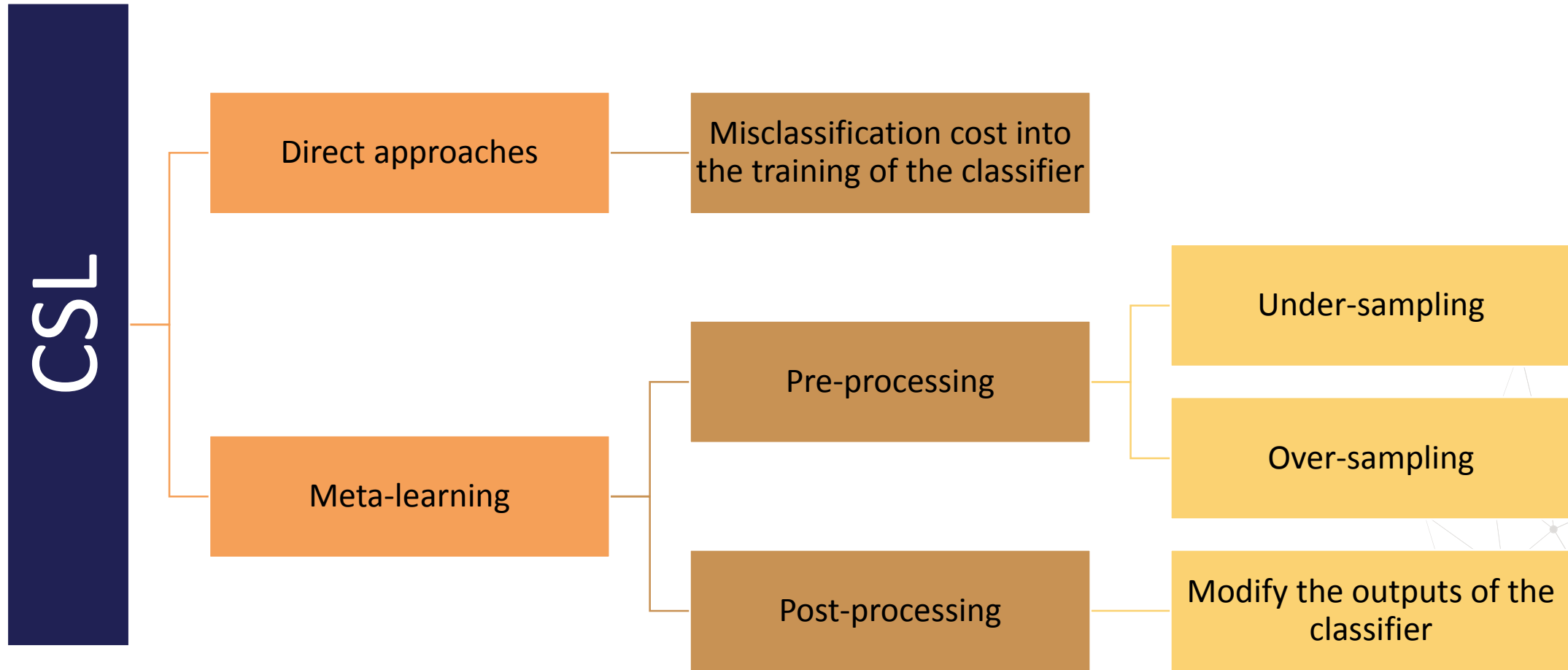




Bayes Conditional Risk

Cost Sensitive Approaches



Cost Matrix

$C(i,j)$ → cost of assigning an observation of class j to class i

$C(0,0)$ and $C(1,1)$ → Cost of correct classification, usually 0

$C(0,1)$ and $C(1,0)$ → Cost of FN and FP, respectively, usually 1

	Real Negative		Real Positive	
Predicted Negative	$C(0,0)$	TN	$C(0,1)$	FN
Predicted Positive	$C(1,0)$	FP	$C(1,1)$	TP

Cost Matrix

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$C(0,1)$ and $C(1,0)$ → Cost of FN and FP, respectively, usually 1

	Real Negative		Real Positive	
Predicted Negative	$C(0,0)$	0	$C(0,1)$	1
Predicted Positive	$C(1,0)$	1	$C(1,1)$	0

Cost Matrix

$C(i,j)$ → cost of assigning an observation of class j to class i

$C(0,0)$ and $C(1,1)$ → Cost of correct classification, usually 0

$C(0,1)$ and $C(1,0)$ → Cost of FN and FP, respectively, usually 1

	Real Negative		Real Positive	
Predicted Negative	$C(0,0)$	0	$C(0,1)$	100
Predicted Positive	$C(1,0)$	1	$C(1,1)$	0

Given a cost matrix, an observation should be classified into the class that has the minimum cost

Conditional Risk – Expected Cost

- $R(i|x)$ is the expected cost of classifying an observation into class i
- $P(j|x)$ is the probability of an observation of being of class j
- $C(i,j)$ is the cost of assigning an observation of class j to class i

$$R(i|x) = \sum_{j=1}^M P(j|x) \cdot C(i, j),$$

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$$R(i|x) = \sum_{j=1}^M P(j|x) \cdot C(i, j),$$

An observation should be classified into the class that has the minimum cost or minimum risk

Conditional Risk – Expected Cost

For observation 1, assuming binary classification, and 1 minority class

- $P(0 | 1) = 0.8$ and $P(1 | 1) = 0.2$
- $C(1,0) = 1$ and $C(0,1) = 10$
- $R(0,1) = P(0, 1) * C(0,0) + P(1 | 1) * C(0,1)$
- $R(0,1) = 0.8 * 0 + 0.2 * 10$
- $R(0,1) = 2$
- $R(1,1) = P(0, 1) * C(1,0) + P(1 | 1) * C(1,1)$
- $R(1,1) = 0.2 * 1 + 0.8 * 0$
- $R(1,1) = 0.2$

$$R(i|x) = \sum_{j=1}^M P(j|x) \cdot C(i, j),$$

Conditional Risk – Expected Cost

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- $C(1,0) = 1$ and $C(0,1) = 10$
- $R(0 | 1) = 2$ and $R(1 | 1) = 0.2$

$$R(i|x) = \sum_{j=1}^M P(j|x) \cdot C(i, j),$$

Even though $P(0 | 1) > P(1 | 1)$, $R(0 | 1) > R(1 | 1) \rightarrow$ we predict class 1

Conditional Risk – Expected Cost

The classifier will classify an instance x into positive class if and only if:

$$R(1 | x) \leq R(0 | x)$$

Conditional Risk – Expected Cost

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- $R(0,1) = P(0, 1) * C(0,0) + P(1 | 1) * C(0,1)$
- $R(1,1) = P(0, 1) * C(1,0) + P(1 | 1) * C(1,1)$

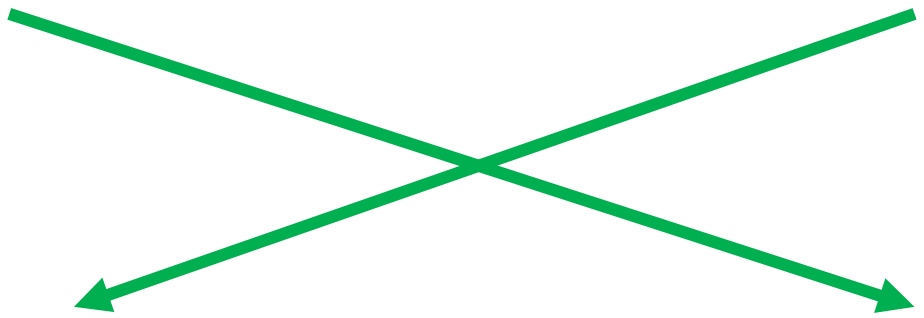

$$P(0 | x)C(1,0) + P(1 | x)C(1,1) \leq P(0 | x)C(0,0) + P(1 | x)C(0,1)$$

Conditional Risk – Expected Cost

The classifier will classify an instance x into positive class if and only if:

$$R(1 | x) \leq R(0 | x)$$

- $R(0,1) = P(0, 1) * C(0,0) + P(1 | 1) * C(0,1)$
- $R(1,1) = P(0, 1) * C(1,0) + P(1 | 1) * C(1,1)$


$$P(0 | x)C(1,0) \leq P(1 | x)C(0,1)$$

Conditional Risk – Expected Cost

$$P(0 | x)C(1,0) \leq P(1 | x)C(0,1)$$

$$(1-P(1 | x)) C(1,0) \leq P(1 | x)C(0,1)$$

$$C(1,0) - P(1 | x) C(1,0) \leq P(1 | x)C(0,1)$$

$$C(1,0) \leq P(1 | x)C(0,1) + P(1 | x) C(1,0)$$

$$C(1,0) \leq P(1 | x)(C(0,1) + C(1,0))$$

Conditional Risk – Expected Cost

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Conditional Risk – Expected Cost

We can transform the risk back to a probability, so now, I can determine the threshold of probability above which I can “confidently” classify an observation as a member of class 1

$$\frac{C(1,0)}{(C(0,1) + C(1,0))} \leq P(1|x)$$

Conditional Risk – Expected Cost

- $C(1,0) = 1$ and $C(0,1) = 10$

$$\frac{C(1,0)}{(C(0,1) + C(1,0))} \leq P(1|x)$$

$$\frac{1}{(10 + 1)} \leq P(1|x)$$

THANK YOU

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