

## Sender–Receiver Exercise 2: Reading for Receivers

Harvard SEAS - Fall 2024

2024-09-19

The goals of this exercise are:

- to develop your skills at understanding, distilling, and communicating proofs and the conceptual ideas in them
- to practice reasoning about updates to dynamic data structures and binary search trees in particular

## 1 The Result

In the previous class (Tuesday 09-17), we saw that insert operations can be performed on a binary search tree (BST) in time  $O(h)$ , where  $h$  refers to the height of the tree. As an in-class exercise, some of you saw that a variety of different operations (search, min/max, next-smaller/next-larger) can also be performed in time  $O(h)$ ; pseudocode for those operations is in the detailed lecture notes. Here you will see how *deletions* can be performed in time  $O(h)$ .

**Theorem 1.1.** *Given a binary search tree  $T$  of height  $h$ , and a key  $K$  stored in the tree, we can delete a matching key-value pair  $(K, V)$  from  $T$  in time  $O(h)$ . Deletion means that we produce a new binary search tree that contains all of the key-value pairs in  $T$  except for one less occurrence of a pair with key  $K$ .*

To prepare for the exercise, we recommend reviewing the lecture notes from Tuesday 09-17 to make sure you are comfortable with BSTs and the simpler operations on them (insert, search, min/max, and next-smaller/next-bigger).

## 2 The Proof