**1. Prove the Cauchy-Schwarz Inequality**

The Cauchy-Schwarz inequality states:

∣∑i=1nuivi∣2≤(∑i=1nui2)(∑i=1nvi2)\left| \sum\_{i=1}^n u\_i v\_i \right|^2 \leq \left( \sum\_{i=1}^n u\_i^2 \right) \left( \sum\_{i=1}^n v\_i^2 \right)​i=1∑n​ui​vi​​2≤(i=1∑n​ui2​)(i=1∑n​vi2​)

**Plan:**

* Write the simplest proof in comments or documentation for theoretical purposes.
* Optionally, create a method in the central framework to numerically validate this inequality.

**2. Reflect on Independence vs. Uncorrelation**

**Plan:**

* Add a section in the theoretical write-up to highlight:
  + **Independence**: Random variables XXX and YYY are independent if their joint probability distribution factors as P(X,Y)=P(X)P(Y)P(X, Y) = P(X)P(Y)P(X,Y)=P(X)P(Y).
  + **Uncorrelation**: XXX and YYY are uncorrelated if Cov(X,Y)=0\text{Cov}(X, Y) = 0Cov(X,Y)=0.
  + Differences: Independence implies uncorrelation, but uncorrelation does not necessarily imply independence.
* Implement a numerical test to compute covariance and test uncorrelation for simulated data.

**3. E-M Simulator Enhancement**

Enhance the existing Euler-Maruyama (E-M) simulator by developing a **unified framework** for managing various types of SDEs.

**Key Features:**

* A central class SDEFramework to manage SDEs.
* Methods for:
  + Setting initial conditions.
  + Specifying f(x,t)f(x, t)f(x,t) (drift) and g(x,t)g(x, t)g(x,t) (diffusion) components.
  + Running the simulation.
* Reusable components for both Homework 3 and Homework 4 SDEs.

**4. Optional: Regression Coefficients**

**Plan:**

* Derive coefficients aaa (intercept) and bbb (slope) using the least squares method: b=Cov(X,Y)Var(X),a=Yˉ−bXˉb = \frac{\text{Cov}(X, Y)}{\text{Var}(X)}, \quad a = \bar{Y} - b\bar{X}b=Var(X)Cov(X,Y)​,a=Yˉ−bXˉ
* Show relationships with R2R^2R2 (coefficient of determination).

A graph on a screen

Description automatically generated