience?

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Do Evaluations Rise With Experience?

Abstract
The same raters assess the merits of applicants, athletes, art, and more using standard criteria. We investigated one important potential contaminant in such ubiquitous decisions:
Evaluations become more positive when conducted later in a sequence.

Do Evaluations Rise With Experience?

Abstract
The same raters assess the merits of applicants, athletes, art, and more using standard criteria. We investigated one important potential contaminant in such ubiquitous decisions:
Evaluations become more positive when conducted later in a sequence.

a) judges' ratings of professional dance competitors rose across 20 seasons of a popular television series,
b) university professors gave higher grades when the same course was offered multiple times, and
c) in an experimental test of our hypotheses, evaluations of randomly ordered short stories became more positive over a 2-week sequence.
d) As judges completed repeated evaluations, they experienced more fluent decision making, producing more positive judgments.

Data description

variable	description
serial	observation number
season	1- 20 – the IV
episode	1-12 within each season (varies)
judgenum	judge number (judges 1-3 are permanent)
judgexp	number of episodes the judge have attended
dancenum	dance within each episode
score	the judge's evaluation of the performance – the DV
finalist	whether or not that team made the top 3 that season
teamID	initials of the performers
ppepisodexp	number of episodes the professional partner has been on the show

	Dependent variable:	
	score	
	(RE)	(FE)
judgexp	0.198^{***}	0.002
	(0.051)	(0.002)
ppepisodexp	-0.005	0.021^{***}
	(0.051)	(0.003)
Constant		6.368^{***}
		(0.170)
Observations	1,570	1,570
R^2	0.388	0.172
Adjusted R^2	0.267	0.171
F Statistic	414.956^{***} (df = 2; 1310)	125.581^{***} (df = 2; 1567)
Note:		*p<0.1; **p<0.05; ***p<0.01

Panel model for log(wage)

- Example is based on A. Colin Cameron and Pravin K. Trivedi, *Microeconometrics Using Stata*, Stata Press, 2010, pages 240-241.
- Let's consider a dataset from Baltagi, B.H., and S. Khanti-Akom. 1999. On efficient estimation with panel data: An empirical comparison of instrumental variables. *Journal of Applied Econometrics 5: 401-406.*
- Tha data were drawn from the Panel Study of Income Dynamics (PSID).
- There are 4,165 individual-year pair observations.

PSID data description

variable name	variable label
lwage	log wage
exp	years of full-time work experience
wks	weeks worked
fem	female or male
ed	years of education
blk	black
id	entity identifier
t	time
exp2	years of full-time work experience squared

Fixed-effects estimates

```
> fixed <-plm(lwage~exp+exp2+wks+ed+fem+blk, data=psid,
              index=c("id", "t"), model="within")
> summary(fixed)

Coefficients:
        Estimate   Std. Error t-value  Pr(>|t|)
exp    1.1379e-01  2.4689e-03 46.0888 < 2.2e-16 ***
exp2  -4.2437e-04  5.4632e-05 -7.7678 1.036e-14 ***
wks    8.3588e-04  5.9967e-04  1.3939    0.1634
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    240.65
Residual Sum of Squares: 82.632
R-Squared:      0.65663
Adj. R-Squared: 0.59916
F-statistic: 2273.74 on 3 and 3567 DF, p-value: < 2.22e-16
```

Why were `ed` and `blk` not estimated?

Fixed effects estimator – Baltagi p. 13

Note that for the simple regression

$$y_{it} = \alpha + \beta x_{it} + u_i + \varepsilon_{it} \tag{8}$$

and averaging over time gives

$$\bar{y}_{i.} = \alpha + \beta \bar{x_{i.}} + u_i + \bar{\varepsilon}_{i.} \tag{9}$$

Therefore, subtracting (8) from (9) gives

$$y_{it} - \bar{y}_{i.} = \beta(x_{it} - \bar{x_{i.}}) + (u_i - u_i) + (\varepsilon_{it} - \bar{\varepsilon}_{i.}) \tag{10}$$

Also, averaging across all observations in (8) gives

$$\bar{y}_{..} = \alpha + \beta \bar{x_{..}} + u_i + \bar{\varepsilon}_{..} \tag{11}$$

where we utilised the restriction that $\sum_{i=1}^{N} u_i = 0$.
In fact only β and u_i are estimable from (8), and not α and u_i separately, unless a restriction like $\sum_{i=1}^{N} u_i = 0$ is imposed. In this case, $\hat{\beta}$ is obtained from regression (10), $\hat{\alpha} = \bar{y}_{..} - \hat{\beta}\bar{x}_{..}$ can be recovered from (11) and $u_i = \bar{y}_{i.} - \hat{\alpha} - \beta \bar{x_{i.}}$ from (9).

R^2-statistics in R

A. Colin Cameron and Pravin K. Trivedi, *Microeconometrics Using Stata*, Stata Press, 2010, p. 264.

- Let $\hat{u}$ and $\hat{\beta}$ be estimates obtained from a panel model. Let $\rho^2(x, y)$ denote the squared correlation between x and y. Then
- Within R^2: $\rho^2 \left\{ (y_{it} - \bar{y}_i), (\boldsymbol{x}_{it}'\hat{\beta} - \bar{\boldsymbol{x}}_i'\hat{\beta}) \right\}$
- Between R^2: $\rho^2 \left(\bar{y}_i, \bar{\boldsymbol{x}}_i'\hat{\beta} \right)$
- Overall R^2: $\rho^2 \left(y_{it}, \boldsymbol{x}_i'\hat{\beta} \right)$

```
> source("static_wide_panels_R2.R")
> R2_stats = static_wide_panels_R2(fixed)

 R-squared:
          within  =  0.6566299
          between =  0.02762965
          overall =  0.04760428
```

Exercise

Let's consider two clothes and shoes expenditure models

$$clothes\&shoes_i = \beta_0 + \beta_1 income_i + \beta_2 kids_i + \varepsilon_i \tag{12}$$

$$clothes\&shoes_i = \beta_0 + \beta_1 income_i + \beta_2 kids_i + \beta_3 month_i + \varepsilon_i \tag{13}$$