**Research: The Fundamental Theorem of Calculus and Its Relation to Density Functions**

**1. The Fundamental Theorem of Calculus**

The Fundamental Theorem of Calculus (FTC) bridges **differentiation** and **integration**:

1. **Part 1**: If F(x)F(x) is the **antiderivative** of f(x)f(x), then: ∫abf(x)dx=F(b)−F(a)\int\_a^b f(x) dx = F(b) - F(a)
2. **Part 2**: If F(x)=∫axf(t)dtF(x) = \int\_a^x f(t) dt, then: F′(x)=f(x)F'(x) = f(x)

This means that integration (area under the curve) and differentiation (rate of change) are inverses of each other.

**2. Density Functions and Cumulative Distribution Functions**

In probability theory:

1. **Probability Density Function (PDF)**:
   * Represents the density of probability over a range of values for a continuous random variable XX.
   * The area under the PDF curve between two points gives the probability: P(a≤X≤b)=∫abf(x)dxP(a \leq X \leq b) = \int\_a^b f(x) dx
2. **Cumulative Distribution Function (CDF)**:
   * Represents the total probability accumulated up to a value xx: F(x)=P(X≤x)=∫−∞xf(t)dtF(x) = P(X \leq x) = \int\_{-\infty}^x f(t) dt
3. **Relation Between PDF and CDF**:
   * By the Fundamental Theorem of Calculus, the derivative of the CDF is the PDF: f(x)=ddxF(x)f(x) = \frac{d}{dx}F(x)

**3. Analogies Between FTC and Probability Theory**

| **FTC** | **Probability Theory** |
| --- | --- |
| Derivative F′(x)=f(x)F'(x) = f(x) | PDF: f(x)=ddxF(x)f(x) = \frac{d}{dx} F(x) |
| Integral ∫abf(x)dx=F(b)−F(a)\int\_a^b f(x) dx = F(b) - F(a) | Probability: P(a≤X≤b)=F(b)−F(a)P(a \leq X \leq b) = F(b) - F(a) |
| F(x)F(x) accumulates area under f(x)f(x) | CDF accumulates probabilities from −∞-\infty to xx |

**4. Importance in Simulation**

In the context of Homework 6:

* **CDF Sampling**: Generating random values from a distribution often uses the inverse of the CDF.
* **Empirical CDF**: During simulations, the cumulative probabilities of observed values approximate the theoretical CDF.

**5. Real-World Example**

For a continuous random variable like the height of people in a population:

* The **PDF** shows the density of heights.
* The **CDF** shows the percentage of people below a specific height.
* By simulating heights and plotting the empirical CDF, we can see how the theoretical CDF is realized in practice.

