SSC MARCH 2020

MATHEMATICS PART-II

Time allowed : 2 hours

Maximum marks : 40

General Instructions :

(i) *All questions are compulsory.*

(ii) *Use of calculator is not allowed.*

(iii) *The numbers to the right of the questions indicate full marks.*

(iv) *In case of MCQ's [Q. No. 1(A)] only the first attempt will be evaluated and will be given credit.*

(v) *For every MCQ, the correct alternative (A), (B), (C) or (D) in front of sub-question number is to be written as an answer.*

(vi) *Draw proper figures for answers wherever necessary.*

(vii) *The marks of construction should be clear and distinct. Do not erase them.*

(viii) *Diagram is essential for writing the proof of the theorem.*

1. **(A) Four alternative answers are given for every sub-question. Select the *correct* alternative and write the alphabet of that answer :** [4]

(i) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

Answer : (B) (3, 4, 5)

(ii) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other externally. What is the distance between their centres?

(A) 4.4 cm (B) 2.2 cm (C) 8.8 cm (D) 8.9 cm

Answer : (C) 8.8 cm

(iii) Distance of point (–3, 4) from the origin is ______.

(A) 7 (B) 1 (C) –5 (D) 5

Answer : (D) 5

(iv) Find the volume of a cube of side 3 cm :

(A) 27 cm^3 (B) 9 cm^3 (C) 81 cm^3 (D) 3 cm^3

Answer : (A) 27 cm^3

(B) Solve the following questions : [4]

(i) The ratio of corresponding sides of similar triangles is 3 : 5, then find the ratio of their areas.

Answer : Ratio of areas of similar triangles = (Ratio of corresponding sides of similar triangles)^2

$$= \left(\frac{3}{5}\right)^2$$

$$\text{Ratio of their areas} = \frac{9}{25}$$

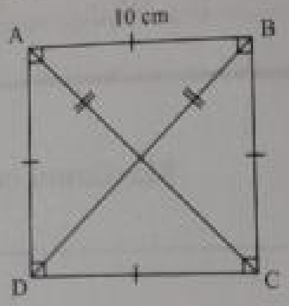
Ans.

(ii) Find the diagonal of a square whose side is 10 cm.

Answer : Let □ABCD is a square (Given)

$l(AB) = l(BC) = l(CD) = l(AD) = 10cm$

In ΔABC,



$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\therefore AC^2 = AB^2 + AB^2$ ($\because AB = BC$)

$\therefore AC^2 = 2AB^2$

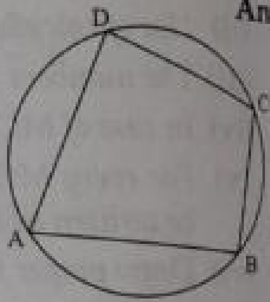
$\therefore AC = \sqrt{2}\,AB$

$= \sqrt{2}(10)cm$ (AB = 10cm)

$\therefore AC = 10 \times 1.414 = 14.14$ cm

$\therefore$ Diagonal of the square AC = 14.14 cm Ans.

(iii) □ABCD is cyclic. If $\angle B = 110°$, then find measure of $\angle D$.



Answer : □ABCD is cyclic

$\therefore m\angle B + m\angle D = 180°$

$\therefore 110° + m\angle D = 180°$ (Given, $m\angle B = 110°$)

$\therefore m\angle D = 180° - 110°$

$m\angle D = 70°$ Ans.

(iv) Find the slope of the line passing through the points A(2, 3) and B(4, 7).

Answer : Suppose $A(2, 3) = (x_1, y_1)$

and $B(4, 7) = (x_2, y_2)$

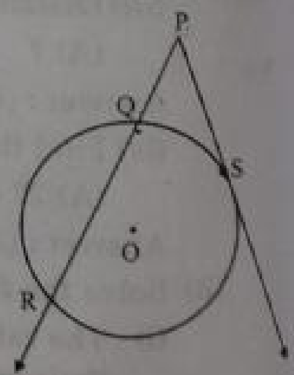
$\therefore$ Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-3}{4-2}$

$= \dfrac{4}{2} = 2$

$\therefore$ Slope of line AB = 2 unit Ans.

2. **(A) Complete and write the following activities (*Any two*) :** [4]

(i) In the figure given, 'O' is the centre of the circle, seg PS is a tangent segment and S is the point of contact. Line PR is a secant. If PQ = 3.6, QR = 6.4, find PS.



Solution : $PS^2 = PQ \times \square$ $\therefore$ **(tangent secant segment theorem)**

$= PQ \times (PQ + \square)$

$= 3.6 \times (3.6 + 6.4)$

$= 3.6 \times \square$

$= 36$

$\therefore PS = \square$ ...**(by taking square roots)**

Answer : Refer figure given in the question

$PS^2 = PQ \times \boxed{PR}$

$= PQ \times (PQ + \boxed{QR})$

$= 3.6\,(3.6 + 6.4)$

$$= 3.6 \times \boxed{10}$$
$$= 36$$
$$\therefore \quad PS = \boxed{6} \quad \text{...(by taking square roots)}$$

(ii) If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$.

Solution :

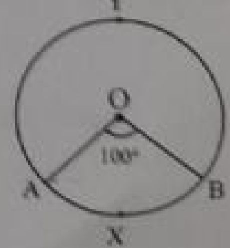
$$1 + \tan^2\theta = \sec^2\theta$$
$$\therefore \quad 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\boxed{}}$$
$$\therefore \quad \tan^2\theta = \frac{625}{49} - \boxed{}$$
$$= \frac{625 - 49}{49}$$
$$= \frac{\boxed{}}{49}$$
$$\tan\theta = \frac{\boxed{}}{7} \quad \text{...(by taking square roots)}$$

Answer :

$$1 + \tan^2\theta = \sec^2\theta$$
$$\therefore \quad 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\boxed{2}}$$
$$\therefore \quad \tan^2\theta = \frac{625}{49} - \boxed{1}$$
$$= \frac{625 - 49}{49}$$
$$= \frac{\boxed{576}}{49}$$
$$\tan\theta = \frac{\boxed{24}}{7} \quad \text{... by taking square roots}$$

(iii) In the figure given, O is the centre of the circle. Using given information complete the following table :

Type of arc	Name of the arc	Measure of the arc
Minor arc	$\boxed{}$	$\boxed{}$
Major arc	$\boxed{}$	$\boxed{}$

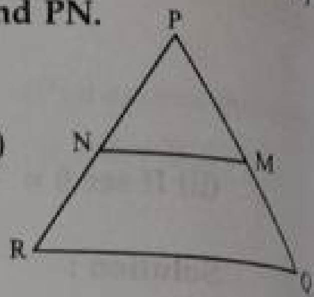


Answer :

Type of arc	Name of the arc	Measure of the arc
Minor arc	$\boxed{AXB}$	$\boxed{100^\circ}$
Major arc	$\boxed{AYB}$	$\boxed{260^\circ}$

(B) Solve the following sub-questions (*Any four*) : [8]

(i) In ΔPQR, NM || RQ. If PM = 15, MQ = 10, NR = 8, then find PN.



Answer : Given NM || RQ

$\therefore \quad \frac{PN}{NR} = \frac{PM}{MQ}$ (Basic proportionality theorem) ...(i)

But PM = 15, MQ = 10, NR = 8 (Given)

$\therefore$ Equation (i) becomes,

$$\frac{PN}{8} = \frac{15}{10}$$

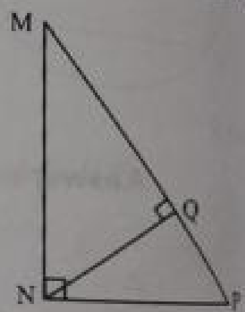
$$\therefore \quad PN = \frac{15 \times 8}{10}$$

$$= \frac{15 \times 4}{5} = 3 \times 4$$

$$\therefore \quad PN = 12 \text{ unit}$$

Ans.

(ii) In ΔMNP, $\angle MNP = 90°$, seg NQ $\perp$ seg MP. If MQ = 9, QP = 4, then find NQ.



Answer : In ΔMNP, $\angle MNP = 90°$, seg NQ $\perp$ seg MP

$\therefore$ According to right-angled triangle geometric mean subtheorem

$$NQ^2 = MQ \times QP$$

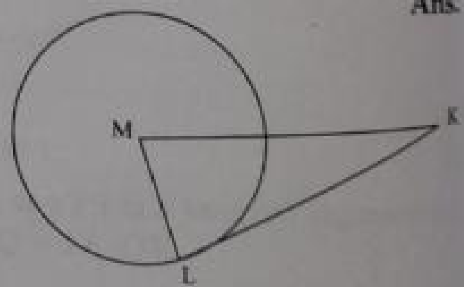
$$= 9 \times 4 = 36$$

$$\therefore \quad NQ = \sqrt{36}$$

$$= 6 \text{ unit}$$

Ans.

(iii) In the figure given above, M is the centre of the circle and seg KL is a tangent segment. L is a point of contact. If MK = 12, KL = $6\sqrt{3}$, then find the radius of the circle.



Answer : In given figure, radius ML $\perp$ tangent segment KL ...(Tangent theorem)

$$\therefore \quad m\angle MLK = 90°$$

In right-angled ΔMLK

$$MK^2 = ML^2 + LK^2 \quad \text{(According to Pythagoras theorem)}$$

$$\therefore \quad (12)^2 = ML^2 + (6\sqrt{3})^2$$

$$\therefore \quad 144 = ML^2 + 108$$

$$\therefore \quad ML^2 = 144 - 108 = 36$$

$$\therefore \quad ML = 6$$

$$\therefore \quad \text{Radius } ML = 6 \text{ unit}$$

Ans.

(iv) Find the co-ordinates of midpoint of the segment joining the points (22, 20) and (0, 16).

Answer : Given points are (22, 20) and (0, 16)

Let, $x_1 = 22$, $x_2 = 0$, $y_1 = 20$, $y_2 = 16$

We Know, $\quad \text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

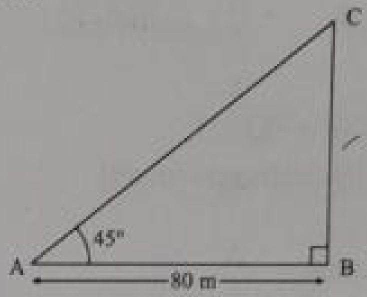
$$= \left(\frac{22+0}{2}, \frac{20+16}{2}\right)$$

$$= \left(\frac{22}{2}, \frac{36}{2}\right)$$

$$= (11, 18)$$ **Ans.**

(v) A person is standing at a distance of 80 metres from a Church and looking at its top. The angle of elevation is of 45°. Find the height of the Church.

Answer : Let the height of church be BC



In ΔABC,

$\therefore \quad \tan 45° = \frac{BC}{AB}$

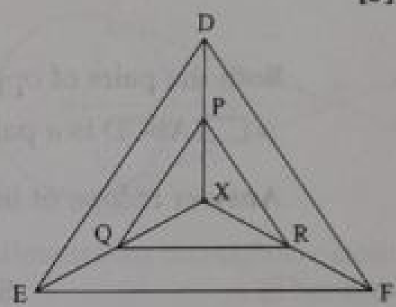
$1 = \frac{BC}{80} \qquad [\because \tan 45° = 1]$

$BC = 80$ m

$\therefore$ The height of the church BC = 80 m. **Ans.**

3. (A) Complete and write the following activities (*Any one*) : **[3]**

(i) In the given figure, X is any point in the interior of the triangle. Point X is joined to the vertices of triangle. seg PQ || seg DE, seg QR || seg EF. Complete the activity and prove that seg PR || seg DF.



Proof : In ΔXDE

PQ || DE ...(Given)

$\therefore \quad \frac{XP}{PD} = \frac{\square}{QE}$ ...(Basic propatianality theorem) ...(i)

In ΔXEF

QR || EF ...(Given)

$\therefore \quad \frac{XQ}{\square} = \frac{XR}{\square}$ ...($\square$) ...(ii)

$\therefore \quad \frac{XP}{PD} = \frac{\square}{\square}$ ...[From (i) and (ii)]

$\therefore$ Seg PR || seg DF ...(By converse of basic proportionality theorem)

Answer : In ΔXDE

PQ || DE ...(Given)

$\therefore \quad \frac{XP}{PD} = \frac{\boxed{XQ}}{QE}$ ...(Basic proportionality theorem) ...(i)

In ΔXEF

QR || EF ...(Given)

$\therefore \quad \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ ...($\boxed{\text{Basic proportionality theorem}}$) ...(ii)

$\therefore \quad \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ ...[From (i) and (ii)]

$\therefore$ Seg PR || seg DF ...(By converse of basic proportionality theorem)

(ii) If A(6, 1), B(8, 2), C(9, 4) and D(7, 3) are the vertices of □ABCD, show that □ABCD is a parallelogram.

Answer : Slope of line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of line AB $= \dfrac{2-1}{8-6} = \boxed{}$ ...(i)

$\therefore$ Slope of line BC $= \dfrac{4-2}{9-8} = \boxed{}$ ...(ii)

$\therefore$ Slope of line CD $= \dfrac{3-4}{7-9} = \boxed{}$ ...(iii)

$\therefore$ Slope of line DA $= \dfrac{3-1}{7-6} = \boxed{}$ ...(iv)

$\therefore$ Slope of line AB $= \boxed{}$ [From (i) and (iii)]

line AB || line CD

$\therefore$ Slope of line BC $= \boxed{}$ [From (ii) and (iv)]

$\therefore$ line BC || line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore$ □ABCD is a parallelogram.

Answer : Slope of line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of line AB $= \dfrac{2-1}{8-6} = \boxed{\dfrac{1}{2}}$ ...(i)

$\therefore$ Slope of line BC $= \dfrac{4-2}{9-8} = \boxed{2}$ ...(ii)

$\therefore$ Slope of line CD $= \dfrac{3-4}{7-9} = \boxed{\dfrac{1}{2}}$ ...(iii)

$\therefore$ Slope of line DA $= \dfrac{3-1}{7-6} = \boxed{2}$ ...(iv)

$\therefore$ Slope of line AB $= \boxed{\text{CD}}$ [From (i) and (iii)]

line AB || line CD

$\therefore$ Slope of line BC $= \boxed{\text{DA}}$ [From (ii) and (iv)]

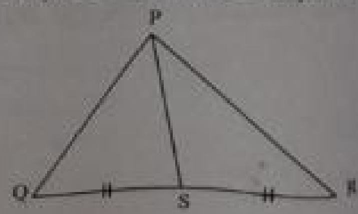
$\therefore$ line BC || line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore$ □ABCD is a parallelogram.

(B) Solve the following sub-questions (*Any two*) : [6]

(i) If ΔPQR, point S is the mid-point of side QR. If PQ = 11, PR = 17, PS = 13, find QR.



Answer : In ΔPQR, point S is the mid-point of side QR.

$\therefore$ Segment PS is median of ΔPQR

According to Apollonius's theorem

$$PQ^2 + PR^2 = 2PS^2 + 2QS^2$$

As per given values,

$\therefore \quad (11)^2 + (17)^2 = 2(13)^2 + 2QS^2$

$\therefore \quad 121 + 289 = 2(169) + 2QS^2$

$\therefore \quad 410 = 338 + 2QS^2$

$\therefore \quad 2QS^2 = 410 - 338 = 72$

$\therefore \quad QS^2 = \dfrac{72}{2} = 36$

$\therefore \quad QS = 6$ unit ...(i)

We know, point S is the mid-point of side QR

$\therefore \quad 2QS = QR$ ($\because$ QS = SR)

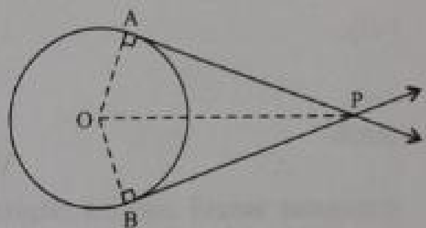
$\therefore \quad QR = 2\ (6)$ [From equation (i)]

$\therefore \quad QR = 12$ unit

$\therefore$ Length of side QR is 12 unit. **Ans.**

(ii) Prove that, tangent segments drawn from an external point to the circle are congruent.

Answer : Point O is the centre of the circle and point P is external to the circle. Segment PA and segment PB are tangent segments to the circle. Point A and point B are touch points of the tangent segments.



Prove : PA ≅ PB

Construction : Draw OA, OB and OP.

Proof : $\because$ Each tangent of a circle is perpendicular to the radius drawn through the point of contact ... (Theorem)

$\therefore$ Radius OA ⊥ AP and, Radius OB ⊥ BP ...(i)

$\therefore$ $m\angle PAO = 90°$ and $m\angle PBO = 90°$

$\therefore$ ΔPAO and ΔPBO are right-angled triangles.

Now in ΔPAO and ΔPBO,

OA = OB ($\because$ Radius of same circle)

$\angle PAO = \angle PBO$ [using (i)]

Hypotenuse OP = Hypotenuse OP ($\because$ common side)

$\therefore$ ΔPAO ≅ ΔPBO (RHS conguruency criterion)

$\therefore$ line PA ≅ line PB ($\because$ corresponding sides of congruent triangles)

Line PA and line PB are tangent. **Hence proved.**

(iii) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

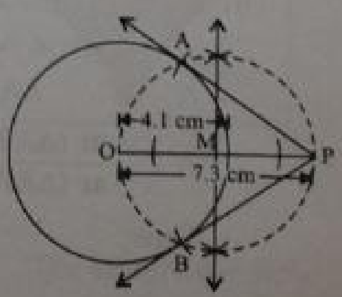
Answer :

Steps of construction :

Step 1 : Draw a circle of radius 4.1 cm with centre O.

Step 2 : Take a point P in the exterior of the circle such that OP = 7.3 cm.

Step 3 : Draw segment OP. Draw perpendicular bisector of segment OP to get its midpoint M.



Step 4 : Draw a circle with radius OM and centre M.

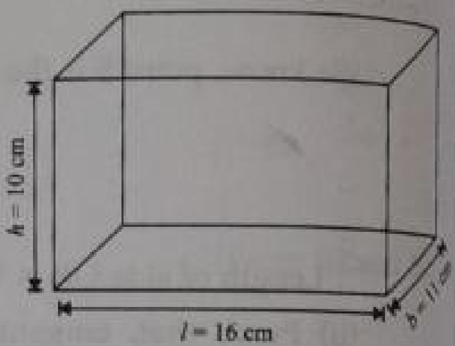
Step 5 : Name the point of intersection of the two circles as A and B.

Step 6 : Join PA and PB.

Thus PA and PB are required tangents.

(iv) A metal cuboid of measures 16 cm × 11 cm × 10 cm was melted to make coins. How many coins were made, if the thickness and diameter of each coin was 2 mm and 2 cm respectively? (π = 3.14)

Answer : We know, Volume of cuboid = $l \times b \times h$



$$= 16 \times 11 \times 10$$

$$= 1760 \text{ cm}^3$$

Now, Volume of coin = (Area of coin) × (thickness)

$$\text{Area of coin} = \frac{\pi d^2}{4}$$

$$= \frac{3.14 \times (2)^2}{4}$$

$$= \frac{3.14 \times 4}{4}$$

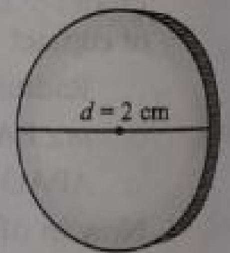
$$= 3.14 \text{ cm}^2$$

$\therefore$ Volume of coin = $3.14 \times t$

$$= 3.14 \times (0.2) \quad ...(t = 2\text{mm} = 0.2 \text{ cm})$$

$$= 0.628 \text{ cm}^2$$

Now, let N number of coins are made from melted cuboid.



$\therefore$ Volume of cuboid = N × volume of coin

$\therefore$ $1760 = N \times 0.628$

$\therefore$ $N = \dfrac{1760}{0.628} = 2802.547$

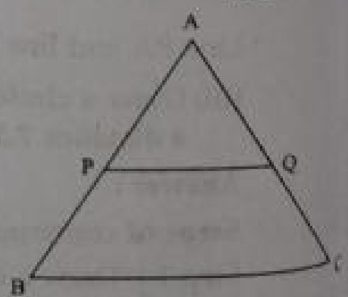
$\therefore$ $N \approx 2802$

$\therefore$ 2802 coins were made. **Ans.**

4. Solve the following sub-questions (*Any two*) : **[8]**

(i) In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that seg PQ || seg BC. If PQ divides ΔABC into two equal parts having equal areas, find $\dfrac{BP}{AB}$.

Answer : In above figure ΔABC, PQ || BC



A – P – B and A – Q – C

and ar(ΔAPQ) = ar($\square$PBCQ)...

In ΔAPQ and ΔABC

$\angle$A = $\angle$A ...(common angle)

$\angle$APQ = $\angle$ABC ...(corresponding angle)

$\therefore$ ΔAPQ ~ ΔABC ...(A-A similarity test)

$\therefore$ $\dfrac{ar(\Delta ABC)}{ar(\Delta APQ)} = \dfrac{AB^2}{AP^2}$ (Theorem of areas of similar triangles) ...(i)

Now,

ar(ΔAPQ) = ar($\square$PBCQ) (Given)

$$\therefore \quad \frac{ar[\square PBCQ]}{ar(\Delta APQ)} = \frac{1}{1}$$

$$\therefore \quad \frac{ar[\square PBCQ] + ar(\Delta APQ)}{ar(\Delta APQ)} = \frac{1+1}{1} = \frac{2}{1}$$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \frac{2}{1} \quad ...(ii) \; [\because ar(\Delta APQ) + ar(\square PBCQ) = ar(\Delta ABC)]$$

$\therefore$ From (i) and (ii)

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \frac{2}{1} = \frac{AB^2}{AP^2}$$

$$\frac{AB}{AP} = \frac{\sqrt{2}}{1} \quad \text{(by taking square roots on both sides)}$$

Let $\quad AB = \sqrt{2}\,x \quad ...(iii)$

and $\quad AP = 1x$

Now, $\quad BP = AB - AP$

$\therefore \quad BP = \sqrt{2}\,x - 1x = (\sqrt{2} - 1)x \quad ...(iv)$

From (iii) and (iv)

$$\therefore \quad \frac{BP}{AB} = \frac{(\sqrt{2}-1)}{\sqrt{2}} \quad \textbf{Ans.}$$

(ii) Draw a circle of radius 2.7 cm and draw a chord PQ of length 4.5 cm. Draw tangents at points P and Q without using centre.

Answer :

Steps of construction :

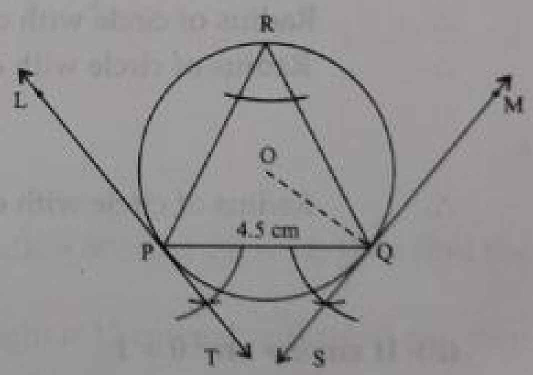
Step 1 : Draw a circle of with centre O and radius 2.7 cm.

Step 2 : Draw a chord PQ of length 4.5 cm

Step 3 : Taking a point R on the major arc QP, join PR and QR.

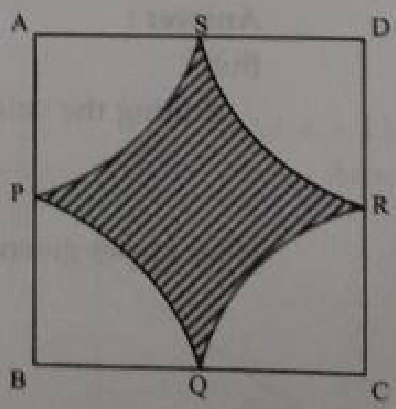
Step 4 : Make $\angle QPT = \angle PRQ$ and $\angle PQS = \angle PRQ$.

Step 5 : Produce TP to L and SQ to M.



Hence, TPL and SQM are the required tangents.

(iii) In the figure given $\square$ABCD is a square of side 50 m. Points P, Q, R, S are midpoints of side AB, side BC, side CD, side AD respectively. Find area of shaded region.



Answer : Area of $\square$ABCD = side^2 ...($\because$ Square)

$= 50^2 = 2500 \text{ m}^2$

There are 4 sectors drawn within square.

In $\square$ABCD with its points A, B, C, D as vertices and radius of each equal to half of side of square *i.e.*, 25m

$$\therefore \quad \text{Area of any one sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 25 \times 25 \quad ...(\text{For square } \theta = 90^\circ)$$

$$= \frac{1}{4} \times 3.14 \times 625$$

$$= 0.785 \times 625$$

$$= 490.625 \text{ m}^2$$

Area of all three sectors = 4 × (Area of one sector)

$$= 4 \times 490.625$$

$$= 1962.5\text{m}^2$$

Now, Area of shaded portion = (Area of square) – (Area of four sectors)

$$= 2500 - 1962.5$$

$\therefore$ Area of shaded portion = 537.5m^2

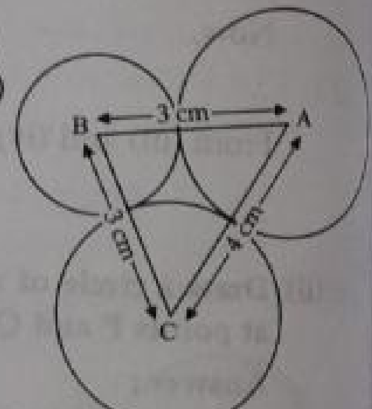
5. Solve the following sub-questions (*Any one*) : [3]

(i) Circles with centres A, B and C touch each other externally. If AB = 3 cm, BC = 3 cm, CA = 4 cm, then find the radii of each circle.

Answer : Suppose radius of circle with centre A is x cm

$\therefore$ Radius of circle with centre B = $(3 - x)$ cm ($\because$ AB = 3cm)

and radius of circle with centre C = $(4 - x)$ cm ($\because$ CA = 4cm)



$\therefore$ $(3 - x) + (4 - x) = BC = 3$

$\therefore$ $3 - x + 4 - x = 3$

$\therefore$ $7 - 2x = 3$

$\therefore$ $2x = 7 - 3$

$\therefore$ $2x = 4$

$\therefore$ $x = 2$

$\therefore$ Radius of circle with centre A = 2cm

$\therefore$ Radius of circle with centre B = $(3 - x)$

$= (3 - 2)$

$= 1$ cm

$\therefore$ Radius of circle with centre C = $(4 - x)$

$= (4 - 2)$

$= 2$ cm

(ii) If $\sin\theta + \sin^2\theta = 1$

show that : $\cos^2\theta + \cos^4\theta = 1$

Answer :

$\sin\theta + \sin^2\theta = 1$ ...(Given)

But $\sin^2\theta + \cos^2\theta = 1$ ...(Standard result)

$\therefore$ Putting the value 1 in given relation we get.

$$\sin\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta$$

$\therefore$ $\sin\theta = \cos^2\theta$ ...(i)

Now as per given relation

$$\sin\theta + \sin^2\theta = 1$$

$\therefore$ $\cos^2\theta + (\cos^2\theta)^2 = 1$ [...From equation (i)]

$\therefore$ $\cos^2\theta + \cos^4\theta = 1$

Hence proved.