

1 INTRODUCTION

The Traveling Salesman Problem (TSP) is an NP-Complete computational problem which has been studied extensively for several decades [?]. In this paper, techniques for obtaining/approximating the solution to the symmetric Traveling Salesman Problem are developed and compared. A branch-and-bound algorithm using a minimum spanning tree to inform a lower bound on subproblems is presented; which, given enough time, produces an exact solution to the problem. The use of approximation algorithms proves more prudent for larger problems while sacrificing solution quality; a fact which motivated the development of a greedy local search algorithm which explores 1-exchange neighbors.

PROBLEM DEFINITION

The symmetric traveling salesman optimization problem is formalized as follows:

Given a connected undirected graph G consisting of n nodes, $\{v_0, \dots, v_{n-1}\}$, with edge weights d_{ij} between nodes i and j . Find a Hamiltonian Cycle, a path P^* where each node has degree 2, with minimum weight.

In this paper, nodes in a 2 dimensional space were considered: $v_i = \begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} \in \mathbb{R}^2 \quad \forall i \in [0, n-1]$

With edge weights calculated using the Euclidean distance: $e_{ij} = ||v_i, v_j||_2 = \sqrt{(v_{ix} - v_{jx})^2 + (v_{iy} - v_{jy})^2} \quad \forall i \neq j, i \in [0, n-1], j \in [0, n-1]$

The solution is an element in the set of vertex sequences which are Hamiltonian Cycles given by:

$$\mathcal{H} = \{(v_i)_{i=0}^{n-1} : v_0 = v_{n-1}, v_i \neq v_j \quad \forall i, j \in [0, n-1]\}$$

The symmetric TSP is given by following optimization problem:

$$P^* = (v_i^*)_{i=0}^{n-1} = \underset{argmin}{(v_i)_{i=0}^{n-1} \in \mathcal{H}} \quad \sum_{i=0}^{n-2} ||v_i, v_{i+1}|| + ||v_0, v_{n-1}||$$

RELATED WORK

ALGORITHMS

Algorithm 1 BnB($\{v_0, \dots, v_{n-1}\}$): Find minimum cost Hamiltonian Cycle for euclidean distances

Data: $\{v_0, \dots, v_{n-1}\}$ set of 2-D points

for all Unordered Pairs $\{i, j\}$ **do**

Construct edge $e = (v_i, v_j, d_{ij})$

Add e to list E of edges in increasing weight order: $E = E \cup \{e\}$

end for

Algorithm 2 2-Opt Local Search($\{v_0, \dots, v_{n-1}\}$): Approximate the minimum cost Hamiltonian Cycle for euclidean distances

Data: $\{v_0, \dots, v_{n-1}\}$ set of 2-D points

for all Unordered Pairs $\{i, j\}$ **do**

Construct edge $e = (v_i, v_j, d_{ij})$

Add e to list E of edges in increasing weight order: $E = E \cup \{e\}$

end for

while Unassigned nodes in v **do**

Assign Nodes to Route based on Greedy Nearest Neighbor implementation

end while

for $i = 1$ to length(Route Matrix) **do**

for $j = i+1$ to length(Route Matrix) **do**

reverse route[i] to route[j] and add it to newroute[i] to newroute[j]

if cost(newroute) < cost(route) **then**

route \leftarrow newroute

end if

end for

end for

EMPIRICAL EVALUATION

DISCUSSION

CONCLUSION