THE RANDOM NETWORK MODEL

Network science aims to build models that reproduce the properties of real networks. Most networks we encounter do not have the comforting regularity of a crystal lattice or the predictable radial architecture of a spider web. Rather, at first inspection they look as if they were spun randomly (Figure 2.4). Random network theory embraces this apparent randomness by constructing and characterizing networks that are *truly random*.

From a modeling perspective a network is a relatively simple object, consisting of only nodes and links. The real challenge, however, is to decide where to place the links between the nodes so that we reproduce the complexity of a real system. In this respect the philosophy behind a random network is simple: We assume that this goal is best achieved by placing the links randomly between the nodes. That takes us to the definition of a random network (BOX 3.1):

A random network consists of N nodes where each node pair is connected with probability p.

To construct a random network we follow these steps:

1) Start with N isolated nodes.

2) Select a node pair and generate a random number between 0 and 1. If the number exceeds p, connect the selected node pair with a link, otherwise leave them disconnected.

3) Repeat step (2) for each of the $N(N-1)/2$ node pairs.

The network obtained after this procedure is called a *random graph* or a *random network*. Two mathematicians, Pál Erdős and Alfréd Rényi, have played an important role in understanding the properties of these networks. In their honor a random network is called the *Erdős-Rényi network* (BOX 3.2).

BOX 3.1

DEFINING RANDOM NETWORKS

There are two definitions of a random network:

***G*(*N*, *L*) Model**
N labeled nodes are connected with L randomly placed links. Erdős and Rényi used this definition in their string of papers on random networks [2-9].

***G*(*N*, *p*) Model**
Each pair of N labeled nodes is connected with probability p, a model introduced by Gilbert [10].

Hence, the $G(N, p)$ model fixes the probability p that two nodes are connected and the $G(N, L)$ model fixes the total number of links L. While in the $G(N, L)$ model the average degree of a node is simply $\langle k\rangle = 2L/N$, other network characteristics are easier to calculate in the $G(N, p)$ model. Throughout this book we will explore the $G(N, p)$ model, not only for the ease that it allows us to calculate key network characteristics, but also because in real networks the number of links rarely stays fixed.