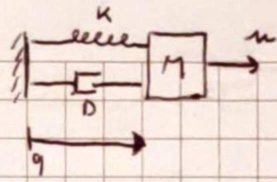
L28 SIMULAZIONI 1

SIMULAZIONE 1 DOF



$$\left\{ M\ddot{q} + Kq + D\dot{q} = u \right.$$

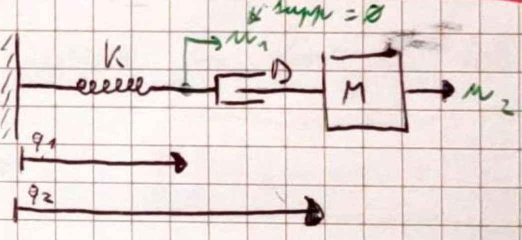
$$x = \begin{array}{c} \begin{bmatrix} \dot{q} \\ q \end{bmatrix} \end{array} \begin{array}{l} x_1 \\ x_2 \end{array} \qquad \dot{x} = \begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} \begin{array}{l} \dot{x}_1 \\ \dot{x}_2 \end{array} \Rightarrow M\dot{x}_1 + Kx_2 + Dx_1 = u$$

$$M\dot{x}_1 = -Dx_1 - Kx_2 + u$$

$$\begin{cases} \dot{x}_1 = -\frac{D}{M}x_1 - \frac{K}{M}x_2 + \frac{u}{M} \\ \dot{x}_2 = x_1 \end{cases}$$

$$A = \begin{bmatrix} -\frac{D}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \emptyset$$

SIMULAZIONE 2 DOF



$$\begin{cases} Kq_1 - D(\dot{q}_2 - \dot{q}_1) = u_1 / \emptyset \\ M\ddot{q}_2 + D(\dot{q}_2 - \dot{q}_1) = u_2 \end{cases}$$

q_1 $\quad q_1 = p_1$ $\quad Kq_1 - D\dot{q}_2 + D\dot{q}_1 = u_1$

$q_2 \ \dot{q}_2$ $\quad q_2 = p_2 \quad \dot{q}_2 = v_2$ $\quad D\dot{q}_1 = u_1 - Kq_1 + D\dot{q}_2$

$\dot{p}_1 = \dot{q}_1 = \quad \dot{v}_2 = \ddot{q}_2$ $\quad \dot{q}_1 = \frac{\quad}{D}$

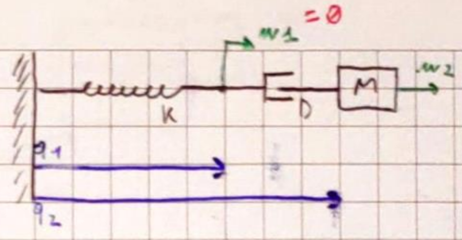
$M\ddot{q}_2 = u_2 - D\dot{q}_2 + D\dot{q}_1$

$\ddot{q}_2 = \frac{\quad}{M}$

$$\begin{cases} \dot{p}_1 = \dfrac{u_1 - Kp_1 + Dv_2}{D} \\ \dot{p}_2 = v_2 \\ \dot{v}_2 = u_2 - Dv_2 + D(u_1 - Kp_1 + Dv_2) \end{cases}$$

SIMULAZIONE 2 DOF

$$\begin{cases} K q_1 - D(\dot{q}_2 - \dot{q}_1) = u_1 \\ M \ddot{q}_2 - D(\dot{q}_2 - \dot{q}_1) = u_2 \end{cases}$$



1) Vedo le derivate massime di ogni variabile e definisco una variabile di stato fino all'ordine $n-1$

$$\dot{q}_1 \text{ max} \Rightarrow \qquad \ddot{q}_2 \text{ max} \Rightarrow \qquad \begin{bmatrix} q_1 \\ \dot{q}_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x \Rightarrow \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{q}_1 \\ \ddot{q}_2 \\ \dot{q}_2 \end{bmatrix}$$

2) Scrivo le equazioni del sistema in funzione delle $\dot{x}_i$:

$$\underbrace{\begin{bmatrix} D \overset{\dot{x}_1}{\dot{q}_1} \\ M \overset{\dot{x}_2}{\ddot{q}_2} \\ \overset{\dot{x}_3}{q_2} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} u_1 - K \overset{x_1}{q_1} + D \overset{x_2}{\dot{q}_2} \\ u_2 - D(\dot{q}_2 - \dot{q}_1) \\ \underbrace{\int \ddot{q}_2}_{x_2} \end{bmatrix}}_{x}$$

$$\rightarrow \dot{q}_1 = \overbrace{\frac{1}{D}\left(u_1 - K \underbrace{q_1}_{x_1} + D \underbrace{\dot{q}_2}_{x_2}\right)}^{\dot{x}_1}$$

$$= u_2 - D\left(\underbrace{\dot{q}_2}_{x_2} - \frac{1}{D}(\downarrow) \right)$$

$$\Rightarrow \begin{cases} \dot{x}_1 = \frac{1}{D}\left[u_1 - K x_1 + D x_2\right] \\ \dot{x}_2 = \frac{1}{M}\left[u_2 - D x_2 + \frac{D}{D}(u_1 - K x_1 + D x_2)\right] \\ \dot{x}_3 = x_2 \end{cases}$$

$$\begin{cases} \dot{x}_1 = -\frac{K}{D} x_1 + x_2 + \frac{u_1}{D} \\ \dot{x}_2 = \cancel{-\frac{D}{M} x_2} + \cancel{\frac{D}{M} x_2} - \frac{K}{M} x_1 + \frac{(u_1 + u_2)}{M} = -\frac{K}{M} x_1 + \frac{u_1 + u_2}{M} \\ \dot{x}_3 = x_2 \end{cases}$$