Coursework: Discrete Event Systems

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An indoor environment is structured according to the map in Fig. 1 shown in the next page.

1 Question: Modeling the map

Model the spacial information provided by the map by means of a finite deterministic automaton G_M. In particular, each room should correspond to a state of the automaton, and transitions between states will only be allowed when two rooms are next to each other by sharing a wall. In particular, 4 types of events are allowed: n,s,e,w, representing a transition from a room to the next one to the north, south, east and west respectively.

2 Question: Modeling the robot

A Robot is able to move in such environment, by performing two types of actions, rotating its heading clockwise by 90 degrees without changing its position (this happens whenever an r event occurs), or moving forward by one unit of space (equal to the size of the room), whenever events n,s,e or w take place. Model, by virtue of a finite deterministic automaton G_R, the robot behaviour. In particular, the automaton should keep track of the robot heading (which is always pointing towards one cardinal direction), making sure that n,s,e or w events may only occur when the heading of the robot is oriented according to the direction of the movement. Choose any initial heading for this automaton, as this will not affect the answers to the subsequent questions.

3 Question: Modeling the robot inside the map

Represent an automaton in Matlab by using 4 variables: a list of events, a list of states, a matrix representing a list of transitions and an initial state. For instance, in order to define the automaton G_M one may let:

```
E1=char('n','s','e','w');
X1=char('r1','r2','r3','r4','r5','r6','r7');
```

to define the set of events and states respectively. Then a $3 \times T$ (with T number

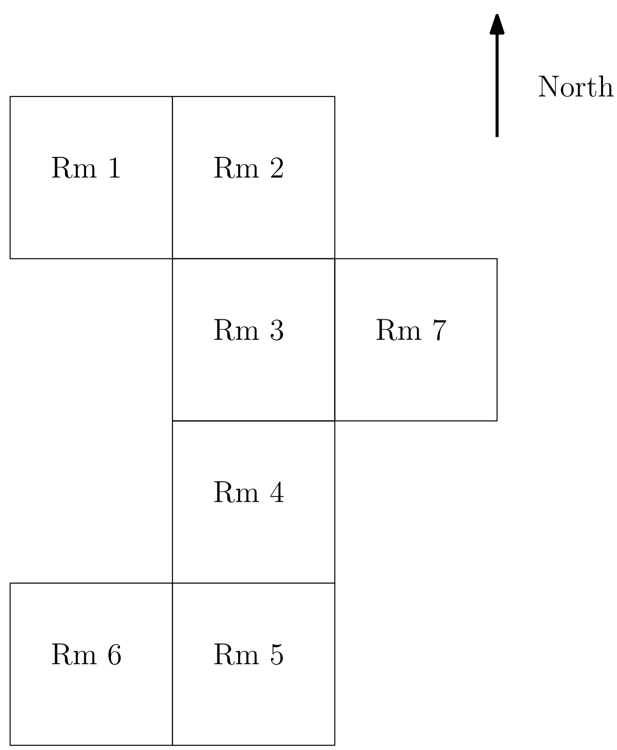

Figure 1: Map of indoor environment

of transitions) integer matrix may be used to represent transitions. In particular `T1=[1,2,3; ...` would represent a transition from state 1 (room 1) to state 2 (room 2) when event 3 (e for east) occurs. The following rows will be constructed accordingly. Build in Matlab both automata G_M and G_R. Write a Matlab Code that implements the parallel composition operator. Model the behavior of the robot inside the environment by building the automaton $G_M||G_R$. List the events, the states and the transition matrix of the automaton $G_M||G_R$.

4 Question: Modeling partial observability

The robot, when moving is unaware of the direction of its heading (no compass information available). In practice this means that events n,s,w,e are partially observable and can only be represented by a unique event m (movement). Build a (potentially) non-deterministic automaton G_N by replacing in $G_M||G_R$ all transitions involving events n,s,e,w by transitions involving event m. Implement a Matlab function that modifies the original automaton, in particular its list of events and of event 'labels' attached to its transitions (column 3 of the matrix), to reflect the partial observability of the n,s,e,w events. List the events, states and transitions of the (potentially) non-deterministic automaton G_N.

5 Question: Estimating position and heading

Whenever switched on, the robot is unaware of its location (room number) and its heading (N,S,W,E). It therefore attempts to reconstruct this information by looking at the sequence of events r and m occurring. Build a Matlab routine that is able to compute the Observer automaton of G_N, given an initial state which is completely uncertain. (Coding Hint: use in order to represent states of the observer automaton vectors of zeroes and ones of length equal to the state-space dimension of the original automaton G_N. In particular, each one in the vector means that the state (of the original G_N automaton) corresponding to that position is included among the possible states in which the system is currently evolving. A zero, instead, represents a state the automaton G_N is currently not in. Given the fact that the initial position and heading of the robot are completely unknown, you should therefore initialize the observer automaton from a state that is a vector of all ones. Then, observing r and m events and following the corresponding transitions, you should be able to construct states which contain fewer ones. Run the routine on G_N.)

1. How many states in the observer automaton ?

2. List the states of the Observer Automaton. (for ease of verification present them as a matrix and also provide the row sum and column sum of this matrix, as well as its first 4 singular values).

3. Where is the robot located (room and heading) if the following sequence of events is recorded m,r,r,m,r,m,r,r,m,r,m ?

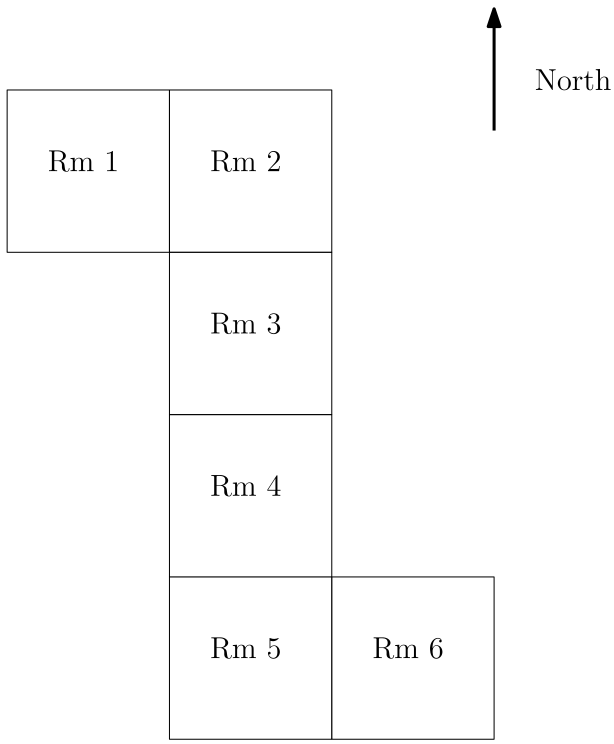

Figure 2: Modified map of indoor environment

4. How can you find out using the Observer Automaton ?

6 Question: Deterministic Interpretation

Look at the Observer automaton computed. Will the robot ever be able to reconstruct exactly its position and heading. Why ? Would your answer be different if the map is modified as in Fig. 2 ? (Hint: if unsure try to re-run your software after modifying the map).

7 Assessment Criteria

The coursework weighs for 25% of the final mark and is assessed according to the following criteria:

1. Technical Soundness of procedure followed;
2. Correctness of software implemented and of proposed results;
3. Clarity of explanation;
4. Insight and depth of interpretation of results;
5. Completeness of results proposed.