# Neural Network Regularization

## Part C - Batch Normalization

CS1090B Data Science II – Spring 2025

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# Feature Normalization

It is a good practice to normalize features before applying the learning algorithm:

Feature vector

Vector of mean feature values

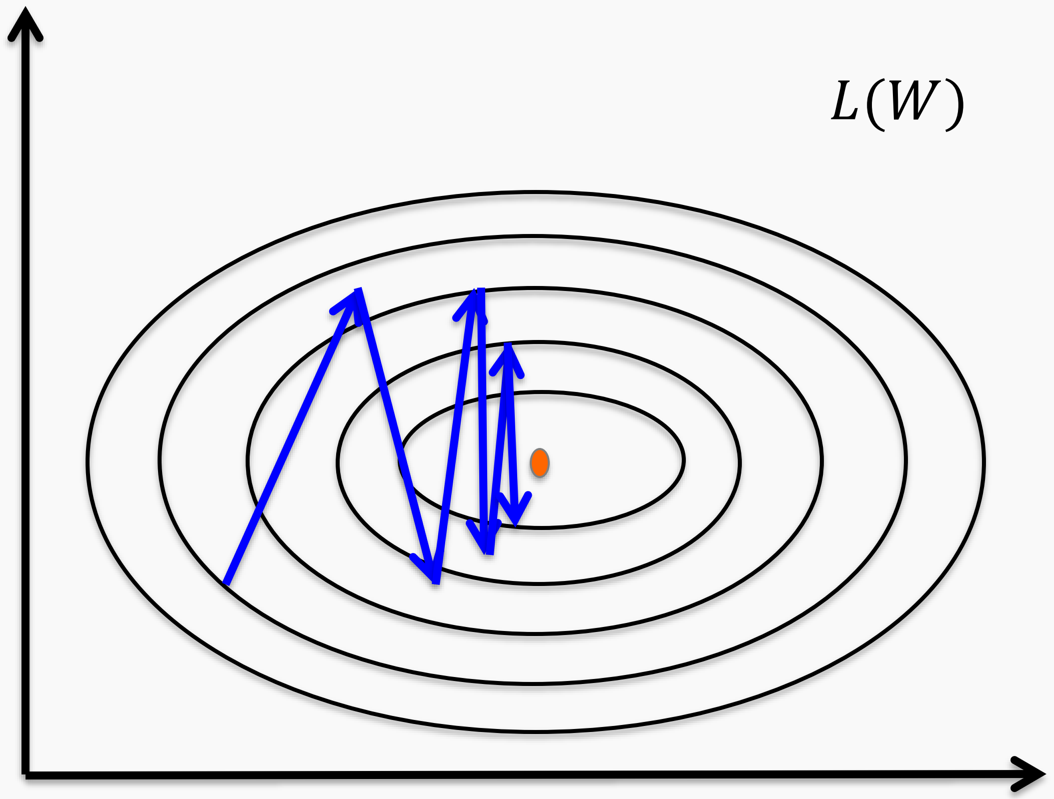
$$\boxed{x' = \frac{x - \mu}{\sigma}}$$

Vector of SD of feature values

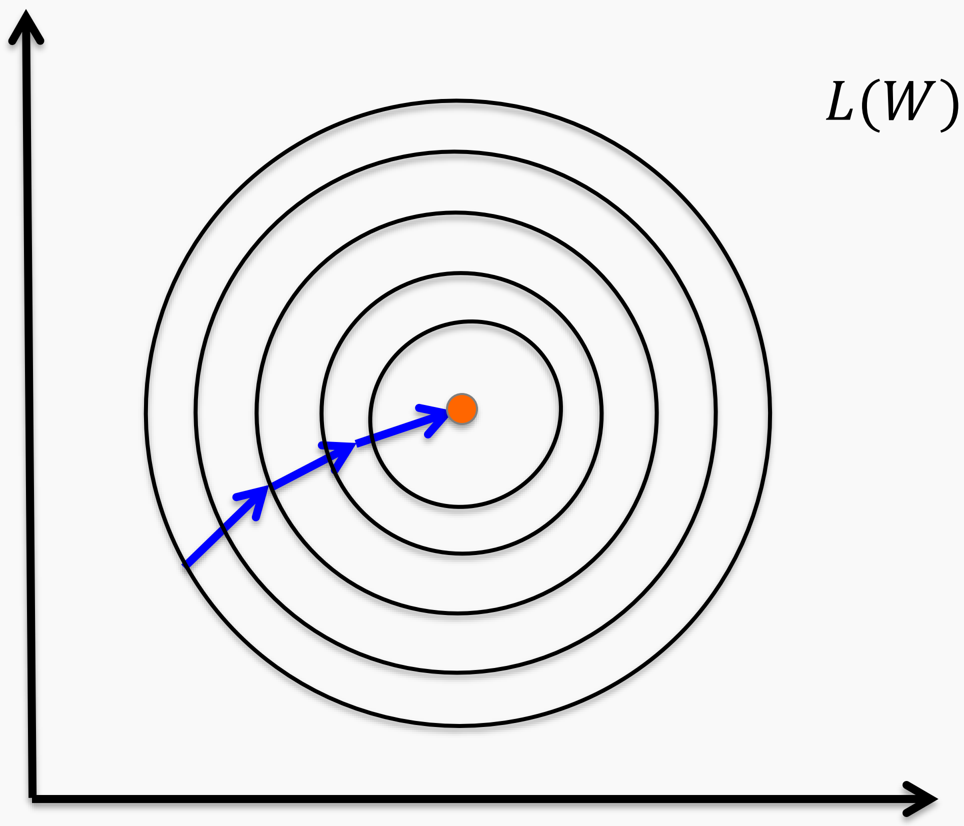
Features in the same scale: mean 0 and variance 1

## Feature Normalization

Speeds up learning



Before normalization



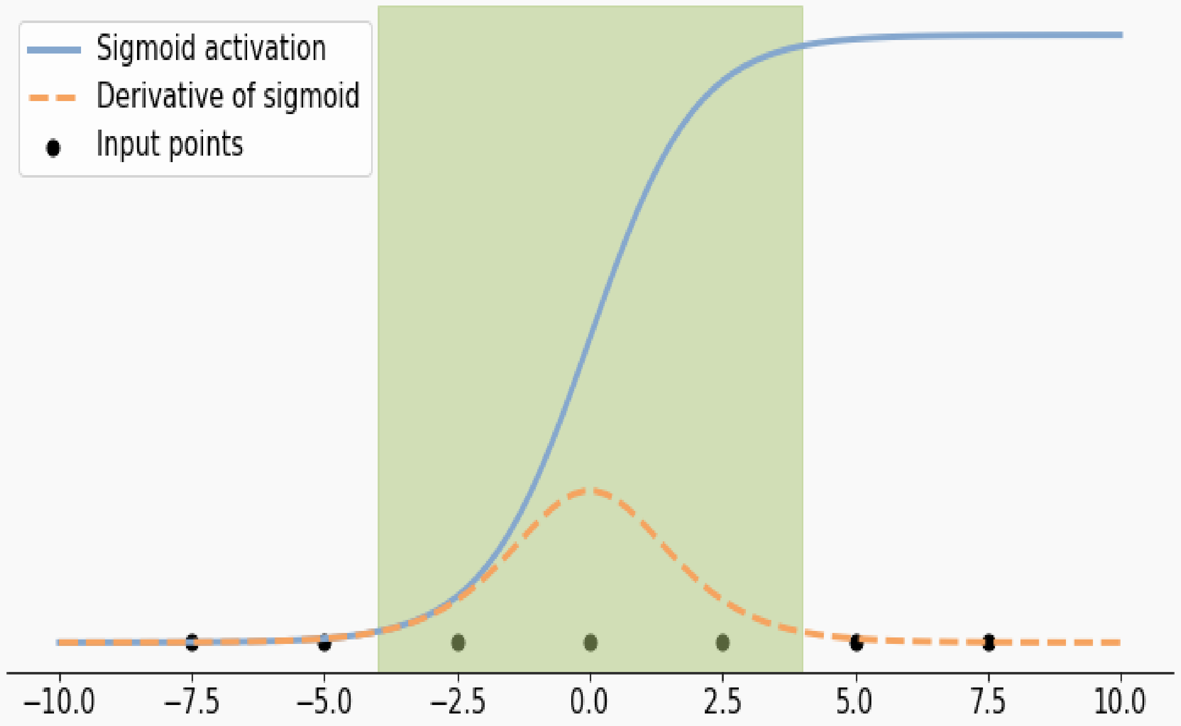
After normalization

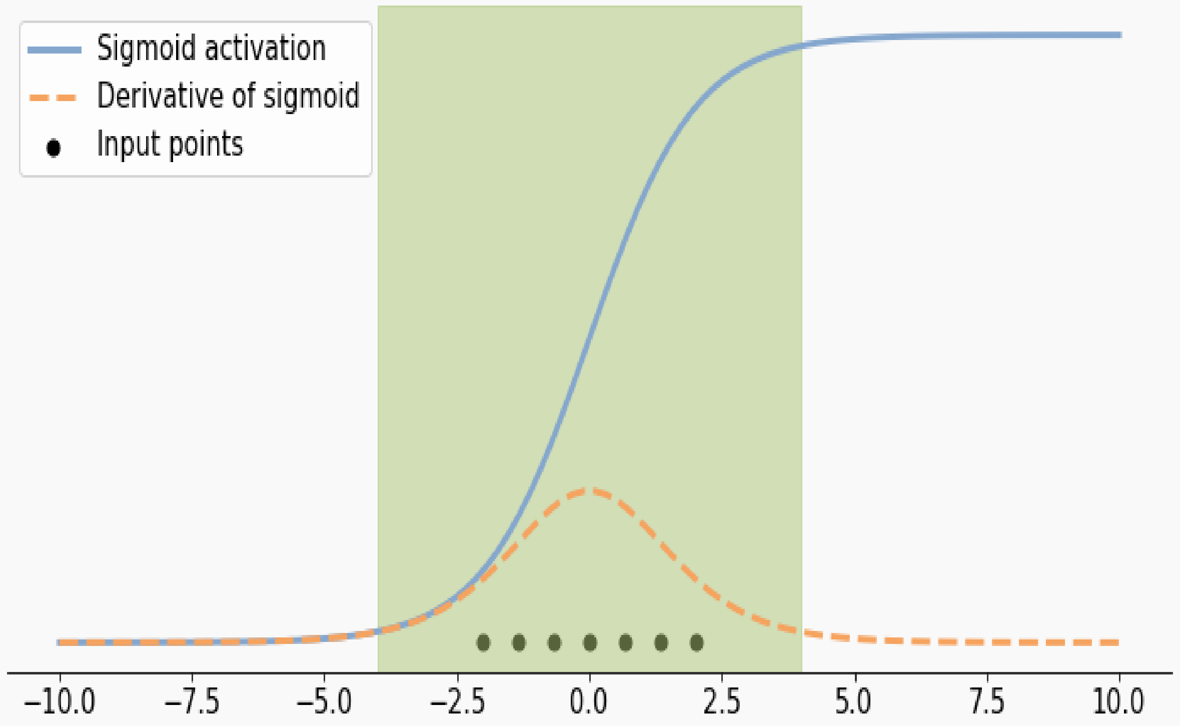
**Note:** This is an ideal case scenario. In reality, the loss landscapes are much more complex.

# How do neural networks learn efficiently?

The distribution that is fed to the layers of a network should be somewhat:

- Normalized - to avoid vanishing gradients





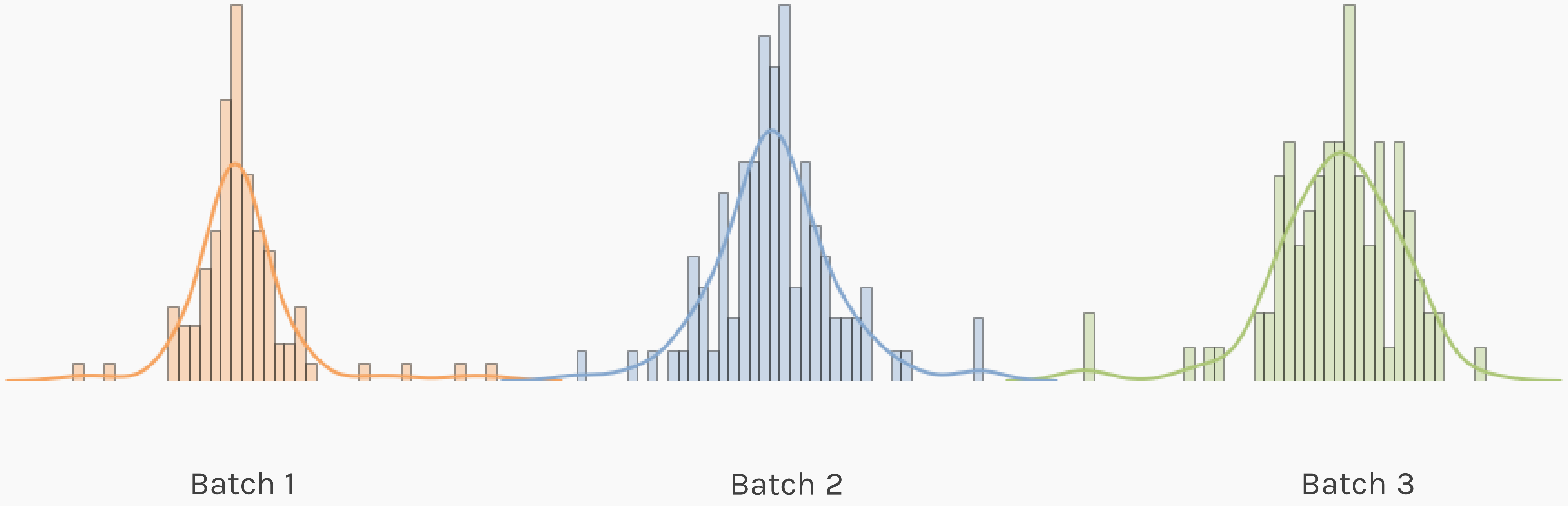
## How do neural networks learn efficiently?

The distribution that is fed to the layers of a network should be somewhat:

- Normalized - to avoid vanishing gradients
- Constant through epochs or batches and data

## Internal Covariance Shift (ICS)

ICS occurs when the input distribution to the hidden layers (hence “internal”) of the neural network end up fluctuating or shifting.



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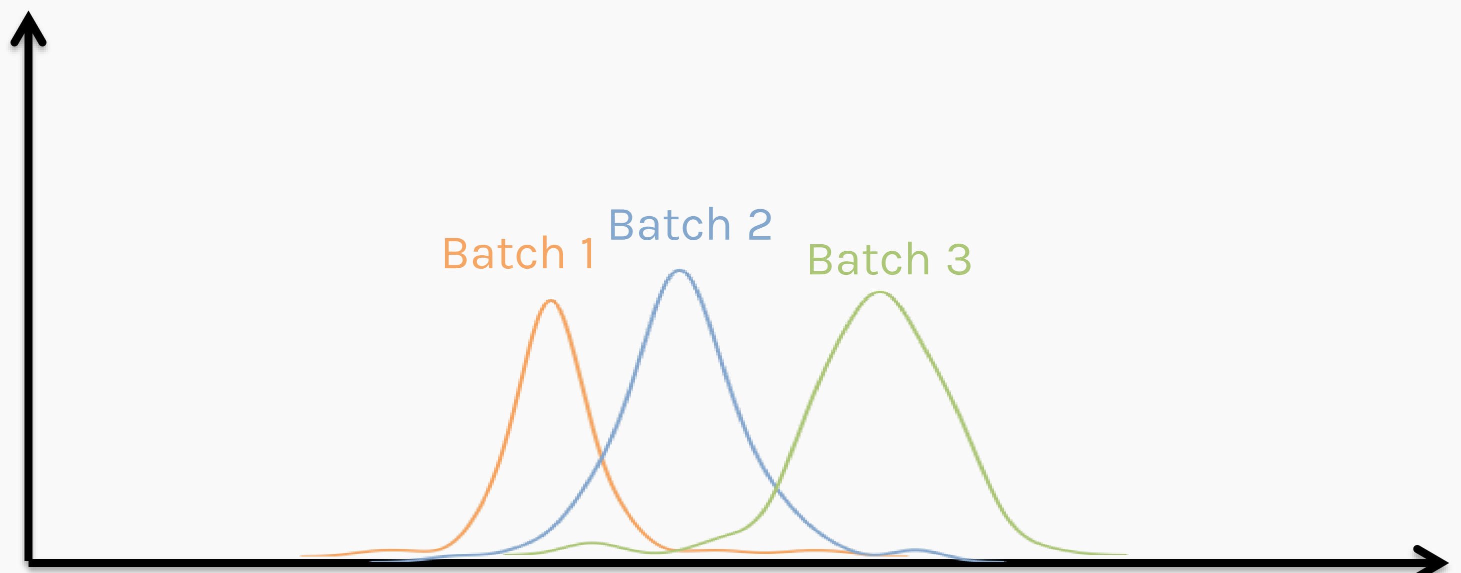
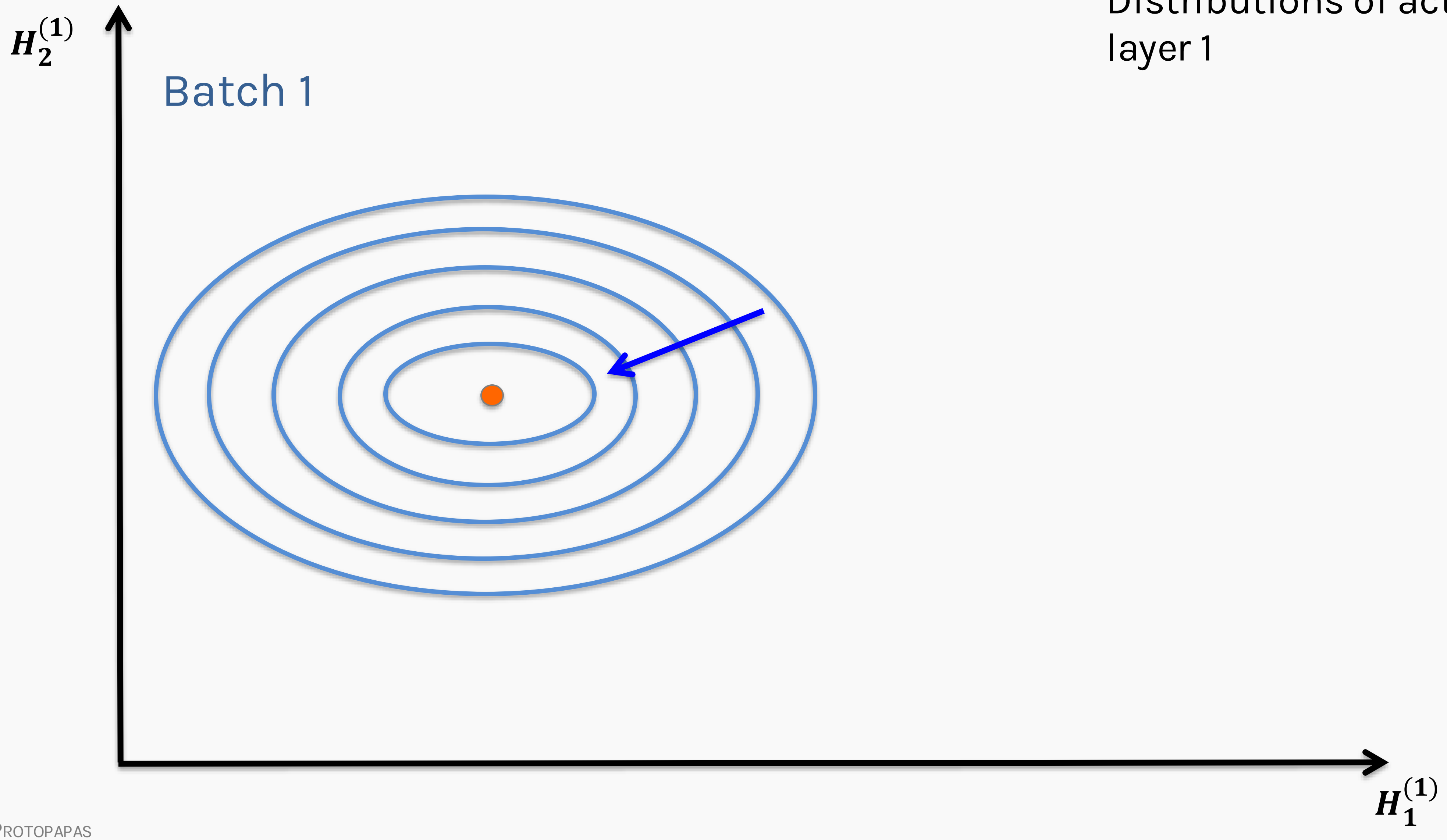


Distribution of the layer outputs across batches

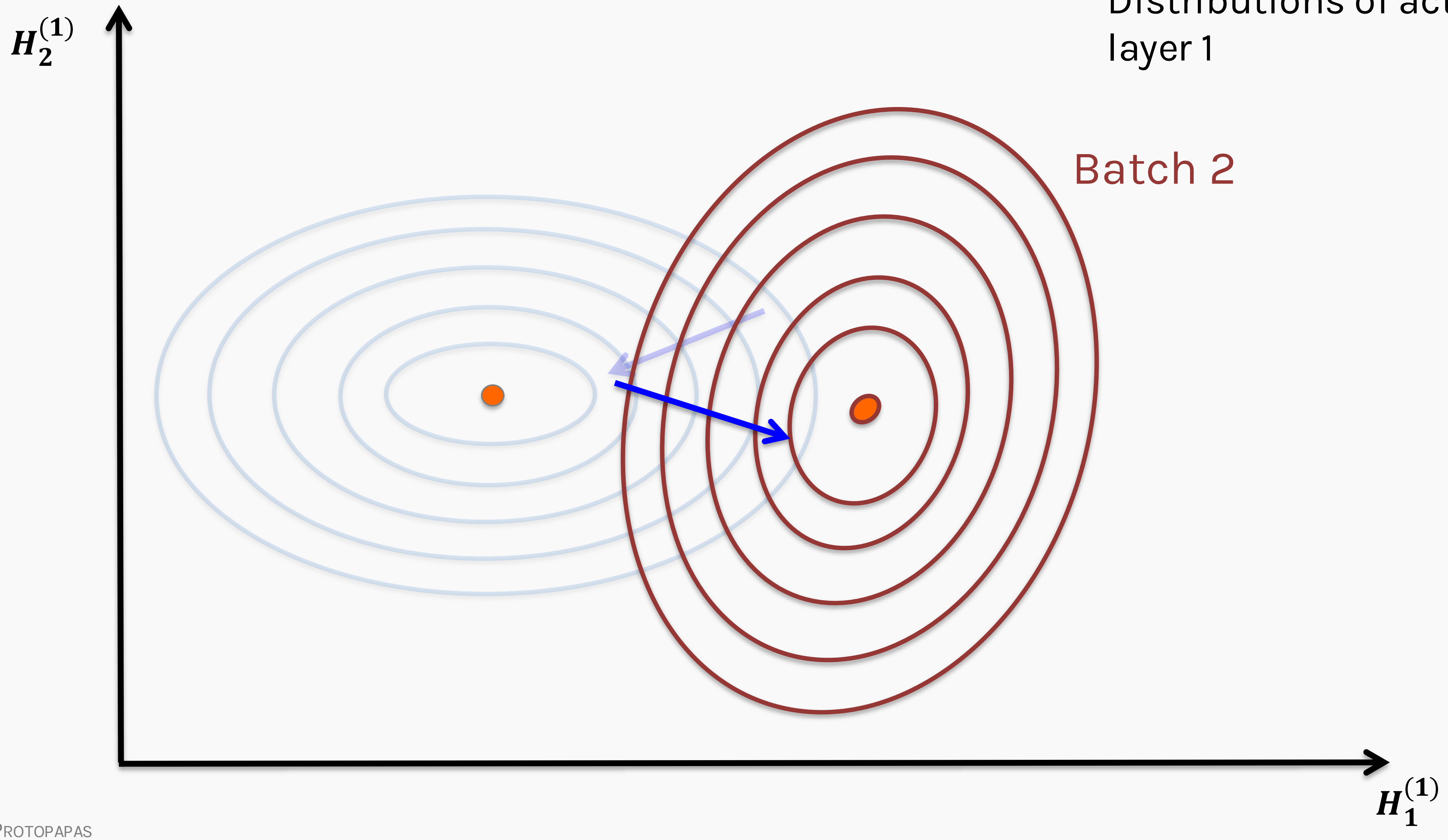
# Internal Covariance Shift



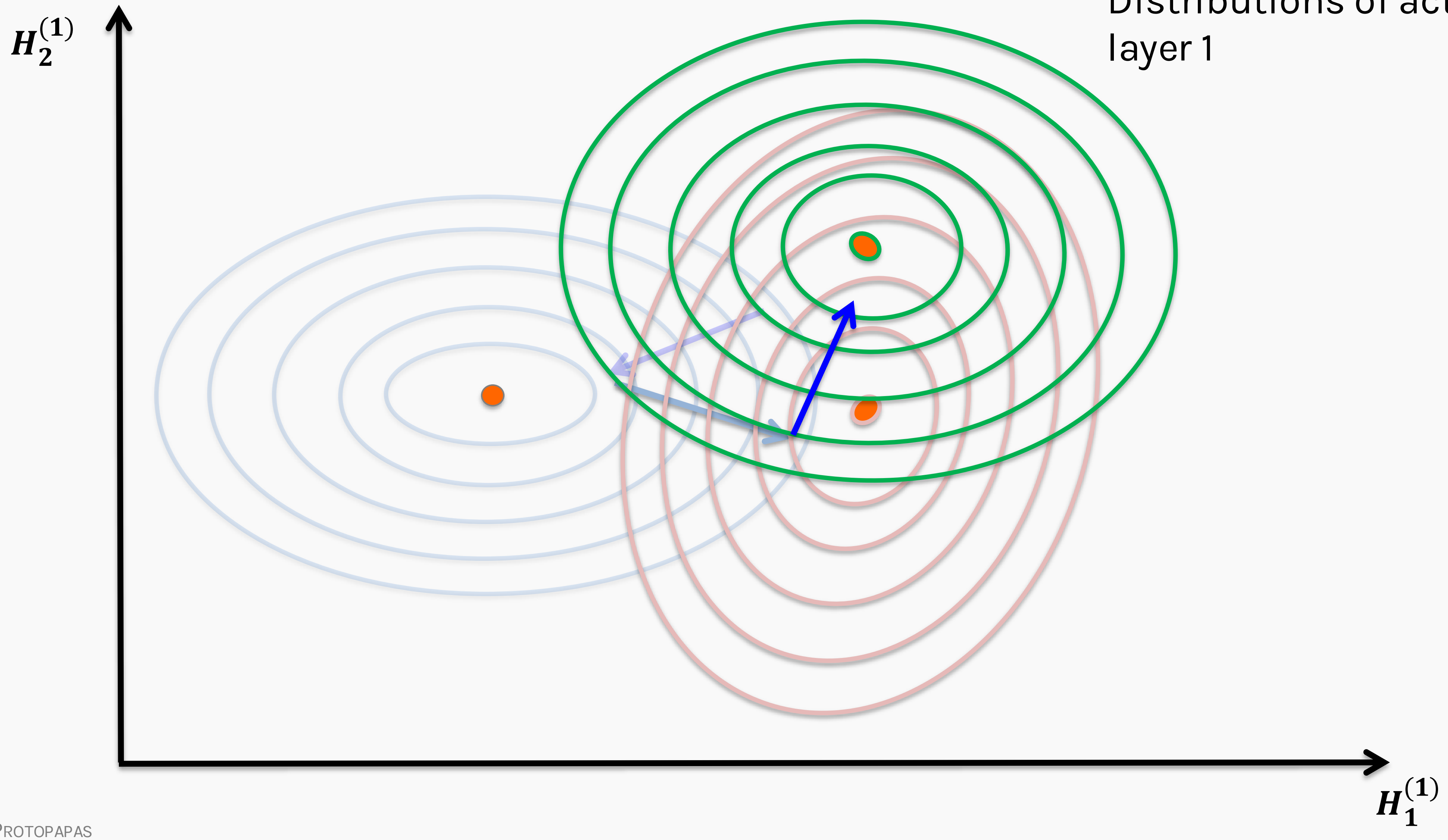
Distributions of activations of hidden layer 1

## Internal Covariance Shift

Distributions of activations of hidden layer 1



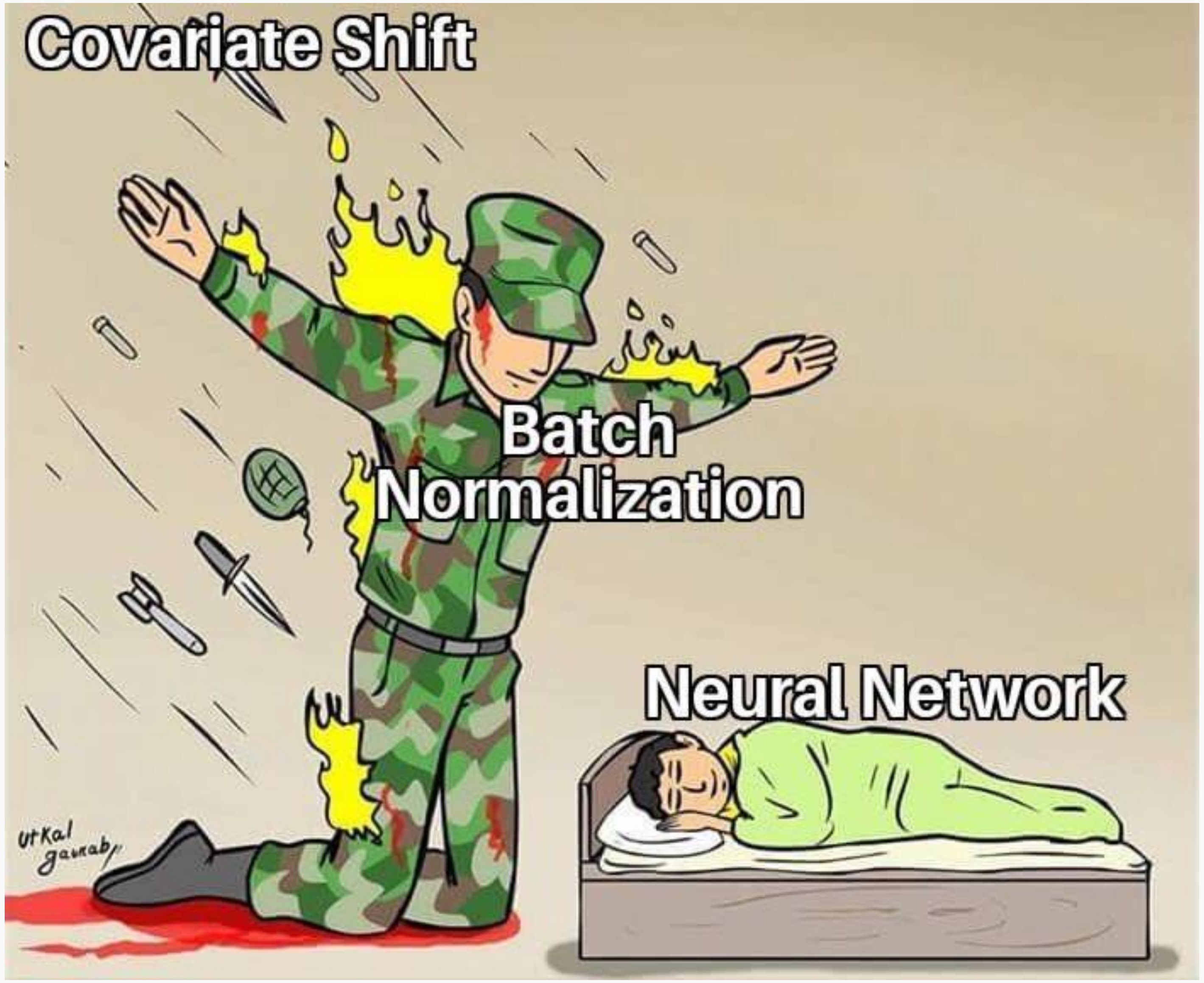
## Internal Covariance Shift



Distributions of activations of hidden layer 1

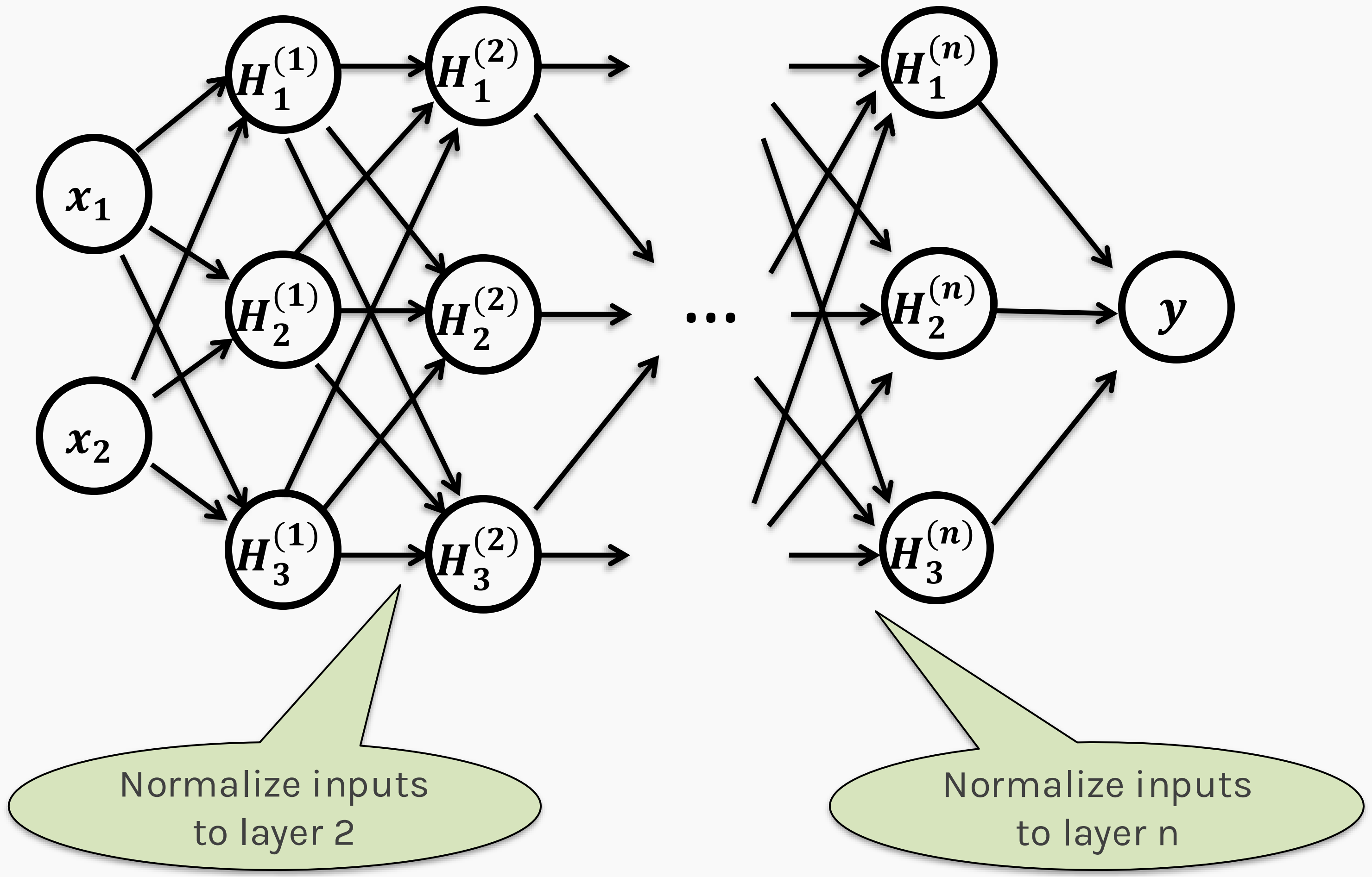
## Internal Covariance Shift

How can this problem be solved?


Covariate Shift
Batch Normalization
Neural Network
Utkal gaurab

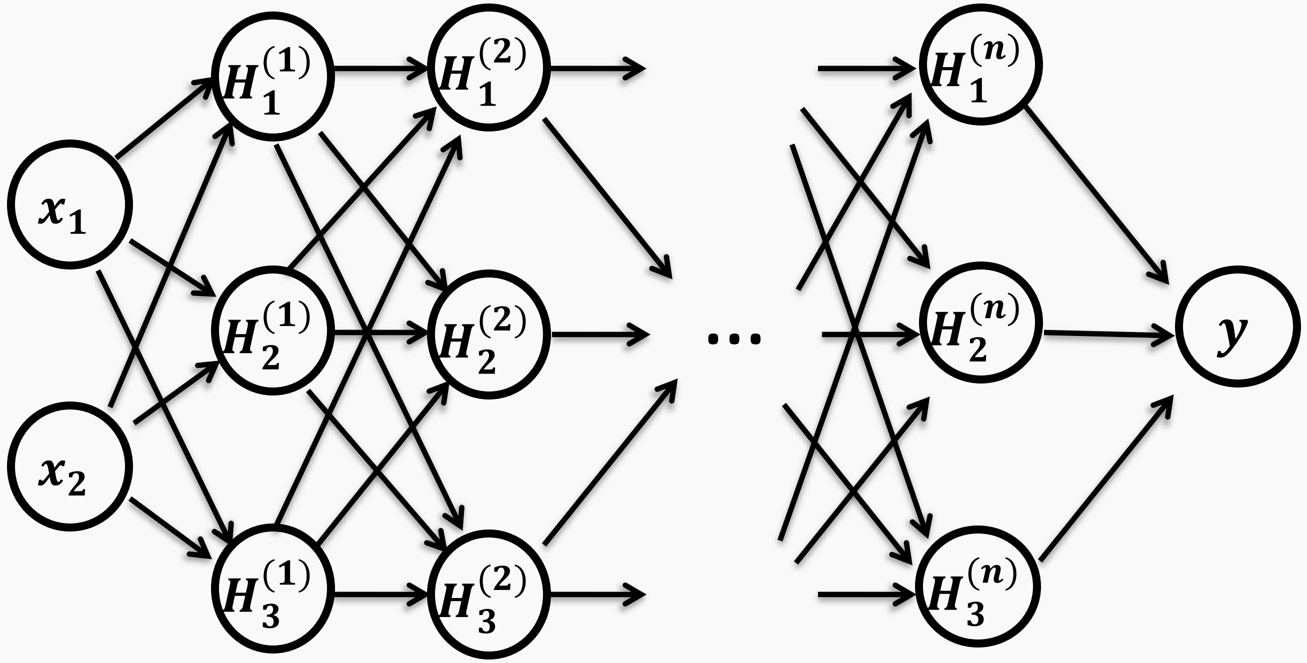
# Internal Covariance Shift Solution

We normalize the inputs to every hidden layer.



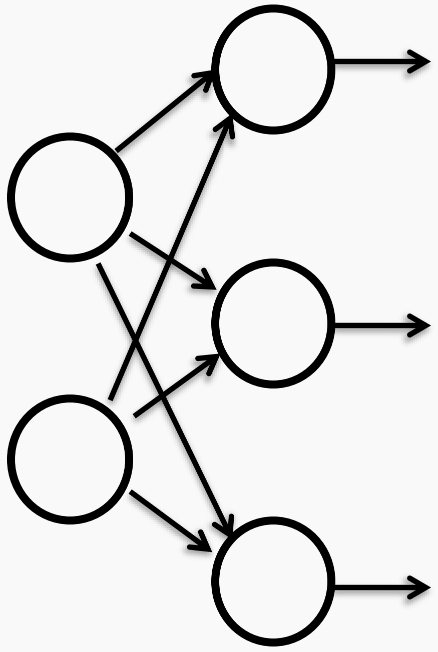
# Internal Covariance Shift Solution

We normalize inputs to every hidden layer.

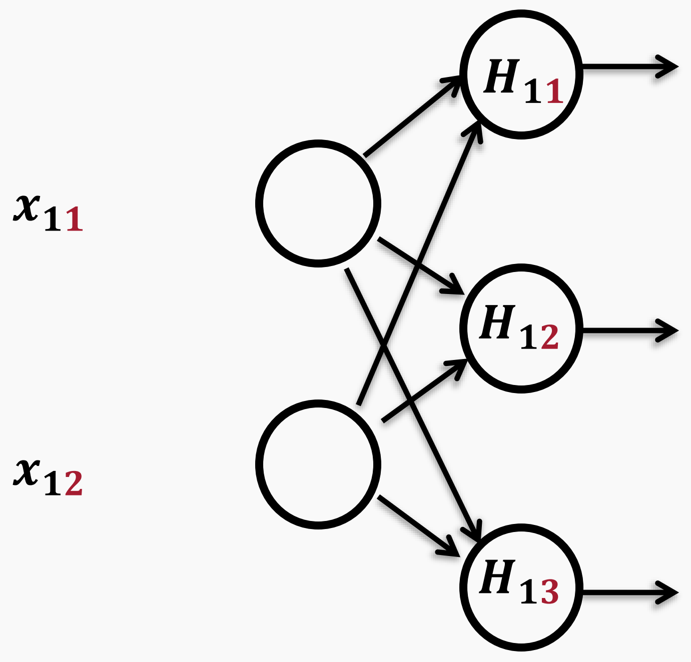


## Batch Normalization

We get the outputs from the first hidden layer.


1st Hidden Layer

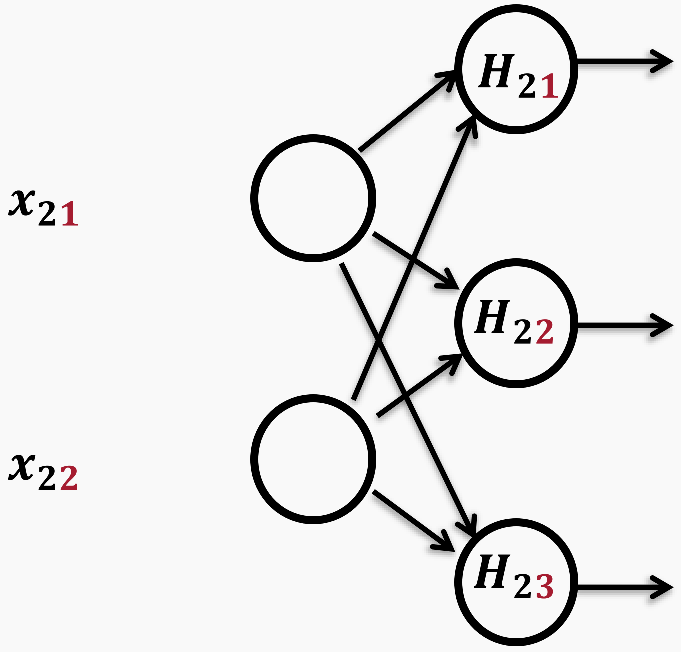
# Batch Normalization



1st Hidden Layer

$$\left[\begin{array}{c} \\ \\ \\ \end{array}\right]$$

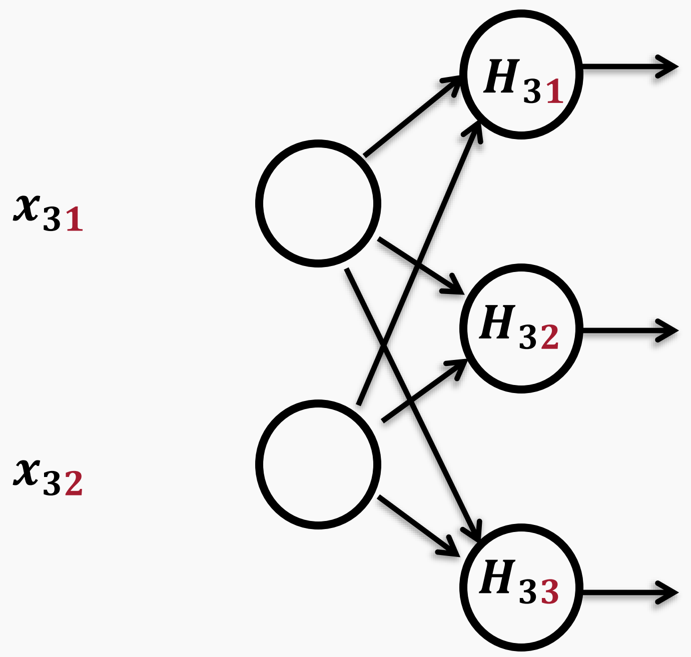
## Batch Normalization



1st Hidden Layer

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ & & \\ & & \end{bmatrix}$$

# Batch Normalization



$3$ data points in batch

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ & & \end{bmatrix}$$

$3$ hidden units activations

We do the same for $N$ data points in batch and $K$ hidden units activations in the next slide

## Batch Normalization

Training time:

Batch of activations for a given layer to normalize

For a given hidden layer

$$\boxed{H = \begin{bmatrix} H_{11} & \cdots & H_{1K} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NK} \end{bmatrix}}$$

*N* data points in batch

*K* hidden units' activations

## Batch Normalization

Training time:

Batch of activations for a given layer to normalize

$$H = \begin{bmatrix} H_{11} & \cdots & H_{1K} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NK} \end{bmatrix}$$

$$H'_{ik} = \frac{H_{ik} - \mu_k}{\sigma_k}$$

## Batch Normalization

Training time:

Batch of activations for a given layer to normalize

$$H = \begin{bmatrix} H_{11} & \cdots & H_{1K} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NK} \end{bmatrix}$$

$$H'_{ik} = \frac{H_{ik} - \mu_k}{\sigma_k}$$

$$\mu_k = \frac{1}{N} \sum_i H_{ik}$$

Mean activations across batch for node k.

## Batch Normalization

When calculating the variance, we add a small constant to the variance to prevent potential divisions by zero.

Training time:

Batch of activations for a given layer to normalize

$$H = \begin{bmatrix} H_{11} & \cdots & H_{1K} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NK} \end{bmatrix}$$

$$H'_{ik} = \frac{H_{ik} - \mu_k}{\sigma_k}$$

$$\mu_k = \frac{1}{N} \textstyle\sum_i H_{ik}$$

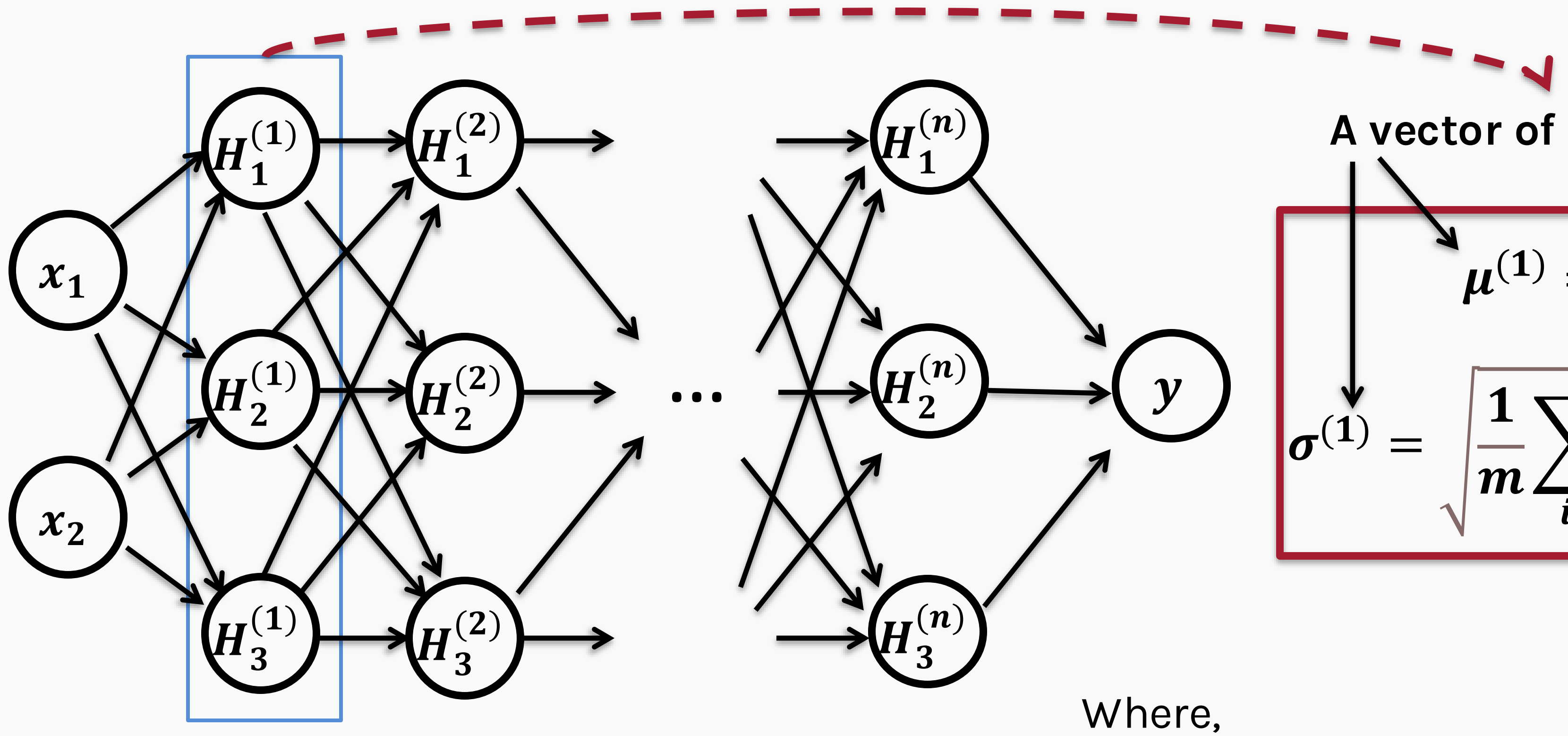
Mean activation across batch for node

$$\sigma_k = \sqrt{\frac{1}{N}\sum_i (H_{ik} - \mu_k)^2 + \delta}$$

Standard deviation across batch, $k$

## Batch Normalization

Batch 1



**A vector of 3 values**

$$\mu^{(1)} = \frac{1}{m}\sum_i H_i^{(1)}$$

$$\sigma^{(1)} = \sqrt{\frac{1}{m}\sum_i \left(H_i^{(1)} - \mu^{(1)}\right)^2 + \delta}$$

Where,

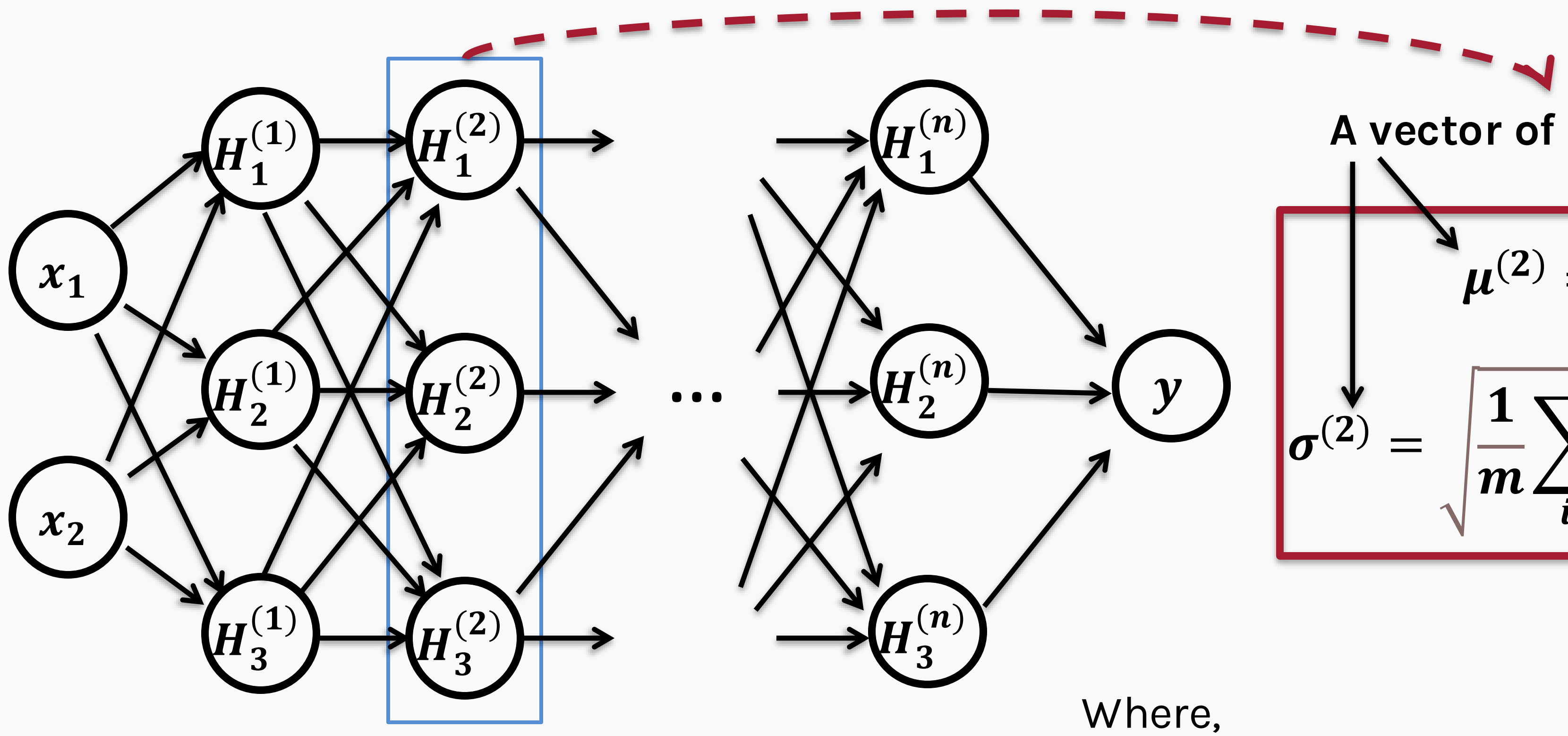
m: Number of training examples in the batch

$H_i^{(1)}$: Hidden layer activation of the first hidden layer for the $i^{th}$ training example

$\boldsymbol{\delta}$: A small constant

# Batch Normalization

Batch 1



A vector of 3 values

$$\mu^{(2)} = \frac{1}{m}\sum_i H_i^{(2)}$$

$$\sigma^{(2)} = \sqrt{\frac{1}{m}\sum_i \left(H_i^{(2)} - \mu^{(2)}\right)^2 + \delta}$$

Where,

m: Number of training examples in the batch

$H_i^{(2)}$: Hidden layer activation of the second hidden layer for the $i^{th}$ training example

$\boldsymbol{\delta}$: A small constant

## Batch Normalization

Training time:

- Normalization can reduce expressive power
- Instead use:

$$\boxed{H''_{ik} = \gamma H'_{ik} + \beta}$$

## Batch Normalization

Training time:

- Normalization can reduce expressive power
- Instead use:

Normalized Activation

Final Transformed Activation

$$\boxed{H''_{ik} = \gamma H'_{ik} + \beta}$$

Learnable parameters

## Batch Normalization

When do we apply batch normalize: before or after activation?

**Before Activation**

## Batch Normalization

We have the equation

$$h^{(2)} = Wa^{(1)} + b$$

where

$a^{(1)}$ : Activation of the first hidden layer

$h^{(2)}$: the output of the second hidden layer w/o activation

If we do batch normalization **after** activation:

$a^{(1)}$ will be affined transformed after this and therefore there is not need to transform again.

## Batch Normalization

We have the equation

$$h^{(2)} = Wa^{(1)} + b$$

where

$a^{(1)}$ : Activation of the first hidden layer

$h^{(2)}$: the output of the second hidden layer w/o activation

If we do batch normalization **before** activation:

$Wa^{(1)} + b$ is very likely to have a symmetric, non-sparse distribution; normalizing it is likely to produce activations with a stable distribution.

## Evaluation

We saw how batch normalization works during training, but what about **prediction**, when we might not have a complete batch!

## Evaluation

### Evaluation time:

- Calculate the running average of the mean and standard deviation.
- For every batch:

Decay parameter

Use this for evaluation

$$\mu_{global} = \alpha\mu_{global} + (1-\alpha)\mu_k$$

$$\sigma_{global} = \alpha\sigma_{global} + (1-\alpha)\sigma_k$$

## Batch Normalization

Evaluation time:

Hidden activations will be a vector as there are no batches.

$$\boxed{H = \begin{bmatrix} H_1 & \dots & H_K \end{bmatrix}}$$

# Batch Normalization

## Evaluation time:

Use the global statistics to normalize the node activations.

$$\boxed{H = [H_1 \quad \dots \quad H_K]}$$

For each hidden node $k$:

$$H'_k = \frac{H_k - \mu_{global}}{\sigma_{global}}$$

Estimated global mean of each unit activation.

Estimated global SD of each unit activation.

Thank you (again)