

Задача 1

$$\int (2x^2 - 2x - 1 + \sin x - \cos x + \ln x + e^x) dx =$$

$$= \frac{2x^3}{3} - \frac{2}{2}x^2 - x - \cos x - \sin x + x \ln x - x + e^x + C$$

Задача 2

$$\int (2x + 6xz^2 - 5x^2y - 3 \ln z) dx = x^2 + 3x^2z^2 - \frac{5x^3}{3}y - 3 \ln z + C$$

Задача 3

Вариант определенно неопределен

$$\int_0^{\pi} 3x^2 \sin(2x) dx = \left| \begin{array}{l} u = 3x^2 \Rightarrow du = 6x dx \\ dv = \sin 2x dx \Rightarrow v = -\frac{1}{2} \cos 2x \end{array} \right| =$$

$$= \left(-\frac{3}{2}x^2 \cos 2x \right) \Big|_0^{\pi} + 3 \int_0^{\pi} x \cos 2x dx = -\frac{3}{2}\pi^2 + 3 \int_0^{\pi} x \cos 2x dx =$$

$$= \left| \begin{array}{l} u = x \Rightarrow du = dx \\ dv = \cos 2x dx \Rightarrow v = \frac{1}{2} \sin 2x \end{array} \right| =$$

$$= \frac{3}{2}\pi^2 - \frac{3}{2}\pi^2 + \left(\frac{3}{2}x \sin 2x \right) \Big|_0^{\pi} + \frac{3}{2} \int_0^{\pi} \sin 2x dx = -\frac{3}{2}\pi^2$$

Задача 4

$$\int \frac{1}{\sqrt{x+1}} dx = \int \frac{1}{\sqrt{x+1}} d(x+1) \Big|_{x+1} = \int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt = 2\sqrt{x+1} + C$$

Задача 6

$$y' + y = x^2 y^3 \quad / : y^3$$

$$\frac{y'}{y^3} + \frac{1}{y^2} = x^2 \quad z = -\frac{1}{y^2} \Rightarrow z' = 2 \frac{y'}{y^3}$$

$$\frac{z'}{2} - z = x^2$$

$$z' - 2z = 2x^2$$

$$z' - 2z = 0 \Rightarrow \int \frac{dz}{z} = \int 2 dx = 0$$

$$\ln|z| - 2x = \ln C \Rightarrow z = Ce^{2x} \Rightarrow z = C(x)e^{2x}$$

$$C'(x)e^{2x} + 2C(x)e^{2x} - 2C(x)e^{2x} = 2x^2 \Rightarrow C'(x) = 2x^2 e^{-2x}$$

$$C(x) = \int 2x^2 e^{-2x} dx = e^{-2x} \left(x^2 + x + \frac{1}{2} \right) + C_2$$

$$y = \frac{1}{\sqrt{e^{2x} \left(x^2 + x + \frac{1}{2} \right) + C_2}}$$

Задача 5

$$(4x^2 + \frac{1}{2}y^2 - 6y)dx + (2y^3 + xy - 6x)dy = 0$$

$$U'_x = 4x^2 + \frac{1}{2}y^2 - 6y \Rightarrow U''_{xy} = y - 6$$

$$U'_y = 2y^3 + xy - 6x \Rightarrow U''_{yx} = y - 6$$

$$\text{then } U = \frac{4}{3}x^3 + \frac{1}{2}xy^2 - 6xy + \varphi(y)$$

$$U'_y = xy - 6y + \varphi'(y) \Rightarrow \varphi'(y) = 2y^3 \Rightarrow \varphi(y) = \frac{1}{2}y^4 + C$$

$$\text{Answer: } \frac{4}{3}x^3 + \frac{1}{2}xy^2 - 6xy + \frac{1}{2}y^4 + C = 0$$