ARPM Project [1] Description

This project models a market consisting of equities and foreign exchange.
A UK investor longs US equity market by selecting i equities from different sectors for portfolio diversification. The investor aims to compute and analyse the distributions of profit and loss dominated in GBP in Δt working days [2].

Step 1: Risk Drivers: $X_{i,t}^{stock} = \log V_{i,t}^{stock}, X_t^{fx} = \log FX_t^{\$ \to £}$

Step 2: Extract Invariants

Equity risk drivers are assumed to follow a GARCH(1, 1) with flexible probability process:

$$\Delta X_{i,t}^{stock} = X_{i,t}^{stock} - X_{i,t-1}^{stock} = \mu_i + \Sigma_{i,t}\epsilon_{i,t}^{stock}$$

$$\Sigma_{i,t}^2 = c_i + b_i\Sigma_{i,t-1}^2 + a_i(\Delta X_{i,t-1}^{stock} - \mu_i)^2$$

Foreign exchange risk drivers are assumed to follow a random walk process: $\epsilon_t^{fx} = X_t^{fx} - X_{t-1}^{fx}$
GARCH(1, 1) parameters are estimated by function **fit_garch_fp**. $(\epsilon_{i,t}^{stock}, \epsilon_t^{fx})\prime = \epsilon_t$ are the i.i.d. invariants.

Step 3: Estimation: Copula-Marginal

Flexible probabilities are specified via time and state conditioning, market state indicator is smoothed and scored $\log VIX$.
Marginal distribution of $\epsilon_{i,t}^{stock}$ is estimated non-parametrically, scenario probability distributed: $\{\epsilon_{i,t}^{stock}, p_{i,t}\}_{t=0}^{\bar{t}}$.
Marginal distribution of ϵ_t^{fx} is estimated parametrically under Student t assumption by function **fit_locdisp_mlfp**.
Grades of equity invariants are obtained by function **cop_marg_sep**: $\{\epsilon_{i,t}^{stock}, p_{i,t}\}_{t=0}^{\bar{t}} \to \{u_{i,t}^{stock}\}_{t=0}^{\bar{t}}$
Grades of FX invariants are obtained analytically by applying Student t CDF: $\{F_{\mu_i,\sigma_i^2,\nu_i}^t(\epsilon_{i,t}^{fx})\}_{t=0}^{\bar{t}} \to \{u_{i,t}^{fx}\}_{t=0}^{\bar{t}}$
Joint estimation of the t copula: firstly, use the quantile function of standard Student t distribution $t(0,1,\nu)$ to standardise invariants $\Phi_\nu^{-1}(u_{(i,t)}) \equiv \tilde{\epsilon}_{i,t}$; then, estimate copula parameters with maximum likelihood by function **fit_locdisp_mlfp_difflength** to obtain correlation matrix ρ and degree of freedom ν; finally, glue together: $CopMarg(\hat{f}_U, \{\hat{f}_{\varepsilon_i}\}_{i=1}^{\bar{i}}) \Leftrightarrow \varepsilon_t \sim \hat{f}_\varepsilon$

Step 4: Projection

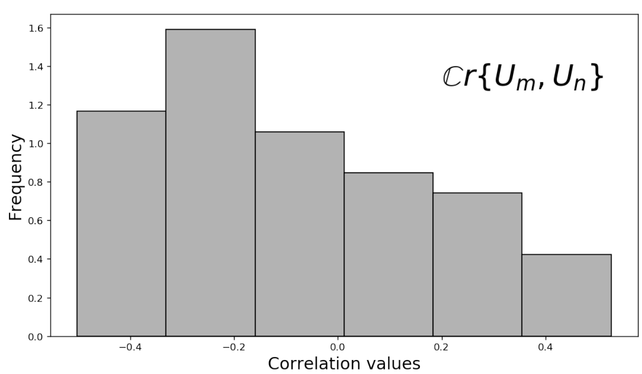
The model generates $J = 10000$ scenarios for the next step shocks.
Project t-copula standardised invariants scenarios: Using function **simulate_t** to simulate standardised invariants scenarios $\tilde{\varepsilon}_{t_m}$ for copula, then computing their grades u_{i,t_m}^j by applying the CDF of a standard Student $t(0, 1, \hat{\nu}^{copula})$.
For equities, feeding the copula scenario u_{i,t_m}^j into historical quantile to obtain projected invariants by function **quantile_sp**.
For FX, projected invariants are obtained by feeding the scenario grades into Φ_ν^{-1}.
Put $\{\epsilon_{i,t+\Delta t}^{stock,(j)}\}_{j=1}^J$ back into GARCH(1, 1) to generate projected equity risk drivers $\{x_{i,t+\Delta t}^{stock,(j)}\}_{j=1}^J$
Put $\{\epsilon_{t+\Delta t}^{fx,(j)}\}_{j=1}^J$ back into Random Walk process to generate projected FX risk drivers $\{x_{t+\Delta t}^{fx,(j)}\}_{j=1}^J$

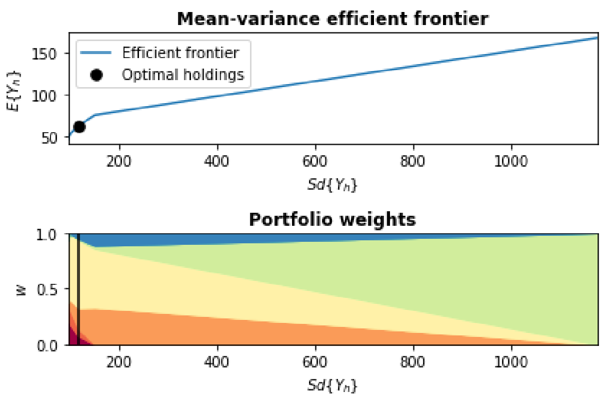
Step 5: Pricing: denominated in GBP £

In each scenario j, profit and loss of each equity denominated in British Sterling is: $\pi_{i,t+\Delta t}^{(j),£} = e^{x_{t+\Delta t}^{fx,(j)} + x_{i,t+\Delta t}^{stock,(j)}} - v_{t_{now}} e^{x_{t_{now}}^{fx}}$

Step 6, 7, 8: Portfolio Construction, Evaluation and Attribution

Step 6 computes the optimal holdings $\boldsymbol{h}^* \equiv (h_1^*, ..., h_i^*)$ that maximises the satisfaction: Sharpe Ratio. Budget is £10000, expected return (0.87%), volatility(4%) and expected performance with optimised holdings are presented (right figure)
Step 7 summaries the satisfaction of PnL by mean-variance trade-off, certainty equivalent (exponential utility function $ut(y) = -e^{-\lambda y}$), VaR (by function **quantile_sp**) , expected shortfall (by function **spectral_index**).
Step 8 applies principal component LFM (by function **pca_cov**) to construct factors $\mathbf{Z}^{PC}$ and loadings $\boldsymbol{\beta}$ for attribution analysis.
The model verifies non-zero correlation among residuals, truncated standard deviation is computed (left figure)





[1] Author, Figo Lau, PhD
[2] This project analyses PnL (£) of 11 equities (\$) in 10 working days. Data time series t: from 01/03/2020 to 17/01/2022.