**Theory/Research**

Illustrate the concept of statistical independence with analogies in probability theory:

1. **Concept:**
   * **Statistical Independence** means that the occurrence of one event does not affect the probability of the other event.
   * For events AAA and BBB: P(A∩B)=P(A)⋅P(B)P(A \cap B) = P(A) \cdot P(B)P(A∩B)=P(A)⋅P(B)
   * This extends to **random variables** XXX and YYY: P(X,Y)=P(X)⋅P(Y)P(X, Y) = P(X) \cdot P(Y)P(X,Y)=P(X)⋅P(Y)
2. **Analogy:**
   * Consider rolling two dice. The result of one die does not influence the result of the other die. If AAA is "Die 1 rolls a 4" and BBB is "Die 2 rolls a 6": P(A)=16,P(B)=16,P(A∩B)=136=P(A)⋅P(B)P(A) = \frac{1}{6}, \quad P(B) = \frac{1}{6}, \quad P(A \cap B) = \frac{1}{36} = P(A) \cdot P(B)P(A)=61​,P(B)=61​,P(A∩B)=361​=P(A)⋅P(B)
3. **Practical Context:**
   * Independence is a core assumption in many statistical methods (e.g., hypothesis testing, regression analysis).
   * Understanding **Donsker’s theorem** and its application in this simulation requires this concept.

**Application/Practice**

Refine the SDE simulator for a **continuous process** with scaling limits.

1. **Simulation Setup:**
   * **Temporal Window:** Subdivide into small intervals Δt=1/n\Delta t = 1/nΔt=1/n.
   * **Jump Magnitudes:** Assign probabilities ppp to make a jump of +Δt+\sqrt{\Delta t}+Δt​ or −Δt-\sqrt{\Delta t}−Δt​.
   * **Connection to Wiener Process:**
     + This represents the **scaling limit** of a random walk as intervals become infinitesimally small.
     + Significance: It approximates the **Wiener process (Brownian motion)** due to the **Donsker invariance principle**.
2. **Implementation Steps:**
   * **Input Parameters:**
     + Total Time (TTT)
     + Intervals (nnn)
     + Initial Position (X0X\_0X0​)
   * **Generate Jumps:**
     + For each time step iii: Xi+1=Xi+Jump,Jump=±ΔtX\_{i+1} = X\_i + \text{Jump}, \quad \text{Jump} = \pm\sqrt{\Delta t}Xi+1​=Xi​+Jump,Jump=±Δt​
     + Use random numbers to determine the jump direction.
   * **Output:**
     + Plot the trajectory of the process over time.

**Testing Inputs for Homework 4**

1. **Basic Simulation:**
   * Total Time: 10
   * Intervals: 1000
   * Initial Position: 0
2. **Longer Time Period:**
   * Total Time: 20
   * Intervals: 2000
   * Initial Position: 5
3. **Higher Granularity:**
   * Total Time: 5
   * Intervals: 50000
   * Initial Position: 0

