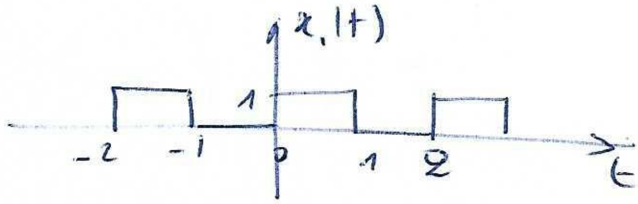
TD N°2

Fondement Multimédia

Exercice N°1

1/ $x_1(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t < 2 \end{cases}$



$$C_n = \frac{1}{2} \int_0^2 x(t)\, e^{-j\frac{2\pi}{2} n t}\, dt$$

$$= \frac{1}{2} \int_0^1 e^{-j\pi n t}\, dt$$

$$= -\frac{1}{2j\pi n} \left[e^{-j\pi n t} \right]_0^1$$

$$= \frac{e^0 - e^{j\pi n}}{2j\pi n}$$

$$\boxed{\begin{aligned} e^{-ja} &= \cos a - j \sin a \\ e^{ja} &= \cos a + j \sin a \end{aligned}}$$

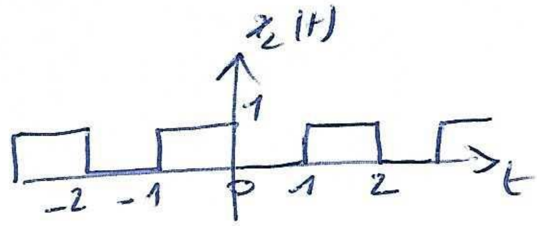
$$= \frac{1 - e^{-j\pi n}}{2j\pi n}$$

$$= \frac{1 - \cos \pi n + j \overbrace{\cancel{\sin \pi n}}^{0}}{2j\pi n}$$

$$= -\frac{j(1 - \cos \pi n)}{2\pi n} \quad \text{donc}$$

donc $C_n = \begin{cases} 0 & n \text{ pair} \\ -\dfrac{j}{\pi n} & \end{cases}$

b/ $x_2(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 \leq t < 2 \end{cases}$



$$C'_n = \frac{1}{2}\int_1^2 e^{-j\pi n t}\,dt$$

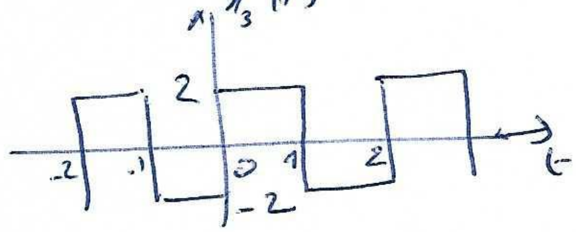
$$= -\frac{1}{2j\pi n}\left[e^{-j\pi n t}\right]_1^2$$

$$= \frac{j}{2\pi n}\left(\cos 2\pi n - j\cancel{\sin 2\pi n}^{\,0} - \cos(\pi n) + j\cancel{\sin(\pi n)}^{\,0}\right)$$

$$= \frac{j}{2\pi n}\left(1 - \cos \pi n\right)$$

$$= \begin{cases} 0 & \text{si } n \text{ pair} \\ \dfrac{j}{\pi n} & \text{si } n \text{ impair} \end{cases}$$

2°) $$x_3(t) = \begin{cases} 2 & 0 \le t < 1 \\ -2 & 1 \le t < 2 \end{cases}$$



$$x_3(t) = 2\left(x_1(t) - x_2(t)\right)$$

$$\sum_{n=-\infty}^{+\infty} C''_n e^{j\frac{2\pi n t}{T}} = 2\left(\sum_{n=-\infty}^{+\infty} C_n e^{j\frac{2\pi n t}{T}} - \sum_{n=-\infty}^{+\infty} C'_n e^{j\frac{2\pi n t}{T}}\right)$$

$$\Rightarrow C'' = 2\left(C_n - C'_n\right)$$

$$= \begin{cases} 0 & \text{si } n \text{ pair} \\ \dfrac{-4j}{\pi n} & \text{si } n \text{ impair.} \end{cases}$$

Exercice 2

$$x(t) = \sum_{n=-\infty}^{+\infty} a_n e^{j\frac{2\pi nt}{T}} \quad , \quad a_n = \frac{1}{T}\int_0^T x(t)\, e^{-j\frac{2\pi nt}{T}} dt$$

1/ $x(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ 2 & 5 \leq t \leq 10 \end{cases}$

1/a) $a_0 = \frac{1}{T}\int_0^T x(t) e^{-j\frac{2\pi 0 t}{T}} dt$

$$= \frac{1}{10}\int_5^{10} 2\, dt = \frac{1}{5}[t]_5^{10} = 1 .$$

$$a_1 = \frac{1}{10}\int_5^{10} 2 . e^{-j\frac{2\pi . 1 t}{10}} dt$$

$$= \frac{1}{5}\int_5^{10} e^{-j\frac{\pi}{5}t} dt .$$

$$= -\frac{1}{\cancel{5}}\frac{\cancel{5}}{j\pi}\left[e^{-j\frac{\pi}{5}t}\right]_5^{10}$$

$$= \frac{e^{-j\pi} - e^{-j2\pi}}{j\pi}$$

$$= \frac{\cos\pi - j\cancelto{0}{\sin\pi} - \cos 2\pi + j\cancelto{0}{\sin 2\pi}}{j\pi}$$

$$= \frac{-2}{j\pi} = \frac{2j}{\pi}$$

2/a) $y(t) = \begin{cases} 1 & 0 \leq t \leq 5 \\ 3 & 5 \leq t \leq 10 \end{cases} \Rightarrow y(t) = x(t) + 1$

$$\sum_{n=-\infty}^{+\infty} b_n e^{j\frac{2\pi n t}{T}} = \sum_{n=-\infty}^{+\infty} a_n e^{j\frac{2\pi n t}{T}} + 1$$

forment une base, on identif[ie] les termes complexes un à un[e]. On a des réels uniquement p[our] $n = 0$.

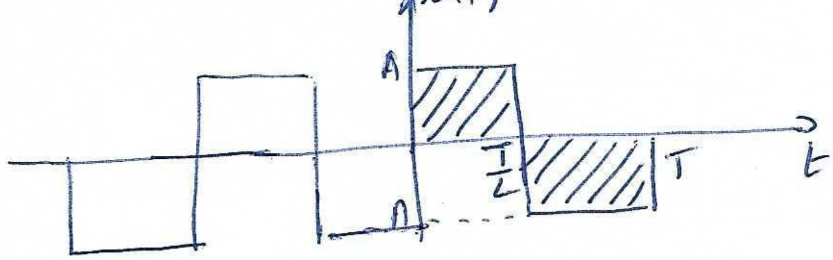
$$\text{donc} \begin{cases} b_0 = a_0 + 1 \\ b_n = a_n \quad \text{pour } n \neq 0. \end{cases}$$

$$\text{d'où} \begin{cases} b_0 = 2 \\ b_1 = \frac{2j}{\pi} \end{cases}$$

$\left(e^{j2\pi n t}\right)_{n=-\infty,+\infty}$ forme une base.

Exercice N°4.

$$x(t) = \begin{cases} A & 0 \leq t < \frac{T}{2} \\ -A & \frac{T}{2} \leq t < T. \end{cases}$$



$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-j\frac{2\pi}{T}nt}$$

$$= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A e^{-j\frac{2\pi}{T}nt}\, dt - \int_{\frac{T}{2}}^{T} A e^{-j\frac{2\pi}{T}nt}\, dt\right)$$

$$= \frac{1}{T}\left(-\frac{AT}{j2\pi n}\left[e^{-j\frac{2\pi}{T}nt}\right]_0^{\frac{T}{2}} + \frac{AT}{j2\pi n}\left[e^{-j\frac{2\pi}{T}nt}\right]_{\frac{T}{2}}^{T}\right.$$

$$C_n = \frac{A\cancel{T}}{j2\pi n \cancel{T}} \left(e^{-j\pi n} + 1 + e^{-j2\pi n} - e^{-j\pi n} \right)$$

$$= \frac{A}{j2\pi n} \left(1 + \cos 2\pi n - j\underbrace{\sin 2\pi n}_{0} \; \cancel{2\cos 2\pi} - j\underbrace{\sin 2\pi}_{0} \right)$$

$$= \frac{A}{j2\pi n} \left(2 - 2\cos(\pi n) \right)$$

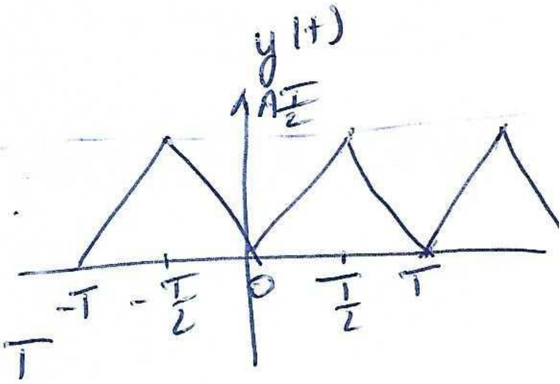
$\boxed{n \in \mathbb{Z} \Rightarrow \cos(2\pi n) \ldots \quad \sin(2\pi n) = \ldots \quad \sin(\pi n) = \ldots}$

si n est pair $\cos(\pi n) = 1 \Rightarrow C_n = 0$

si n est impair $\cos(\pi n) = -1 \Rightarrow C_n = \frac{4A}{j2\pi n} = -j\frac{2A}{\pi n}$

$$C_n = \begin{cases} 0 & \text{si } n \text{ pair} \\ -j\frac{2A}{\pi n} & \text{si } n \text{ impair} \end{cases}$$

2°/ $\left(x(t) = \frac{dy(t)}{dt} \right)$



$$y(t) = \begin{cases} At & 0 \leq t < \frac{T}{2} \\ -At + AT & \frac{T}{2} \leq t < T \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{j\frac{2\pi n}{T}t}$$

$$y(t) = \sum_{n=-\infty}^{+\infty} d_n e^{j\frac{2\pi n}{T}t}$$

Relation entre C_n et d_n ?

$$x(t) = \frac{dy(t)}{dt}$$

$$= \frac{d}{dt} \sum_{n=-\infty}^{+\infty} d_n e^{j\frac{2\pi n t}{T}}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} d_n \frac{d}{dt} e^{j\frac{2\pi n}{T}t}$$

$$= \sum_{n=-\infty}^{+\infty} d_n \, j\frac{2\pi n}{T} e^{j\frac{2\pi n}{T}t}$$

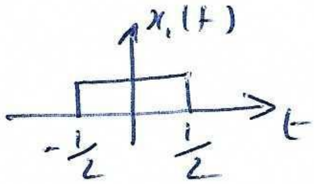
$$= \sum C_n e^{j\frac{2\pi n}{T}t}$$

donc $C_n = j\frac{2\pi n}{T} d_n \implies d_n = \frac{T}{2\pi j n} C_n$

$$d_n = \begin{cases} 0 & \text{si } n \text{ pair} \\ -\frac{T}{j2\pi n}\frac{j2A}{\pi n} & \text{si } n \text{ impair} \end{cases} = \begin{cases} 0 & \text{si } n \text{ pair} \\ -\frac{AT}{\pi^2 n^2} & n \text{ impair} \end{cases}$$

Exercice N°3 :

1/a- $x_1(t) = \text{rect}(t)$



$$X_1(\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t} dt$$

$$= \left[-\frac{1}{j\omega} e^{-j\omega t}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

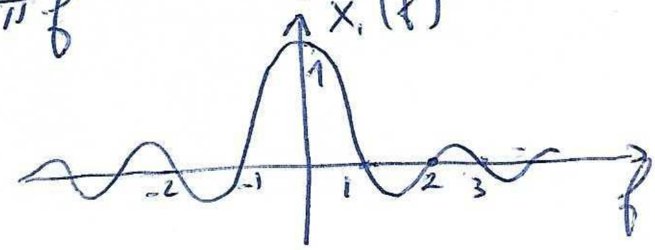
$$= \frac{1}{j\omega}\left(e^{-j\frac{\omega}{2}} + e^{j\frac{\omega}{2}}\right)$$

Rappel

$$\boxed{e^{jx} - e^{-jx} = 2j\sin x}$$

$$= \frac{2}{\omega}\sin\omega/2$$

$$= \frac{\sin\omega/2}{\omega/2} = \frac{\sin(\pi f)}{\pi f} = \text{sinc}\, f.$$



b°/ $x_2(t) = \mathrm{tri}(t)$.

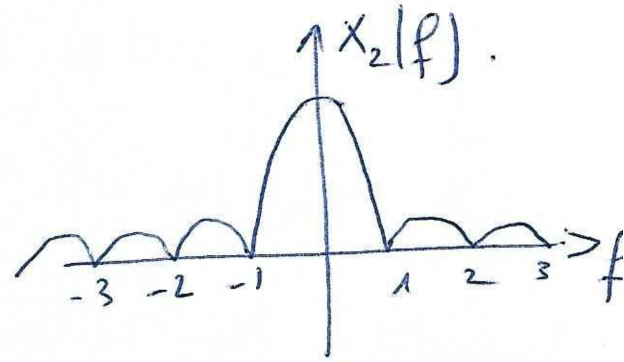
$= \mathrm{rect}(t) * \mathrm{rect}(t)$ (convolution)

$= x_1(t) * x_1(t)$

$$\Rightarrow X_2(\omega) = X_1(\omega) \cdot X_1(\omega)$$

$$= \frac{\sin^2\left(\frac{\omega}{2}\right)}{\frac{\omega^2}{4}}$$

$$X_2(f) = \mathrm{Sinc}^2(f)$$



2 − a/ $x_3(t) = \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$ $(T > 0)$

$= \frac{1}{T}\, x_1\left(\frac{t}{T}\right)$.

Propriété de Dilatation (changement d'échelle).

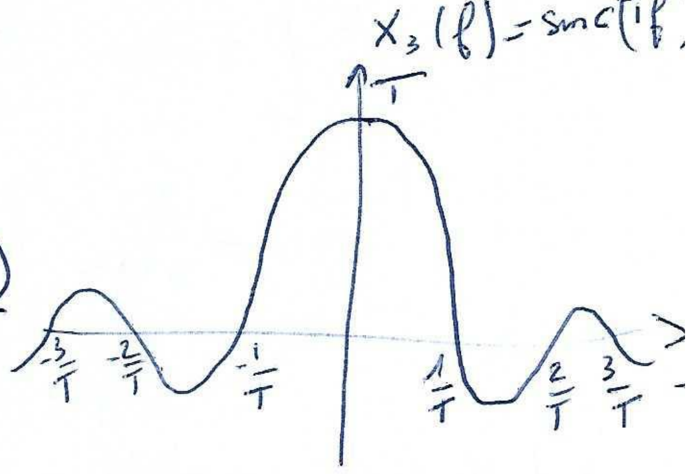
$$x(at) \xleftrightarrow{TF} \frac{1}{|a|}\, x\left(\frac{\omega}{a}\right)$$

$$X_3(f) = \frac{1}{T} T\, X_1(Tf)$$

$$= \mathrm{Sinc}(Tf)$$

$$= \frac{\sin(\pi T f)}{\pi f}$$

$$X_3(\omega) = \frac{\sin\left(T\frac{\omega}{2}\right)}{\frac{\omega}{2}}$$



b/ $x_4(t) = \mathrm{tri}\left(\frac{t-2}{10}\right)$

$= x_2\left(\frac{t-2}{10}\right)$

$X_4(f) = 10\, e^{-j2\pi f \times 2} X_2(10f)$

$= 10\, e^{-j4\pi f} \operatorname{sinc}^2(10f)$.

échelle $x_{41}(t) = \mathrm{tri}\left(\frac{t}{10}\right) = x_2\left(\frac{t}{10}\right) \xleftrightarrow{TF} X_{41}(f) = 10\, X_2(10f)$

$= 10 \operatorname{sinc}^2(10f$

translation $x_4(t) = \mathrm{tri}\left(\frac{t-2}{10}\right) = x_{41}(t-2) \xleftrightarrow{TF} X_4(f) = e^{-j2\pi f \times 2} X_4$
décalage temporel

$= e^{-j4\pi f} 10 \sin$

3 - a/ $x_5(t) = \begin{cases} e^{-3t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

$X_5(\omega) = \int_{-\infty}^{+\infty} e^{-3t} e^{-j\omega t} dt$

$= \int_0^{\infty} e^{-(3+j\omega)t} dt$

$= \left[-\frac{1}{3+j\omega} e^{-(3+j\omega)t} \right]_0^{\infty}$

$\int_a^b e^{\alpha t} = \frac{1}{\alpha} \left[e^{\alpha t} \right]_a^b$

$= -\frac{1}{3+j\omega} (0 - 1)$

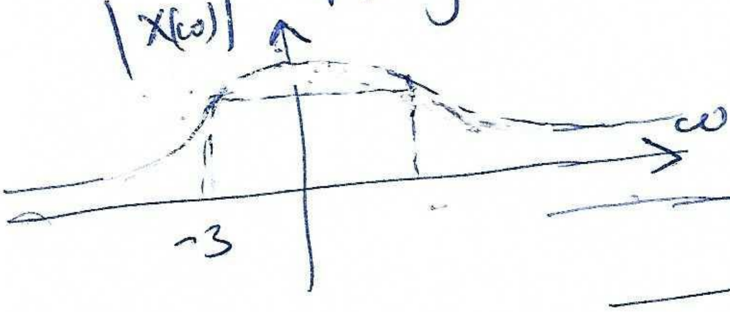
$= \frac{1}{3+j\omega} = \frac{3-j\omega}{9-\omega^2}$.

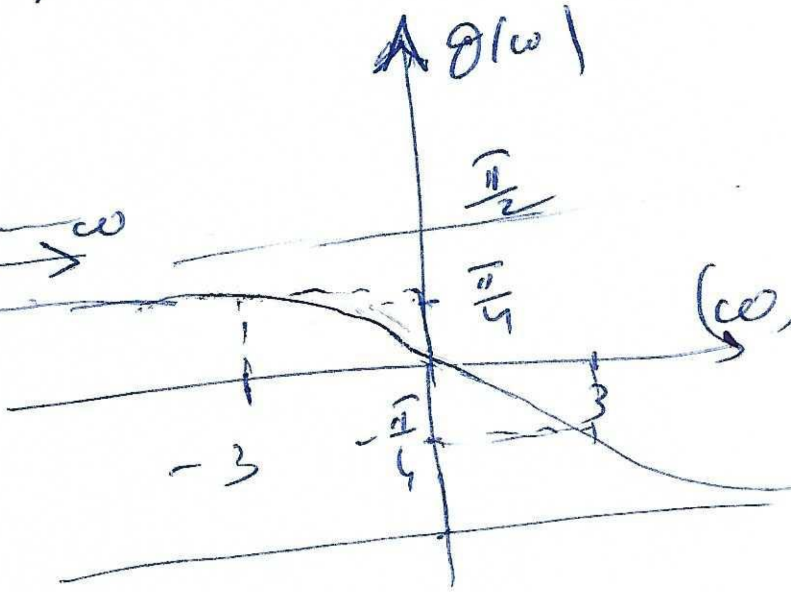
b/ $X_6(\omega) = e^{-|4t|}$

$x_5(t) = e^{-3t}u(t)$

$X_5(\omega) = \frac{1}{3+j\omega}$

$|X_5(\omega)| = \frac{1}{\sqrt{3^2+j\omega}}$, $\theta(\omega) = -\mathrm{arctg}\,\frac{\omega}{3}$



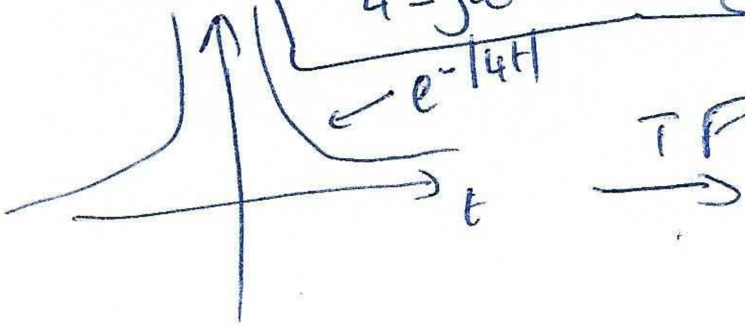


b) $x_6(t) = e^{-|4t|} = \begin{cases} e^{-4t} & t>0 \\ e^{4t} & t<0 \end{cases}$

$X(\omega) = \int_{-\infty}^{0} e^{4t}e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-4t}e^{-j\omega t}$

$= \int_{-\infty}^{0} e^{(4-j\omega)t}\,dt + \int_{0}^{\infty} e^{-(4+j\omega)t}\,dt$

$= \frac{1}{4-j\omega} + \frac{1}{4+j\omega} = \frac{2\times 4}{4^2+\omega^2} = \frac{8}{16+\omega^2}$



TF