Misallocation in the Market for Inventors*

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Abstract

How much misallocation is there in the market for inventors? How costly is it for growth? And what implications does it have for policy and firm dynamics? This paper introduces a novel growth model where firms search for inventors in a frictional labor market, and where the efficiency of innovation depends on the quality of the match between the inventor and firm. The model features both knowledge spillovers and congestion in the labor market, hence the resources invested in search are generally suboptimal, but it is unclear ex-ante whether this level of investment is too low or too high. I use the model to quantify this tradeoff and to study what policies can rectify the inefficiencies in this market. Crucially, I document novel evidence that inventors are 40% more productive when moving across firms, a finding *not* driven by differences in firm innovativeness. Introducing search frictions to an otherwise familiar model of growth helps rationalize why some firms grow quickly whereas others stagnate: most firms never successfully match with an inventor and never get the chance to innovate, and those that do need to innovate rapidly to generate as much growth as we observe. I also find that search frictions and the resulting misallocation of inventors across firms are of first-order importance for growth: shutting down search frictions and moving to a world where inventors are perfectly allocated across firms increases growth from 2.0 to 2.6 percent. Finally, I find that firms invest too little in searching for inventors, and that policies that correct this underinvestment by subsidizing search costs can increase growth at a lower cost than traditional R&D subsidies.

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1 Introduction

This paper studies the labor market for inventors and asks what role policy should play in shaping this market. Innovative activity combines the skills of inventors with the knowledge of the firm they work in, and it does not come as a surprise that many high-profile inventors make headlines when they moved across firms (among them Peter Rawlinson from Tesla to Lucid and Jeffrey Wilcox from Intel to Apple and back)[1]. I therefore open up what has heretofore been treated as a black box in the growth literature and show that the allocation of inventors has first-order consequences for firm dynamics and growth.

I make three contributions. The first is theoretical: I introduce a novel growth model where firms need to engage in costly search for inventors before coming up with new products. This model builds on both the growth and search literatures, and it inherits some of the efficiency properties of both. As in all models of growth, firms benefit from knowledge spillovers and therefore do not internalize the benefit of making better matches and innovating more quickly. However, when firms decide how much effort to devote to searching for inventors, they do not internalize the benefits to the inventors of being in a better match or the costs to rival firms of losing their inventors. Search frictions can, in general, mean that firms expend either too much or too little effort to meet with workers. On the one hand, the fact that they are willing to move suggests that they expect greater returns to their innovative effort at their new firms than their old. On the other hand, the resources that their old firms invested[2] to recruit them in the first place went to waste—-as usual in search models, the privately optimal level of search effort may be greater or lower than the social optimum. These considerations are missing in standard models of growth, where the inputs into innovation are instead undifferentiated and hired in a frictionless market.

My second contribution is empirical: I document novel facts about inventor mobility across firms, and show that inventors move frequently in their typically-short careers. I also use a matched-pairs design related to that in Prato (2022) and compare similar inventors who did or did not move across firms. I find that mobility across firms is associated with 40% more patenting. I use these novel facts as moment targets for the model and replicate them using simulated data.

[1] Details about Rawlinson and Wilcox's stories can be found here and here, respectively.

[2] Gavazza, Mongey, and Violante (2018) provides a useful decomposition of recruiting costs, and finds that the two that comprise the largest share of such costs are payments to job boards or search engine aggregators (for example, purchasing a license to Monster's database) and payments to agencies or third-party recruiters.

My third contribution is quantitative: I show that my model has a rich set of implications for growth, policy, and firm dynamics. First, I use the estimated model to quantify the impact of the misallocation of inventors on growth. I find that removing this misallocation altogether could raise growth from 2.0 percent to 2.6 percent, which shows that search frictions in the market for inventors are enormously costly for growth. Intuitively, the model finds such large costs because inventor mobility is both very frequent and also associated with an enormous increase in patenting. I then show that introducing search frictions to a standard growth model in the vein of Klette and Kortum (2004) has significant implications for firm dynamics, and can help rationalize why some firms grow to enormous sizes very quickly. Most firms in the calibrated model never get the chance to match with an inventor and therefore never get to innovate, and therefore the firms that do match therefore need to innovate very quickly to rationalize the rate of growth and creative destruction that we see in the data. In stark contrast to most firm-dynamics models, which struggle to generate firms like Amazon or Microsoft that become innovation leaders within one generation, the calibrated model can generate *more* dispersion in patenting among young firms (less than 30 years old) than we see in the data, even in the absence of any sort of firm heterogeneity. Finally, I use the model to conduct a series of counterfactuals and study novel growth policies. I ask whether firms invest too much or too little to meet with inventors, and whether policy can correct the inefficiencies in this market. I find that this level of investment is too low, and that subsidies for this investment would be welfare-enhancing. I show that R&D subsidies in their current form do not address this inefficiency, and that a policy of targeting search costs could deliver the same growth as R&D subsidies at less than one-fourth the cost.

1.1 Literature and Contribution

This paper builds on and contributes to several literatures. First, this paper introduces a model of endogenous growth and firm dynamics in the tradition of Klette and Kortum (2004). A standard feature of such models is that firms can hire inventors (and other inputs into idea creation) on a frictionless spot market. My contribution is to instead model a frictional market for inventors and show that such a market structure has markedly different consequences for policy and firm dynamics.

Conversely, this paper contributes to the literature on frictional search by applying it to inventors and disciplining it with patent data. Other papers bridging the search and Schumpeterian growth traditions are Engbom (2020) and Lentz and Mortensen (2012), but these papers model frictional search for production workers as opposed to inventors.

A growing line of research uses patent data to study inventors. Prato (2022) and Arkolakis, Peters, and Lee (2020) show that the migration of inventors across nations is a major factor in driving growth, whereas my paper studies the smaller-scale but more frequent mobility of inventors across firms. Akcigit, Caicedo, Miguelez, Stantcheva, and Sterzi (2018) studies the allocation of inventors across research teams in a model where ideas are sold on a competitive market. van der Wouden and Rigby (2020) studies inventor mobility from 1900 to 1975 and also finds that inventor mobility is associated with an increase in patenting. I perform a similar empirical exercise and use it to discipline a structural model and perform policy counterfactuals.

This paper also takes a step towards addressing a well-known difficulty within the firm dynamics literature: standard models of firms struggle to produce firms that grow extremely quickly. Jones and Kim (2018) notes that workhorse random-growth models of firm dynamics do not generate firms like Microsoft or Google that grow to enormous sizes in a short span of time, and note that it is typically necessary to add some kind of heterogeneous growth rate across firms. They cite the "Luttmer Rocket" of Luttmer (2011), a short-lived period when certain firms have better ideas before reverting to the mean, as one way to match the speed at which superstar firms grow large. Introducing search frictions to the Schumpeterian growth framework also helps address this problem: in the calibrated model, most newborn firms never match to an inventor, and therefore never spin off any new product lines. The few firms that do match innovate and expand into new product lines very quickly, which gives them the opportunity to match with more inventors down the line. Adding search frictions for inventors therefore helps generate *endogenously* heterogeneous growth rates, especially for newborn firms with relatively few product lines. The calibrated model can easily generate new firms that scale up incredibly quickly, something which standard models struggle to achieve.

Celik (2022) considers a complementary growth model where not everyone can be an inventor due to limited training, and finds that individuals are misallocated across occupations: some of the best potential inventors are shut out, whereas some mediocre individuals from rich families become inventors. This paper and my own show that the misallocation of innovative resources is a major source of lost growth.

Finally, this paper contributes to the literature on innovation policy, in the vein of Atkeson and Burstein (2019). Another related and complementary paper is Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), which studies R&D policy in a model that has heterogeneous and dynamic firm types but frictionless markets for homogeneous skilled labor. In both that paper and mine, the key source of inefficiency is the presence of a knowledge spillover: holding all else equal, raising productivity at other firms increases the returns to innovation at one's own firm. In both my paper and Acemoglu et al. (2018), growth depends on how the inputs into allocation (inventors or skilled labor) are allocated across firms, and in both papers the equilibrium allocation is generally not optimal.

2 Model

Time is continuous. There is a stand-in household with a measure M of workers, who inelastically supply their labor to profit-maximizing firms. An exogenous share ν of these individuals work in the frictional market for inventors[3], and the remaining share $1-\nu$ can freely move across four frictionless sectors: final goods production, new firm creation, "headhunting" for inventors, and R&D. Inventors become non-inventors at a rate δ_i, and non-inventors become inventors at a rate $\delta_r = (\nu\delta_i)/(1-\nu)$.[4]

There is perfect risk-sharing in the household. The household derives logarithmic utility from the final good Y and discounts the future at rate ρ. Consumption of the final good is the only source of utility in this model and there are no aggregate shocks, hence we can write the household's discounted utility U as:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \log(Y_t)dt.$$

[3] I denote this share ν of individuals as "inventors" regardless of whether they are currently matched or not. The assumption of an exogenous measure of inventors is a reasonable simplification in light of the findings in Bell, Chetty, Jaravel, Petkova, and Van Reenen (2019) that the children of inventors are far more likely to be inventors themselves. See Celik (2022) for a model that endogenously generates a fixed set of inventors.

[4] This is consistent with a steady-state share ν of inventors and helps to rationalize the fact that most inventors only patent for a few years.

2.1 Production

The production side of the model is deliberately standard. As in Klette and Kortum (2004), the final good is a CES aggregate of an exogenous measure 1 of varieties. Each variety i may be produced by multiple firms f, and each firm may produce more than one good. Different varieties $i' \neq i$ are imperfect substitutes, with elasticity of substitution σ, but the same variety produced by different firms $f' \neq f$ are perfect substitutes. I find it convenient to denote $[fi]$ as the "division" within firm f that produces good i. Production of the final consumption good Y_t is as follows, where $y_{[fi]t}$ is the production of each division:

$$Y_t = \left(\int_0^1 \left(\sum_f y_{[fi]t} \right)^{\frac{\sigma}{\sigma-1}} di \right)^{\frac{\sigma-1}{\sigma}}.$$

Division $[fi]$ has access to a linear labor-only technology with productivity $z_{[fi]t}$:

$$y_{[fi]t} = z_{[fi]t} l_{[fi]t}.$$

I follow Garcia-Macia, Hsieh, and Klenow (2019) and assume that each division must pay an infinitesimal operating cost of ϵ to produce. This ensures that only the division with the highest z in each product line i will produce, and that it will charge a homogeneous markup $\mu = \sigma/(\sigma - 1)$[5]:

I denote $z_{it} = \max_f z_{[fi]t}$ as the efficiency of the leading producer of product line i, $F_t(z_{it})$ as the CDF of product lines with efficiency z at time t, $Z_t = \left(\int_0^1 z_{it}^{\sigma-1} dF_t(z_{it}) \right)^{\frac{1}{\sigma-1}}$ as the usual measure of "average productivity" , and the "relative efficiency" of a division as $\hat{z} \equiv (z/Z)^{\sigma-1}$.

I denote the total mass of workers involved in production as l_t. Final goods output Y is $Z_t l_t$, wages for production workers w are $Y_t/(\mu l_t)$, and product-level gross profits π can be written as:

$$\pi_{[fi]t} = (\mu - 1) \underbrace{\left(\frac{z_{[fi]t}}{Z_t} \right)^{\sigma-1}}_{\text{Relative Productivity } \hat{z}} w_t L_t.$$

In what follows, I drop time subscripts where there is no possibility of confusion.

[5] If two "divisions" belonging to different firms f and f' can produce the same good i, and both decide to pay the operating cost, they compete in Bertrand fashion. If $z_{[f'i]t} < z_{[fi]t}$, then division $[f'i]$ will make no profits from production. Because of the operating cost, the less-productive division will not enter, and the more-productive division will charge a fixed markup.

2.2 New Firm Creation

Aside from creative destruction from inventors at incumbent firms, which is the main focus of this paper, growth can also come from creative destruction by entrants. There is free entry into new firm creation: any of the $M(1-\nu)$ non-inventors can at any time decide to try to start up a new firm, thereby converting their endowment of time into a flow rate of new firms.[6] The rate ϕ_s at which a worker can start up firms depends on the quality of the entry technology ι and the degree of congestion $\psi > 0$ in entry: if a measure E of new firms are created, the productivity of the entry technology scales down by E^{ψ}, similar to Peters and Zilibotti (2021).

$$\phi_s = \iota / E^{\psi}.$$

Upon successfully entering, the entrepreneur[7] draws a random product line i and improves on the relative productivity $\hat{z}_{[fi]t}$ of the leading producer of i by an additive factor λ. The entrepreneur creates a new firm f' with a single unmatched division to produce i. I assume that the "creatively destroyed" division $[fi]$ exits and lays off their inventor.[8] The representative household's valuation of the new startup is the net present value of its future profits.

Because innovations are cumulative, entrants do not internalize the impact of their efforts on future innovators. This means that there is an externality and that the market allocation will generally be inefficient.

I denote $N(\hat{z})$ as the value of an unmatched division with relative productivity $\hat{z}$, and $dF(\hat{z})$ as the PDF of product lines across relative productivity space. I can now write CD, which is both the expected value of creatively destroying a product line and the expected value of entry conditional on success:

$$CD = \int N(\hat{z}' + \lambda) dF(\hat{z}').$$

[6] Making the cost of entry be in units of labor rather than in units of the final good is important for achieving balanced growth.

[7] Note that entry is a separate technology from innovation in this model: entrepreneurs are drawn from the mass $M(1-\nu)$ of non-inventors, not the mass $M\nu$ of inventors.

[8] This simplification greatly increases the tractability of the model and can be microfounded if I assume that the frontier division, armed with the knowledge of how to produce i, can copy any innovation made by the laggard, reducing their expected profits from innovation to zero.

Because there are no frictions in or barriers between the labor market for final goods producers and the entrepreneurial market, entry E must be such that the returns to entry exactly balance out the opportunity cost of working in final goods production:

$$\phi_s CD = w.$$

The mass of workers e in entry must also be consistent with the level of entry E:

$$\phi_s e = E.$$

2.3 Innovation

Firms can only innovate if they match a division to an inventor. For tractability, I assume that each division within a firm can have at most one inventor.

I assume that inventors and research spending cannot be transferred across divisions within the same firm, and that search effort is division-specific. Combined with the fact that each variety is infinitesimal, this means that each division within a firm can make decisions without reference to any other division and that firm values are additively separable in division values.

All innovation in this model is creative destruction, which works the same for incumbents as for entrants: when the inventor and division innovate, they draw a random product i', improve on its relative productivity by the same step size λ, and spin off a new unmatched division within the same firm to produce the improved product line. The expected value of an innovation is the same CD for an incumbent as it is for an entrant.

The rate of innovation is an increasing function of the resources r (in units of labor) which the division provides to the inventor. Research effort r is a stand-in for all kinds of R&D expenditures (lab technicians, equipment, etc). The rate of innovation is also increasing in the time-invariant division-inventor match quality $q_{[fi]j}$, which acts as TFP for the innovation process. This notion of match quality is new to the literature:

I parameterize the rate of incumbent innovation ϕ_n very simply[9]:

[9] Standard models in the tradition of Klette and Kortum (2004), such as Peters (2020), also assume a firm-level innovation production function that features constant returns to scale in *both* resources r and the number of product lines controlled by the firm.

$$\phi_n(q,r) = \underbrace{q}_{\text{Match Quality}} \cdot \underbrace{r^{\gamma}}_{\text{R\&D}}.$$

Note that $\hat{z}$ does not show up in the value of innovation, hence the benefit Ω of matching to an inventor is constant across all values of $\hat{z}$.

2.4 Compensating Inventors and Investing in Research

To model the bargaining between the division and the worker, I assume the inventor and division can successfully bring the idea to market with probability 1, or the inventor can exercise their outside option and successfully bring the idea to market with probability $\chi < 1$. This captures the possibility that the inventor may try to leave the firm and steal the idea. If the division offers the inventor a share $\chi + \epsilon$ of the value of the patent, the inventor will always give the patent to the firm and it will always be brought to market.[10]

Divisions therefore anticipate that inventors will keep some share of the value of innovation, which induces a hold-up problem for the division. It must pay w for each unit of r to compensate those workers for their opportunity cost. The division therefore chooses r to maximize the following:

$$\max_r (1-\chi) q r^{\gamma} CD - wr$$

Note that divisions do not compensate inventors with a flat wage, but rather only reimburse them upon a successful innovation occurring.[11] They cannot pre-commit to a piece rate w_i and will therefore never pay inventors more than a share χ (as they would profitably deviate by cutting the payment) or less than χ (as the inventor would not share the idea with them.)

The aggregate rate of innovation per product line is equal to the rate of creative destruction, summed across entrant and incumbent innovation. I denote it as Δ.

[10] This is isomorphic to imposing an exogenous piece rate of χ. I do not use a more-standard Nash bargaining setup as it generally cannot be solved as a convex optimization problem in a model with on-the-job search—see Shimer (2006). My approach still avoids the need to carry around an extra state, and as I will show later, it also helps avoid the need to solve for the inventors' value functions altogether. The key difference between this assumption and a typical Nash bargaining setup with bargaining parameter χ is that it breaks the link between compensation and the expected duration of the match, which is the major source of difficulty in using Nash with on-the-job search.

[11] Celik and Tian (2022) and Koh, Santaeulàlia-Llopis, and Zheng (2020) document that inventors face high-powered incentives in the real world, which makes this assumption more defensible.

$$\Delta = \phi_s e + \int_{\hat{z},q} \phi(q) dF^V(\hat{z},q)$$

2.5 Matching to Inventors

I now explain how inventors are matched to divisions. To look for inventors, divisions hire headhunters h from the mass of non-inventors, paying them a wage w to bid them away from production, entry, and research. Effective search effort is $m(h) = h^\eta$, and if $\eta < 1$ this implies diminshing returns to search at the level of the division. Divisions can search regardless of whether they are already matched or not; i.e. they have the option to fire their current inventor and replace them. Search is random, and matched inventors are just as likely to be contacted as unmatched. Unmatched inventors earn an outside option of 0, and the value of being unmatched is U.

There is a constant-returns-to-scale aggregate meeting function that converts search effort (the number of unmatched divisions weighted by their search effort h^η) to meetings H. This meeting technology is a Cobb-Douglas function of aggregate search effort S and the total mass of matched and unmatched inventors νM, with a search efficiency parameter α and an elasticity with respect to search effort θ. I denote $dF^N(\hat{z})$ as the mass of unmatched divisions with relative productivity $\hat{z}$, and h as their search effort; and I denote $dF^V(\hat{z},q)$ as the mass of matched divisions with relative productivity $\hat{z}$ and match quality q, and h_q as their search effort. Note that, because innovation is unrelated to $\hat{z}$, so are the returns to search.

I denote m^* and $m^*(q)$ as the optimal search effort of unmatched divisions and matched divisions with match quality q, respectively. I am now ready to write out the aggregate meeting rate H

$$H = \alpha \left(\underbrace{\int_{\hat{z}} m^* dF^N(\hat{z}) + \int_{\hat{z},q} m^*(q) dF^V(\hat{z},q)}_{\text{Search Effort } S} \right)^\theta (\nu M)^{1-\theta}$$

I next introduce some helpful notation. First, it will be useful to write out the "contact rates", or the number of meetings per unit of search effort and the number of meetings per inventor, which I respectively denote C_s and C_i:

$$C_s = \frac{H}{S},\ C_i = \frac{H}{\nu M}$$

Upon meeting, the division and inventor draw a match quality q from a uniform distribution with support $[l, h]$. They then decide whether to break any current match that they have and match with each other. If both agree to the match, the match retains its quality q throughout its duration.

The expected value Ω of meeting a new inventor is as follows for an unmatched division:

$$\Omega = \underbrace{\left(1 - \int_{\hat{z}',q'} \frac{dF^V(\hat{z}',q')}{\nu M}\right)\left(\int_q \left(V(\hat{z},q) - N(\hat{z})\right) dF^q(q)\right)}_{\text{Meeting Unmatched Inventor}}$$
$$+ \underbrace{\int_{\hat{z}',q'} \int_{q>q'} \left(V(\hat{z},q) - N(\hat{z})\right) dF^q(q) \frac{dF^V(\hat{z}',q')}{\nu M}}_{\text{Meeting Matched Inventor}}$$

For a matched division with states $\hat{z}, \tilde{q}$, this instead becomes:

$$\Omega(q) = \underbrace{\left(1 - \int_{\hat{z}',q'} \frac{dF^V(\hat{z}',q')}{\nu M}\right)\left(\int_{q>\tilde{q}} \left(V(\hat{z},q) - V(\hat{z},\tilde{q})\right) dF^q(q)\right)}_{\text{Meeting Unmatched Inventor}}$$
$$+ \underbrace{\int_{\hat{z}',q'} \int_{q>\max[q',\tilde{q}]} \left(V(\hat{z},q) - V(\hat{z},\tilde{q})\right) dF^q(q) \frac{dF^V(\hat{z}',q')}{\nu M}}_{\text{Meeting Matched Inventor}}$$

Divisions balance the costs of hiring labor for search against the benefits of making new matches. In equilibrium, labor hired for headhunting must be paid the same market-clearing wage w as workers used in final production, and the returns to search (the benefit of meetings Ω times meetings per unit of search H_s) must equal their cost. Hence, we get the following equality for all divisions across all states:

$$m'(h)\Omega C_s = w$$

$$m'(h)\Omega(q) C_s = w$$

Matches break at an exogenous rate δ_m.

2.6 Hamilton-Jacobi-Bellman Equations

In what follows I consider a transformed version of the economy on a balanced growth path: wages w and average productivity Z grow at a constant rate g, the distribution of divisions across relative productivities $\hat{z}$ and match qualities q is constant, and policies are constant.

The value of an unmatched division has five parts: flow profit from its product, the possibility of creative destruction, the capital gains from drifting down in "relative productivity", the gains from meeting inventors Ω, and the costs of generating those meetings.

$$\rho N(\hat{z}) = \max_{h} \underbrace{\pi(\hat{z})}_{\text{Flow Profit}} - \underbrace{\Delta N(\hat{z})}_{\text{Creative Destruction}} - \underbrace{g(\sigma-1)\frac{\partial N(\hat{z})}{\partial \hat{z}}}_{\text{Drift Down}} + \underbrace{m(h)\Omega C_s - wh}_{\text{Search}}$$

The value of a matched division has six parts: flow profits, the risk that the match breaks, the possibility of creative destruction, drifting down, innovation, and the risk that their inventor will meet another division.

$$\rho V(\hat{z},q) = \max_{r,h} \underbrace{\pi(\hat{z})}_{\text{Flow Profit}} - \underbrace{(\delta_m+\delta_i)(N(\hat{z})-V(\hat{z},q))}_{\text{Match Break}} - \underbrace{\Delta V(\hat{z},q)}_{\text{Creative Destruction}} - \underbrace{g(\sigma-1)\frac{\partial V(\hat{z},q)}{\partial \hat{z}}}_{\text{Drift Down}}$$

$$+ \underbrace{m(h)\Omega(q)C_s - wh}_{\text{Search}} + \underbrace{(1-\chi)qr^{\gamma}CD - wr}_{\text{Innovation}}$$

$$+ \underbrace{\int_{q'>q} C_i\left(N(\hat{z})-V(\hat{z},q)\right)\frac{\int_{\hat{z}} h^{\eta}dF^N(\hat{z}) + \int_{\hat{z}}\int_q^{q'} h_{\tilde{q}}^{\eta}dF^V(\hat{z},\tilde{q})}{S}dF^q(q')}_{\text{Inventor Finds New Division}}$$

2.7 Equilibrium

A balanced growth path consists of detrended values V, N, W, U, policies e, h, r, X_m, X_u, wages w, distributions F, F^V, F^N, and a growth rate g such that divisions maximize profits, workers maximize their utilities, and policies are consistent with the distributions and growth rate being constant over time.

2.8 Discussion of Assumptions

The stark, simplifying assumptions that I have imposed on inventor compensation and matching (no outside option, a fixed piece rate χ, and equal meeting rates for employed and unemployed) make the model far more tractable. Crucially, they allow me to solve the model without solving for the inventors' value functions. To show this, I begin with the following helpful proposition:

Proposition 1. *Assume γ (the curvature of the innovation cost function) and χ (the inventor's bargaining share) are both strictly between 0 and 1. Further, assume that inventors and divisions only observe their own current and potential match quality q, and do not observe whether the other division or inventor is already matched. If an inventor and division meet and draw potential match quality q, and if γ strictly less than 1:*

1. *If both are unmatched, the meeting converts to a match.*
2. *If only one party is matched, the meeting converts to a match if and only if the new match quality q is higher than the quality of the party's current match.*
3. *If both are matched, the meeting converts to a match if and only if the new match quality q is greater than the quality of either party's current match.*

Proof. First, recall that inventors have no outside option, and that the search technology is equally good for matched or unmatched inventors and divisions. A matched division has the option to set $r = 0$ and search as intensely as if it was unmatched, and it has the option to accept and reject the same potential matches as if it was unmatched. Hence, being matched is at least weakly better than being unmatched for a division. We can go further and show that this dominance must be strict: if $0 < \gamma < 1$, then as $r \to 0$ the division's profits from innovation $(1-\chi)qr^{\gamma}CD - wr$ become positive. Hence, a matched division can do strictly better than an unmatched one by keeping the same search-and-match policy but engaging in a small amount of research.

Because a matched division will always engage in a positive amount of research, it will always pay out a positive amount in compensation to the inventor in expectation. A matched inventor can do strictly better than an unmatched one by accepting and rejecting the same future offers but taking the division's compensation in the meantime. Therefore, both divisions and inventors strictly prefer being matched to being unmatched.

Because both the argmax and maximum of $(1-\chi)qr^{\gamma}CD - wr$ are increasing in q, divisions and inventors both strictly prefer high q to low q. Hence, they will leave a lower-q match if and only if a higher-q match is available.

$\square$

Recall that the *only* decision inventors need to make is whether to accept a match or not, and the *only* source of welfare in this model is consumption (there are no taste shocks or moving costs.) Hence, characterizing the acceptance sets of both the firms and workers using Proposition 1 allows me to solve and estimate the model without solving for worker value functions.

3 Data

In order to discipline my model, I target a series of novel moments about inventor-firm dynamics. My main source of data is PatentsView, the US Patent Office's digitized repository of all US patent applications since 1976. A major value-added of PatentsView relative to the raw patent data is their effort to disambiguate the names of inventors and the individuals or organizations to whom the patents are assigned: the raw data only includes alphanumeric names rather than a consistent identifier. I take PatentsView's disambiguation as a given and note that my results will be affected to some degree any time that two inventors are incorrectly misclassified as the same person or one inventor is wrongly classified as two.

The key patent-level variables that I use are the inventor, the assignee (the firms and individuals who were assigned ownership rights to the patent), and the filing date of the patent (I use filing rather than grant dates as the former are presumably closer to when the patent was created.)

I subset the data to only include US-based inventors, and I only focus on patents with at least one corporate assignee. I therefore do not need to be concerned with whether non-US patents or "backyard" patents are comparable to US corporate patents. Excluding "backyard" innovation also means that my dataset is more comparable to my model, which does not have such innovations.

I provide a few key summary statistics next:

Description	Value
Number of unique patents	3.3m
Number of unique inventors	1.4m
Number of unique assignees	200k

3.1 Key Moments

In this section I introduce some of the moments that I use to discipline my model.

Inventors in my sample have, on average, 5.66 patents across 1.6 assignees, and their first and last patents are separated by an average of 5.7 years.[12] This is consistent with a move once every 3.5 inventor-years or so, indicating about as much mobility among inventors as among the general population.[13]

As I do not have access to the actual dates of employment for each inventor, I cannot directly measure job-to-job or employment-unemployment transitions. I must instead use the dates when their patents were filed. In some cases I find that inventors patent for firm A, then firm B, then A again in rapid succession. This suggests that an inventor's final patents at their old firm may be filed after their first patents at their new firm. Therefore, I code an employee as having moved in year t if the plurality of their patents were assigned to a different firm than the last time when they had non-zero patents.[14]

I compute two moments to help discipline the dynamics of the inventor labor market. First, as a proxy for transitions into unemployment, I calculate the average number of years between an inventor's first and last patent with the same assignee.[15] I find that this average time gap is around 1.8 years. Second, as a proxy for job-to-job flows, I calculate the share of patents granted to inventors whose assignee (employer) at year t is different from their employer in the last year when they had a nonzero number of patents. I find that this share is about 20 percent. Analogously, 50 percent of patents in year t are awarded to inventors who had not patented in $t-1$, including

[12] If an inventor only patents in one year, they count as a zero towards this mean.

[13] US Bureau of Labor Statistics (2022) finds that Baby Boomers had 2.1 jobs from ages 45 to 54, and had many more job changes at younger ages.

[14] If an employee has one patent at Microsoft in 2012, then one patent at Apple in 2014, I code them as having moved in 2014. If an employee has one patent for Apple in 2012, then one for Apple and two for Google in 2014, I still code them as having moved in 2014 even if their patent at Apple was filed later in 2014 than their two patents at Google.

[15] If an inventor only patents in a single year for that assignee, they count as a 0 towards this mean.

inventors who had never patented at all.

I summarize these moments in this table:

Description	Value
Inventor “career length" (years)	5.7
Average patents per inventor	5.7
Average assignees per inventor	1.6
Average inventor-firm “tenure (years)	1.8
Share of patents to “new" inventors	50
Share of patents to “poached" inventors	20

3.2 Matched Pairs

I follow Prato (2022) and discipline the effects of mobility on innovation using a matched-pairs design, pairing inventors that move with similar inventors who do not move. My units of observation are inventor-years where the inventor files for a patent.[16] My treatment observations are years in which an inventor moved and had a patent, and my placebo observations are years in which inventors had a patent but did not move.[17] I calculate “moves” in the same way as in the last section: I code an inventor as having “moved” in year t if the plurality of the patents they filed in that year were assigned to a different firm than in the last year when they had nonzero patents.

As in Prato (2022) I match treatment years to placebo years based on the cumulative number of patents that both inventors had at $t-1$, the inventors’ first year with a patent, the current year, and the fact that both had non-zero patents in year t. The last condition is important as it means that inventors patent in both the treatment and placebo years—that is, I am comparing treatment years with at least 1 patent to placebo years with at least 1 patent. I am not comparing individuals who move (and who, by definition, continue to patent) with individuals who stay but may stop patenting. Aside from these matching conditions, my large sample size gives me the opportunity to use stricter matching conditions than Prato (2022). I therefore choose my placebos from the list of inventors that never change firms, and I also match treatment to placebo based on their most recent

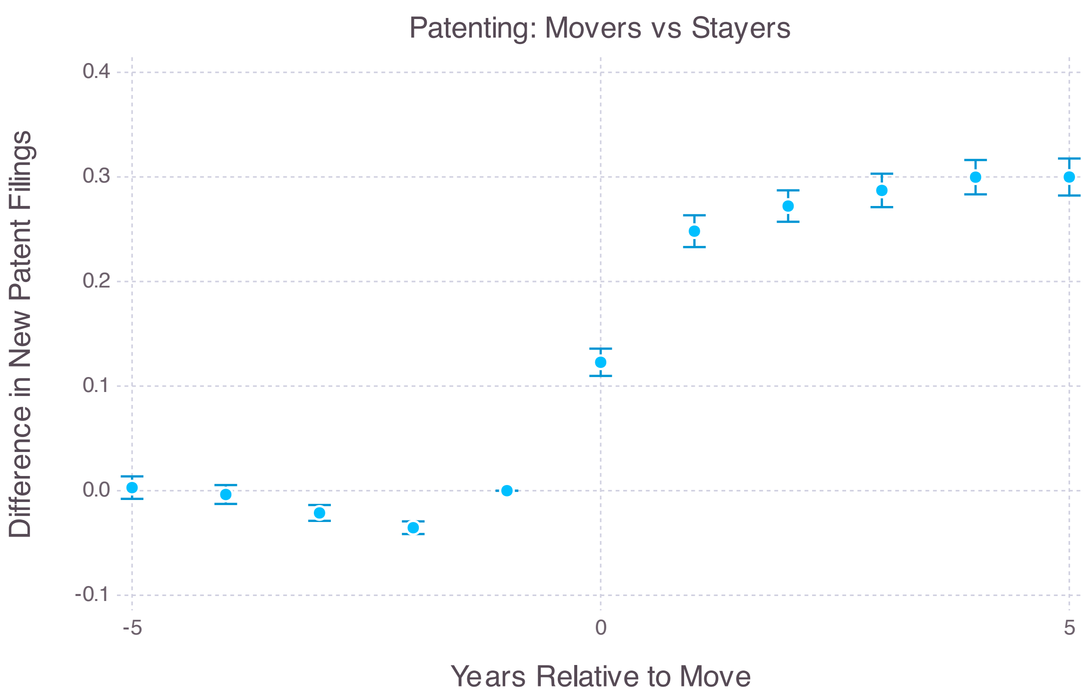
[16] In Prato (2022), the units of observation are inventors. The differences is that I do not restrict my sample to the first time that I observe an inventor move.

[17] I allow one inventor-year to act as a placebo for multiple treatments.

employer before t and on the exact number of patents that both filed in the year $t-1$. Naturally, I also choose both treatment and placebo from the set of inventors who patent in more than one year. Ultimately, I am left with a sample of over 75,000 inventor-years with a move matched to a placebo.

Note that, because my placebos are chosen from the set of inventors who stay at the same firm and keep patenting after $t-1$, the original firm must continue to patent after the treatment inventor leaves. This should exclude many instances when a "move" across firms was in fact a merger or acquisition.

For each treatment and placebo year, I record the number of patents that inventors receive in the five years before and after that year. I take the difference of these leads and lags between the treatment and placebo, and plot the difference below.



This matched-pairs comparison provides striking suggestive evidence that mobility across firms is associated with more patenting, but it has no causal or structural interpretation in and of itself. Both the treatment and placebo are selected conditional on continuing to patent, which may introduce subtle selection effects. More importantly, I do not know if the treatment moved straight from job-to-job or if they transited through unemployment. Consider an inventor who moved up the match quality ladder, lost their latest job, and then quickly found another one at the bottom of the match quality grid. Such an inventor would show up as a “treated” inventor in this analysis, and would bias down the estimates relative to the true average treatment effect of meeting and agreeing to move to a new employer. But I can run the same regressions in both my model and the data, and use this as an informative moment to discipline the underlying structural parameters.

To summarize these results in a single number, I calculate the average change in the percentage difference in patents between the treatment and placebo as follows:

$$\frac{\text{Mean Patents of Treatment, } t >= 0}{\text{Mean Patents of Placebo, } t >= 0} - \frac{\text{Mean Patents of Treatment, } t < 0}{\text{Mean Patents of Placebo, } t < 0}$$

I find that this number is around 0.4—that is, the average treatment inventor has the same number of patents as the average placebo inventor in the five years before the move, but has 40 percent more patents in the six years during and after the move.

3.2.1 Robustness

As mentioned before, the results of this this matched-pairs design should not be interpreted as any sort of causal effect or local average treatment effect. Nevertheless, I now offer a few robustness checks that suggest that these results are truly informative about the mechanism in my model.

For each treatment-placebo pair, I replace raw patent counts with citation-weighted patent counts and find that the results are qualitatively similar. I also ask whether my results are driven by inventors moving to systematically more-productive firms, rather than movement along a match quality “ladder” as in my model. To do so, I take the same treatment-placebo pairs and replace their own patent counts (both raw and citation-weighted) with the average number of patents per inventor at the firms that they worked at.[18] I find that the raw differences go in the opposite direction, whereas the cite-weighted differences go in the same direction but are much smaller quantitatively. I plot these robustness checks in the next graph: the upper row reports individual patents and citations whereas the lower row reports firm-level average patents and citations; and the left column reports raw patent counts whereas the right reports citation-weighted counts. The upper-left figure is my baseline empirical design, included here for easy comparison.

[18] If an inventor patents at Apple in 2014 and Microsoft in 2016, I replace their patents with the average inventor-level patent count at Apple in 2014 and Microsoft in 2016, but keep them at 0 in 2015.

I also find that my results are driven by an increase in patenting among treated inventors, rather than a fall in patenting among the placebos. To show this, I calculate the ratio of patents in the pre-treatment period to patents in the post-treatment period for the placebo and treatment and demonstrate that both are well above 1:

$$\frac{\text{Mean Patents of Placebo, } t >= 0}{\text{Mean Patents of Placebo, } t < 0} \sim 1.73$$

$$\frac{\text{Mean Patents of Treatment, } t >= 0}{\text{Mean Patents of Treatment, } t < 0} \sim 1.18$$

Note that the upward trend among the placebos is mechanical: not all inventors had been patenting for 5 years as of the year of the move.

4 Quantitative Exercise

I now explain how I map the model to data, with the goal of quantifying the losses caused by search frictions in the market for inventors, asking how or whether policy can recover those losses, and testing how well the model can replicate key firm dynamics facts.

4.1 Calibration

The model has fourteen parameters: the elasticity of substitution across varieties σ, the rate of discounting ρ, the innovation curvature parameter γ, the inventor compensation term χ, the firm entry congestion curvature ψ, the elasticity of the meeting function θ, the lower and upper bounds of the match quality distribution l and h, the share of the population ν that can be an inventor, the exogenous match break rate δ_m, the efficiency of the search technology α, the step size λ, the efficiency ι of the firm entry technology, and the measure of workers M.

4.1.1 Externally-Calibrated Parameters

I take parameters from the literature wherever possible.

I set the elasticity of substitution σ, discounting ρ, and curvature in the innovation production function γ function to standard values of 0.05, 4, and 0.5, as in (for example) Peters (2020).

I set the elasticity of the meeting technology θ to a standard value of 0.5, as in Fujita and Moscarini (2017) or Gavazza et al. (2018).

I follow Peters and Zilibotti (2021) in imposing a small degree of congestion ($\psi = 0.1$) in the entry technology for firms. As in that paper, this is purely for computational convenience.[19]

For the curvature of the match cost technology η, I follow Kaas and Kircher (2015) and set it to 0.5.

4.1.2 Internally-Calibrated Parameters

I match the remaining nine parameters by replicating key moments in both the data and the model. For a subset of moments, I calculate analogues using the steady state and do not simulate. For the remainder of these moments, I simulate a panel of 30,000 inventors over 40 years and calculate the same moments in the data as in the model. I estimate all parameters jointly but give a heuristic explanation of which moments map to which parameters in this section.

[19] This ensures that the amount of labor demanded for entry will never have a corner solution.

I scale the highest and lowest match qualities ($q = q_l$ and $q = q_h$) and the entrant productivity ι inside the solution algorithm to ensure that GDP growth is 2 percent.

I set λ to target an exit of entrant firms of 14 percent, a value I take from Peters and Walsh (2022).[20]

I target h/l, the relative gap between the most and least productive inventors, so that the model can reproduce the 40 percent increase in patenting that I find with my matched-pairs design. I run the same code in the simulated data as in the real data: I code an inventor as having moved in year t if the plurality of their patents in t were assigned to a different firm than in the last year when they had a patent.

To calibrate ν, the exogenous share of workers who are capable of being an inventor, I target the share of workers who patent in a given year. In the data, I divide the number of individuals with a patent in each of the years from 1976 to 2022 by the size labor force in that year and take the average of that value. On average, fourteen in every ten thousand workers have a patent each year.

I calibrate the match break rate δ_m to match the 1.8 years separating an inventor's first and last patent within the same firm.[21]

To discipline the rate δ_i at which inventors lose their creativity and become non-inventors, I target the average length of time between an inventor's first and last patent.

I use α, the efficiency of the meeting technology, to target the 20 percent of patents in year t that were awarded to inventors who move to a new firm in t.[22] This is a rough proxy for the number of new matches in the economy and is therefore informative about the rate at which meetings happen.

To discipline the "bargaining parameter" χ, I use evidence collected in Koh et al. (2020) that labor comprises roughly 60 percent of the cost of investment in intellectual property. The closest analogue to these labor costs in the model are inventor salaries, as I interpret r as labor used to produce physical inputs for research. I therefore calculate the ratio of inventor pay to inventor pay

[20] Growth in this model is equal to $\lambda\Delta/(\sigma-1)$, and the exit rate of entrant firms (which start with a single product) is approximately Δ. In order to reconcile 2 percent growth, an elasticity of substitution of 4, and a creative destruction rate of 14 percent, λ must be 3/7. The fact that I am rescaling the values of the q grid and ι to hit the growth rate means that the model will produce a creative destruction rate of almost exactly 14 percent if I fix λ to roughly 3/7. Note that, because I solve the model on a discretized grid for $\hat{z}$, the "effective" value of λ will be slightly lower than 3/7, as steps are bounded by the top of the grid, and so the creative destruction rate will be marginally higher than 14 percent. In practice, the inaccuracies that this introduces are tiny.

[21] As with the same moment in the data, an inventor who only patents in a single year counts as 0 towards this mean.

[22] I use the same approach as in the matched-pairs design: I count an employee as having moved if the plurality of their patents in year t were awarded to a different firm than the last year when they had a patent, and I do this in both in the real and simulated data.

plus the cost of r and target a value of 0.6.

I use the new-firm creation parameter ι to target the share of employment in firms less than one year old, which was roughly 1.4 percent in 2018 according to Hong and Werner (2020). Because an entrant firm consists of a single division and operates a single product line at first, I approximate this moment without simulation by multiplying the mass of entrants by the average employment in a newly-created division.

I have normalized the measure of product lines to 1, but I cannot similarly normalize the measure of workers M. Because νM controls the number of inventors per product line, and because q is rescaled to ensure GDP growth is 2 percent, M helps the model rationalize the number of patents per inventor. That is, growth in this economy roughly depends on the number of inventors per product line times the average number of innovative steps per inventor. M scales the first of these two numbers so that the model can rationalize the second while still producing 2 percent GDP growth.

4.1.3 Other Parameters

I add two additional "parameters" for the full quantitative implementation.

First, not every innovation is recorded as a patent, and my estimates will be biased if I map innovations directly onto patents. Arundel and Kabla (1998) find that 35 percent of innovations are recorded as patents in their sample of large European industrial firms, which I use as a benchmark. I therefore set $\kappa = 0.35$ as the share of ideas that are patentable, and only record a random 35 percent of innovative steps as patents in my simulated data.

Second, I add an R&D subsidy, which I model as a fixed percentage subsidy s_r for all expenditures on r workers and compensation to inventors. The problem of a matched division choosing r becomes:

$$\max_r (1 - \chi(1 - s_r))qr^\gamma CD - wr(1 - s_r)$$

The government finances these subsidies in the model with taxes on the labor income of non-inventors. As non-inventors supply their labor inelastically, such taxes are non-distortionary, and I do not need to loop over taxes to balance the budget.

I treat s_r, the effective R&D subsidy, as a parameter to be estimated. Tyson and Linden (2012) report that in 2008, the R&D tax credit cost 11 billion dollars, and direct government support for corporate research cost another 26 billion. With US GDP at about 15 trillion that year — see US Bureau of Economic Analysis (2022) — I target a 0.0025 GDP share of corporate research subsidies.

4.2 Results

In Table 4.2 I present the results of the estimation.

	Description	Value	Target	Data	Model
$q = l$	Lowest match quality	1.4	GDP growth	2%	2%
$q = h$	Highest match quality	1.73	Patenting change after move	40%	37%
λ	Step size	0.43	Entrant exit	14%	14%
ν	Inventor share	0.002	Share of workers with a patent	0.14%	0.14%
δ_m	Exog. match break	1.1	Inventor-firm match duration (years)	1.85	1.77
δ_i	Inventor exit	0.12	Inventor "career" lengths (years)	5.7	5.5
α	Search efficiency	9.4	Share of patents to "poached" inventors	0.2	0.22
χ	Inventor share	0.54	Inventor pay/research costs	0.6	0.63
ι	Firm creation efficiency	0.03	Employment share of startups	1.4%	1.4%
M	Measure of workers	8.17	Average patents per year	1.0	1.0
s_r	Research subsidies	0.18	Subsidies/GDP	0.0025	0.0026
ρ	Discounting	0.05	Standard, i.e. Peters (2020)		
σ	Elasticity of substitution	4	Standard, i.e. Peters (2020)		
γ	Innovation cost curvature	0.5	Standard, i.e. Peters (2020)		
α	Meeting elasticity	0.5	Standard, i.e. Fujita and Moscarini (2017)		
η	Search cost curvature	0.5	Kaas and Kircher (2015)		
κ	Patentable share of ideas	0.35	Arundel and Kabla (1998)		
ψ	Entry congestion	0.1	Set externally—see Peters and Zilibotti (2021)		

In the next table I focus on the subset of moments reported in Section 4.2: average inventor career lengths, average number of assignees and patents in the sample, average inventor-firm tenure lengths, and the share of patents awarded in year t to inventors who had not patented in

$t-1$. Some of these moments overlap with the main list of calibration targets above, and others do not. I put a low but non-zero weight on this second set of moments in the calibration, so these are not untargeted.

Description	Data	Model
Inventor "career length" (years)	5.7	5.5
Average patents per inventor	5.7	5.6
Average assignees per inventor	1.6	1.9
Average inventor-firm "tenure" (years)	1.8	1.77
Share of patents to "new" inventors	0.5	0.52
Share of patents to "poached" inventors	0.2	0.22

The model performs well on all targets, even though the addition of the second set of moments means that it has too few degrees of freedom to match all moments perfectly.

4.2.1 Other Moments

In the model, 15.5 percent of GDP is paid out as investments in innovation (compensation to r or to inventors). Atkeson and Burstein (2019) report a wide range of reasonable estimates for the right GDP share of such investments, from the 3 percent of value added in research and development on the low end to the estimated 13 percent of non-farm output in the broader measure of Corrado, Hulten, and Sichel (2009), with their preferred value of 6 percent of value added splitting the difference. Hence, the model's inferred value of investments in innovation is towards the upper range of plausible estimates. As there is significant uncertainty about the true share of output that acts as an investment in innovation, I leave this as an untargeted moment.

Gavazza et al. (2018) find that search costs roughly amount to one month of pay per hired employee, or 8e-2 years' worth of compensation. The analogous value in the model is 1e-8, a far smaller number. Inventors are obviously quite different from the typical worker (in the calibrated model, their pay is around 60 times greater), so it is not clear that it makes sense to extrapolate the number in Gavazza et al. (2018).

4.2.2 The Allocation of Inventors

63 percent of potential inventors are matched in equilibrium. This surprisingly low number is an artefact of my choice not to let them take other jobs when not working as an inventor.[23]

The and the average match quality conditional on being matched is 1.4, and the average value not conditional on matching (setting $q = 0$ for unmatched inventors) is 1.1. This compares to a minimum (conditional on being matched) of 1.16 and a maximum of 1.52. Below, I display the distribution of matched inventors across the match quality distribution.

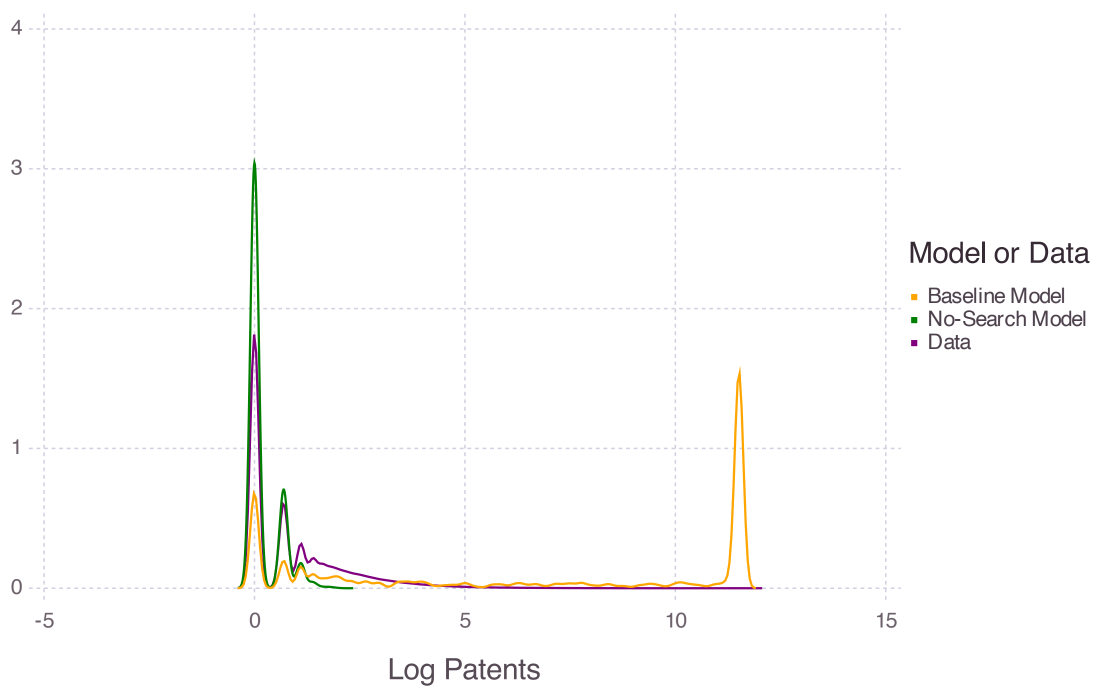
4.3 Implications for Firm Dynamics

This model offers new insights on the determinants of firm growth and innovation: most product lines never match with an inventor before being creatively destroyed, which means that the remaining product lines need to innovate rapidly to make up the difference. Most newborn firms never match more than a handful of their products to inventors, but the small handful that do then spin off many "daughter" product lines that give them more chances to match with inventors

[23]Without this assumption, it is possible that certain values of q would be unacceptably low for an unmatched inventor. This would make it challenging to separately identify α and the lowest value that q could take.

down the line. Some newborn firms therefore die without having innovated at all, whereas a small handful expand rapidly.

To show how introducing search frictions can lead to dramatic dispersion in innovation across firms, I simulate a 30-year panel of newborn firms, born at random dates within that time window, and tally up all the patents that these firms receive by the end of that window. To speed up the simulation, I do not allow firms to innovate after they receive 120,000 patents.I compare the distribution of patents across entrant firms to the distribution generated by a benchmark model where the intensity of innovation across product lines is the same, and to the distribution of patents awarded to firms that first patented in 1990 or later in the real data. In stark contrast to the benchmark model, and indeed to the bulk of the Schumpeterian growth literature, the model with search frictions generates *more* dispersion in patenting than we see in the data for these young firms[24]:



Recall that this model has no firm- or inventor- level permanent heterogeneity, only an idiosyncratic match quality term. It does not need such features, or other modeling devices such as the "Luttmer Rocket", to rationalize why some firms go from small startups to innovation powerhouses

[24]Note that many firms actually hit the cap of 120,000 lifetime patents, which explains why the distribution appears to have a second major peak at the far right end of the graph

in a very short span of time. Note also that this is an untargeted moment—the model can generate at least as much dispersion in firm-level innovation as we see in the data without explicitly targeting it. Introducing search frictions for inventors to the Schumpeterian growth framework therefore helps make sense of some puzzling, hard-to-match firm dynamics facts without needing to "bake in" unexplained heterogeneity.

To help build more intuition for this result, and to understand what parameters are most relevant, note that in the standard framework of Klette and Kortum (2004) new divisions creatively destroy just under one rival division on average before being creatively destroyed themselves.[25] In the absence of any sort of firm heterogeneity, the exact number of "child" divisions follows a Poisson distribution, with a mean and variance of slightly less than 1. In my model, divisions need to match to an inventor before they innovate, and the odds of innovating before being creatively destroyed are quite low at just under 11 percent. Conditional on innovating at all, the number of innovations per division has a mean and variance of just over 7. Hence, introducing search frictions for inventors is a way of adding heterogeneity in growth rates, which especially drives dispersion for young firms with only a handful of inventors.

The most important combination of parameters driving these results is $M\nu$, the number of workers per product line times the share of workers who can be inventors. At the current calibration, this number is extremely low at 0.02.[26] Hence, very few divisions get to innovate, and those that do then innovate rapidly.

4.4 Counterfactuals

In the next sections I study the costs of the misallocation of inventors, and ask whether policy can recover some of these costs.

4.4.1 The Cost of Search Frictions

In order to establish a benchmark or upper bound for the cost of the misallocation of inventors, I begin by increasing the efficiency of the search technology α until the average match quality q of a

[25] Without entrant firms, which are a relatively small source of growth, new divisions would need to creatively destroy exactly one rival on average to keep the mass of product lines fixed at 1.

[26] Note that inventors cycle in and out of the economy, and matches tend to break very quickly. This helps explain why 11 percent of divisions get to innovate before being destroyed even though not all of them could

matched inventor gets close to h, and until almost all inventors are matched. I find that growth in this counterfactual economy increases from 2.0 to 2.6 percent, an enormous gain with significant consequences for welfare.

4.4.2 Subsidizing Search

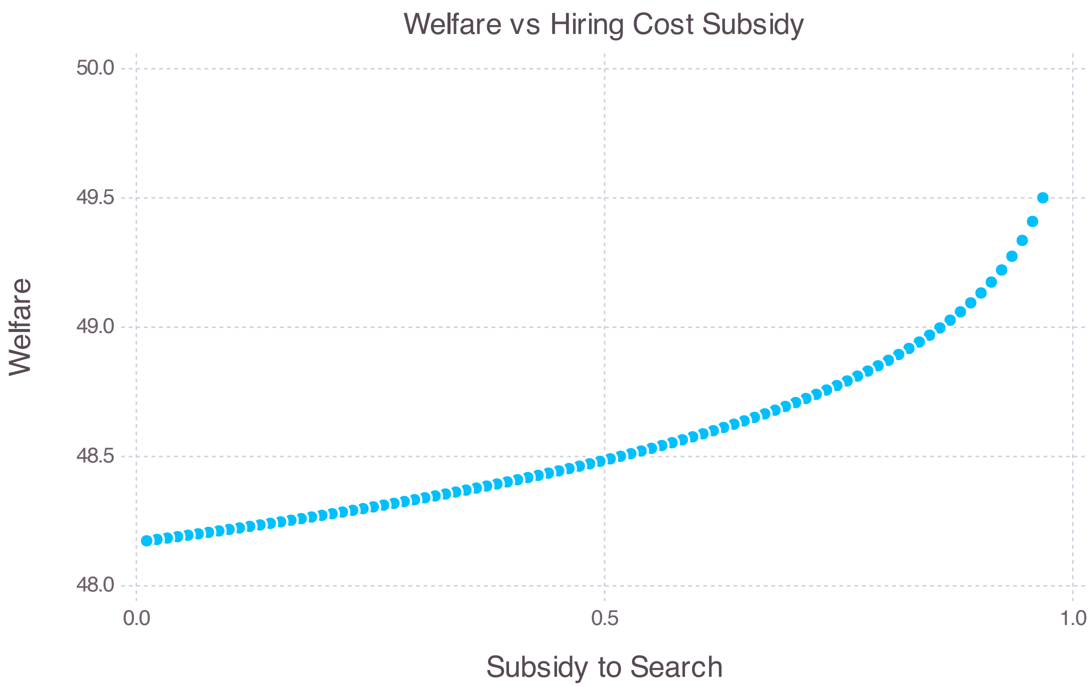
Even though reducing search frictions in the market for inventors would significantly enhance growth, it does not follow that the economy under-invests in inventor search h at the current value of α, or that there is a useful role for policy. The economy faces a simple dynamic tradeoff: On the one hand, a unit of labor that is allocated to the meeting technology increases the rate at which firms match with inventors, which will increase the average match quality q and therefore the growth rate of technology and consumption. On the other hand, that unit of labor (and, more significantly, the increased units of labor allocated to r in response to a larger number of better-matched inventors) cannot be allocated to producing the final good, so there is less consumption holding fixed the level of technology. Because there is congestion in the market for inventors, the resources expended in search may even be suboptimally high.

I therefore study a subsidy (or tax) s_h for every dollar that firms expend to match with inventors. The government finances this with a tax τ_i on the wage income of non-inventors—recall that, as non-inventors supply their labor inelastically, taxing them (or subsidizing them, if the optimal s_h is negative) induces no inefficiency.

The division's problem of how much to invest in meeting inventors therefore becomes:

$$\max_h m(h)\Omega H_s - wh(1 - s_h)$$

I compare different balanced growth paths by the net discounted present value of consumption $U = \int_{t=0}^{\infty} e^{-\rho t} \log(Y_0 e^{gt})$ starting at the same level of technology Z. I plot this measure of welfare against the subsidy s_h below, for values of s_h between 0 and 0.95, and find that it is strictly increasing over this interval. The share of labor allocated to search is still only a fraction of a percent even when the government pays 95 percent of search costs, and the taxes needed to balance the budget are similarly tiny at barely half a percent of labor income. The share of labor allocated to research increases because more inventors are matched and because average match quality improves somewhat.



The key takeaway from this section is that knowledge spillovers dominate congestion spillovers, and so firms under-invest in searching for inventors. A policy of subsidizing search costs is not unprecedented—to give one example, the US Patent Office runs a service that matches small inventors to lawyers working pro bono.[27] Other services such as JOE for economists or the medical matching process can be thought of as institutions designed to lower search costs. One takeaway from this model is that the private sector will systematically underinvest in such friction-reducing technologies.

4.4.3 Are R&D Subsidies Sufficient?

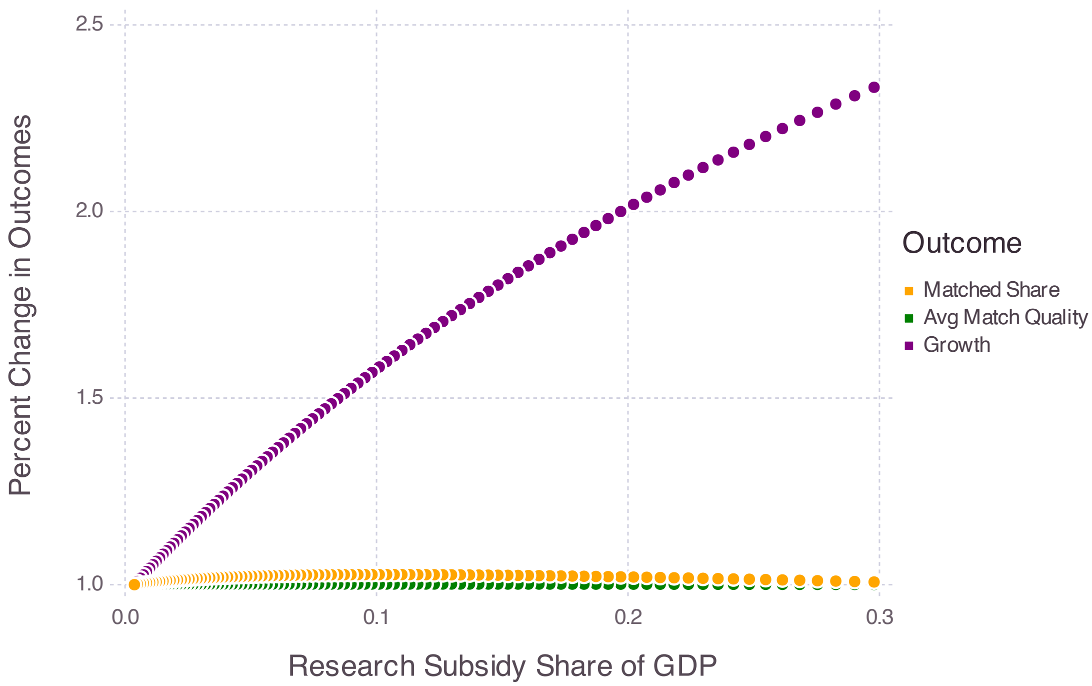
This model has two margins of investment: firms can invest in their "match capital" by increasing h, and they can invest in their "idea capital" by increasing r. I summarize the various subsidies to the latter form of investment as s_r, a stand-in for the R&D tax credits and other research subsidies. While one policy instrument cannot attain full efficiency in a model with two choice variables, it is possible that these R&D subsidies could bring the economy fairly close to the optimum by

[27] This program is described in more detail at https://www.uspto.gov/patents/basics/using-legal-services/pro-bono/patent-pro-bono-program.

themselves: subsidizing r and inventors' salaries increases the returns to search effort, and can in principle indirectly raise the average match quality to the point where there would be minimal gains from any further improvements.

In practice, I find that the indirect effects of R&D on investment in search are largely canceled out by their indirect effect on creative destruction, and that raising R&D subsidies does not meaningfully improve the allocation of inventors. This intuition points to a novel reason why R&D subsidies may yield disappointing results in the real world: the additional creative destruction coming from more innovation by rival firms destroys "match capital" and lowers the innovative potential of the economy. In contrast to standard Schumpeterian growth models, such capital is costly to build up and is totally lost to the economy when a division becomes obsolete. To be clear, I find that R&D subsidies *do* increase growth in this model, but only by reallocating a huge share of the workforce towards r and not by increasing the efficiency of innovation through better matching. In the next graph, I plot the growth rate of the economy, the share of matched inventors, and the average match quality of inventors conditional on being matched at all for R&D subsidies going all the way to 30 percent of GDP, a number I consider unreasonably high.[28] Growth more than triples for extreme values of the subsidy, but match quality and the share of matched inventors stay flat.

[28]Note that there is no curvature in the supply of labor to the r sector.



4.4.4 Comparing the Subsidies

I next compare the R&D subsidy s_r to the hypothetical search subsidy s_h. I set $s_r = 0$ and solve for the new steady state, finding that growth falls marginally. I then search for the value of s_h that restores growth to its former level, and find that it can do so at a much lower cost to the government. The cost disparity remains in favor of s_h even if I inflate search costs in my model to match those in Gavazza et al. (2018)—even with such an extreme correction, subsidies to search restore growth at one-fifth the cost of traditional R&D subsidies.

5 Conclusion

In this paper, I study and characterize the labor market for inventors. I document a series of novel facts about inventor labor market flows and find that the frictions in this market are of first-order importance for growth. I demonstrate that adding search frictions to the Schumpeterian growth framework can help rationalize why some firms grow to enormous sizes very quickly whereas others stay small, and I use the calibrated model to study various counterfactual policies.

I show that conventional R&D subsidies leave this margin of lost growth on the table, and that novel policies that target these search frictions can increase the rate of growth at a low cost to the government.

References

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018). Innovation, reallocation, and growth. *American Economic Review 108(11)*, 3450–91.

Akcigit, U., S. Caicedo, E. Miguelez, S. Stantcheva, and V. Sterzi (2018). Dancing with the stars: Innovation through interactions. Technical report, National Bureau of Economic Research.

Arkolakis, C., M. Peters, and S. Lee (2020). European immigrants and the united states' rise to the technological frontier.

Arundel, A. and I. Kabla (1998). What percentage of innovations are patented? empirical estimates for european firms. *Research Policy 27*(2), 127–141.

Atkeson, A. and A. Burstein (2019). Aggregate implications of innovation policy. *Journal of Political Economy 127*(6), 2625–2683.

Bell, A., R. Chetty, X. Jaravel, N. Petkova, and J. Van Reenen (2019). Who becomes an inventor in america? the importance of exposure to innovation. *The Quarterly Journal of Economics 134*(2), 647–713.

Celik, M. (2022). Does the cream always rise to the top? the misallocation of talent in innovation. *Journal of Monetary Economics (forthcoming)*.

Celik, M. A. and X. Tian (2022). Agency frictions, managerial compensation, and disruptive innovations.

Clemens, M. A. (2011, September). Economics and emigration: Trillion-dollar bills on the sidewalk? *Journal of Economic Perspectives 25*(3), 83–106.

Corrado, C., C. Hulten, and D. Sichel (2009). Intangible capital and us economic growth. *Review of income and wealth 55*(3), 661–685.

Engbom, N. (2020). Misallocative growth. Manuscript.

Fujita, S. and G. Moscarini (2017). Recall and unemployment. *American Economic Review 102(7)*, 3875–3916.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2019). How destructive is innovation? *Econometrica 87*(5), 1507–1541.

Gavazza, A., S. Mongey, and G. L. Violante (2018). Aggregate recruiting intensity. *American Economic Review 108(8)*, 2088–2127.

Hong, S. and D. Werner (2020). Trends in Startups? Share of Jobs in the U.S. and Eighth District. *The Regional Economist 28*(1).

Jones, C. I. and J. Kim (2018). A schumpeterian model of top income inequality. *Journal of Political Economy 126*(5), 1785–1826.

Kaas, L. and P. Kircher (2015). Efficient firm dynamics in a frictional labor market. *American Economic Review 105*(10), 3030–60.

Klette, T. J. and S. Kortum (2004). Innovating firms and aggregate innovation. *Econometrica 112(5)*, 986–1018.

Koh, D., R. Santaeulàlia-Llopis, and Y. Zheng (2020). Labor share decline and intellectual property products capital. *Econometrica 88*(6), 2609–2628.

Lentz, R. and D. T. Mortensen (2012). Labor market friction, firm heterogeneity, and aggregate employment and productivity. *Unpublished Manuscript*.

Luttmer, E. G. (2011). On the mechanics of firm growth. *The Review of Economic Studies 78*(3), 1042–1068.

Peters, M. (2020). Heterogeneous markups, growth, and endogenous misallocation. *Econometrica 88(5)*, 2037–2073.

Peters, M. and C. Walsh (2022). Population growth and firm-product dynamics. Manuscript.

Peters, M. and F. Zilibotti (2021). Creative destruction, distance to frontier, and economic development. *The Economics of Creative Destruction - A Festschrift in Honor of Philippe Aghion and Peter Howitt*.

Prato, M. (2022). The global race for talent: Brain drain, knowledge transfer, and economic growth.

Shimer, R. (2006). On-the-job search and strategic bargaining. *European Economic Review 50*(4), 811–830.

Tyson, L. and G. Linden (2012). The corporate r&d tax credit and us innovation and competitiveness: Gauging the economic and fiscal effectiveness of the credit. *Center for American Progress*, 21–2.

US Bureau of Economic Analysis (2022). Gross domestic product. retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDP.

US Bureau of Labor Statistics (2022). Number of jobs, labor market experience, marital status, and health: Results from a national longitudinal study. retrieved from BLS; https://www.bls.gov/news.release/pdf/nlsoy.pdf.

van der Wouden, F. and D. L. Rigby (2020). Inventor mobility and productivity: a long-run perspective. *Industry and Innovation 28(6)*, 677–703.

6 Appendix: Solving the Model

I use a discretized version of the model, hence why I write sums rather than integrals in this section.

6.1 Pseudocode

- Step 0: Initialize the model with a guess for distributions F, F^V, F^N; general equilibrium objects π, g, Δ, w; values V, N; and policies r, h, e as described below.

  - Re-scale the $\hat{z}$ grid so $Z = 1$.
  - Guess that $\min[\nu M \cdot 0.5, 0.5]$ inventors are matched, i.e. that either half of inventors or half of all divisions are matched (this ensures that, no matter what I put for M and ν, I will not guess that more than a measure 1 of inventors or divisions are matched). Guess that matched divisions are uniformly distributed across the $\hat{z}, q$ grid, and that unmatched divisions are uniformly distributed across the $\hat{z}$ grid.
  - A reasonable first guess for V and N, conditional on the guess for the number of workers in production L^p, would be the present discounted value profits accruing to a division with productivity $\hat{z}$ assuming no innovation, drift from growth, or creative destruction. I increase these values by 10% for V to take into account that matched division have the option to innovate, and I increase them by a further multiplicative factor equal to $0.1 \cdot q/q_l$ for each value of q to take into account the greater value of a division with a good match.
  - Guess that 10 percent of non-inventor workers will work in producing the research good (this is r).
  - Guess that 5 percent of non-inventor workers will work in entry (this is e).
  - Guess that 0.1 percent of non-inventor workers will work in search (this is h), and that search effort is uniform for all divisions.

- Step 1: Holding fixed the distributions, update policies, values, and GE objects.

  - Step 1a: Holding fixed policies r, h, e and values V, N, solve for the GE terms Δ, w, π, g.

– Step 1b: Re-scale every element of the q grid and ι by a common term κ until g is 2%[29], *unless* performing a counterfactual. Check if the growth rate had converged; i.e. check if it had already been 2% before rescaling.

– Step 1c: Holding fixed values and GE terms, solve for policies r, h, e.

  * If divisions demand $R > 0.6M(1-\nu)$ units of labor as an input into research, scale down their demands by a factor $0.6M(1-\nu)/R$ such that no more than 60% of the non-inventor labor force is used for research.

– Step 1d: Holding fixed policies and GE terms, update values V and N using an implicit step, and check if they have converged.

– Step 1e: Repeat 1a-1d until values and the growth rate converge, or until the maximum number of inner-loop iterations is reached.

- Step 2: Update the distributions using the policies recovered in Step 1.

  – If $\sum_{\hat{z},q} F^V(\hat{z},q) > M\nu$, i.e. there are more matched divisions than inventors, scale down the mass of matched divisions until only 90% of inventors are matched, and scale up the mass of unmatched divisions until the overall mass is again 1.0.

- Step 3: Repeat 1 and 2 until values, distributions, and the growth rate converge, or until the maximum number of outer-loop iterations is reached.

6.2 Solving for GE objects

To calculate the rate of creative destruction Δ, I sum up innovative activity among matched divisions and entrants.

$$\Delta = e\iota/E^{\psi} + \int_{\hat{z},q} \phi(q) dF^V(\hat{z},q)$$

This allows me to get the growth rate g, which is $\Delta\lambda/(\sigma-1)$.

[29] I follow Engbom (2020) in holding fixed the growth rate and scaling the parameters to ensure that this growth rate is attained. I find, as he did, that the solution algorithm is more stable if I let the parameters vary but keep the growth rate fixed than vice versa.

Note that both Δ and g can be scaled if we multiply ι and every value of q by a multiplicative $\kappa = 0.02/g$. Doing so would rescale innovation such that the growth rate would be 2 percent if policies were held fixed.

Wages and flow profits are as usual in CES models of monopolistic competition:

$$w = Y/(\mu l)$$

$$\pi = (\mu - 1)\hat{z}wL$$

6.3 Updating Policies

Solving for entry e is straightforward conditional on knowing the value of creative destruction $CD = \sum N(\lambda\hat{z})F(\hat{z})$. Recall the three key equations for this problem:

$$\phi_s = \iota/E^{\psi}$$

$$\phi_s CD = w$$

$$\phi_s e = E$$

Combining the first and second equation immediately yield:

$$E = \left(\frac{\iota CD}{w}\right)^{\frac{1}{\psi}}$$

Combining this with the first and third yields:

$$e = \frac{1}{\iota}\left(\frac{\iota CD}{w}\right)^{\frac{1-\psi}{\psi}}$$

Now I turn to the problem of the matched division choosing r. Conditional on CD, w, and τ_r, the division's effort level r can be solved in closed-form:

$$\max_r (1 - (1 - \tau_r)\chi)qr^{\gamma}CD - (1 - \tau_r)wr$$

$$r = \left(\frac{(\gamma(1-(1-\tau_r)\chi)qCD)}{(1-\tau_r)w} \right)^{\frac{1}{1-\gamma}}$$

Finally, I calculate the number of workers hired as headhunters. Conditional on the value of meeting Ω and on the contact rate per "unit of matching" C_s, search effort can be solved as follows for unmatched and matched firms, respectively:

$$\eta h^{\eta-1}\Omega C_s = w(1-s_h)$$

$$h = \left(\frac{w(1-s_h)}{\eta\Omega C_s} \right)^{\frac{1}{\eta-1}}$$

$$\eta h(q)^{\eta-1}\Omega(q)C_s = w(1-s_h)$$

$$h(q) = \left(\frac{w(1-s_h)}{\eta\Omega(q)C_s} \right)^{\frac{1}{\eta-1}}$$

With r, h, e solved, labor used in production l is $M(1-\nu)$ minus labor used for all three other functions.

6.4 Updating Values

I use a discretized version of the HJB equations, with $\hat{z}$ and q on grids. I approximate the derivative of V and N with respect to $\hat{z}$ using a standard upwinding approach:

$$\frac{\partial V(\hat{z}_n)}{\partial \hat{z}} \sim \frac{V(\hat{z}_n) - V(\hat{z}_{n-1})}{\hat{z}_n - \hat{z}_{n-1}}$$

I stack V and N into a single vector S and construct an intensity matrix across states A that summarizes all the flows across these states. This matrix takes policies as given and incorporates the costs of taxation and wages incurred when a division or inventor moves across states. I denote S as the stacked vector of values and f as the flow utility coming from profits, research costs, and effort.

$$S_{t+\epsilon} = S_t + \epsilon(f + AS_t - (\rho - g)S_t)$$

This can be rewritten as:

$$\underbrace{\left[\left(\frac{1}{\epsilon} + \rho - g\right) I - A\right]}_{T} S_{t+\epsilon} = \frac{1}{\epsilon} S_t + f$$

Finally we get the updated vector of values:

$$S_{t+\epsilon} = T^{-1}\left(\frac{1}{\epsilon} S_t + f\right)$$

I set an extremely large value for ϵ, check for convergence, and *then* update the value of S by taking a convex combination of $S_{t+\epsilon}$ and S_t.

6.5 Updating Discretized Kolmogorov Forward Equations

It is sufficient to characterize the distribution of divisions across states $(\hat{z}, q)$—knowing this distribution automatically pins down the distribution of workers. Denoting G as the stacked vector of distributions across states and K as the transition matrix across states, we get:

$$G_{t+\epsilon} = G_t - \epsilon K' G_t$$

It is possible to solve for the stationary distribution directly by iterating on this equation repeatedly or by looking for the solution to:

$$K'G = 0$$

I find the following approach works well: first, scale every element of K' by some factor s such that the sum of absolute values in each row is much less than 1.0 (recall that only the diagonal elements of K' are negative). Next, add I to K'. Then look for the solution to:

$$(I + sK')G = G$$

$(I + sK')$ is a stochastic matrix where all elements are positive and the rows sum to 1, and I can find G using off-the-shelf routines for finding the stationary distribution of a stochastic process.

The matrix K used to update the distribution is different from the one used to update the values (A). First, a creatively destroyed division does not derive value from the new division created to replace it, but the "destroying" division does. I do not keep track of the "destroying" distribution in A, but I do in K. Second, K does not incorporate taxes.

Notice one slightly unusual thing about K: the elements of the distribution actually show up directly in the matching probabilities. To solve for G, I hold K fixed and update till G converges, then I recalculate K and re-solve for G. I effectively iterate on solving for K and G until both converge.

6.6 Extreme Values of $\hat{z}$

I solve the model on a discretized grid of $\hat{z}$, which does not have a natural upper or lower bound. In this section, I describe a few adjustments that I make to the model to deal with this discretization.

First, at the lowest point of the $\hat{z}$ grid, I set the terms $\partial V(\hat{z}, q)/\partial \hat{z}$ and $\partial N(\hat{z})/\partial \hat{z}$ to 0.

Second, I modify the value of creative destruction CD. If a division or unmatched inventor creatively destroys a rival division with $\hat{z} > \hat{z}_{\text{max}}$, the highest value of $\hat{z}$ on the grid, they spin off a new unmatched division with productivity $\epsilon \to 0$ greater than the rival, not λ greater. Hence, the value of CD becomes:

$$CD = \sum N(\min[\hat{z}_{\text{max}}, \lambda + \hat{z}])F(\hat{z})$$

Finally, when updating g, I setΔ to 0 for the highest value of the grid.

7 Appendix: Deriving Growth and Drift Rates

In this section I offer heuristic explanations of how to calculate g and where the term $g(\sigma - 1)\partial N(\hat{z})/\partial\hat{z}$ in the HJBs comes from. To do so, the following derivations about $\hat{z}$ will be useful.

$$\hat{z} = \left(\frac{z}{Z}\right)^{\sigma-1}$$

$$Z = \left(\int_{\hat{z}} z^{\sigma-1} F(z)\right)^{\frac{1}{\sigma-1}}$$

$$Z = \left(\int_{\hat{z}} \hat{z} Z^{\sigma-1} F(z)\right)^{\frac{1}{\sigma-1}}$$

$$1 = \left(\int_{\hat{z}} \hat{z} F(z)\right)^{\frac{1}{\sigma-1}} = \int_{\hat{z}} \hat{z} F(z)$$

The growth rate g can be calculated by calculating the percent change in Z over an interval of time divided by the length of the interval, and then taking the limit of that expression as the length of the interval goes to zero.

$$g = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{Z_{t+\epsilon} - Z_t}{Z_t}$$

Recall:

$$Z_t = \left(\int_z z^{\sigma-1} F_t(z)\right)^{\frac{1}{\sigma-1}} = Z\left(\int_{z_t} \frac{z^{\sigma-1}}{Z^{\sigma-1}} F_t(z)\right)^{\frac{1}{\sigma-1}} = \left(\int_z \hat{z} F_t(z)\right)^{\frac{1}{\sigma-1}}$$

At each point of the productivity grid, some share of the mass at that grid point jumps from $\hat{z}$ to $\lambda + \hat{z}$ at a rate of Δ. Over a time step ϵ, this means that:

$$Z_{t+\epsilon} = \left(\int_z (\hat{z} + \epsilon\Delta\lambda) F_t(z)\right)^{\frac{1}{\sigma-1}}$$

Dividing the numerator and denominator by Z_t, dropping time subscripts, and we get:

$$g = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{\left(\int_z (\hat{z} + \epsilon\Delta\lambda) F_t(z)\right)^{\frac{1}{\sigma-1}} - 1}{1}$$

Simplifying a bit:

$$g = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{\left(\int_z \hat{z} F_t(z) + \int_z \epsilon \Delta\lambda F_t(z)\right)^{\frac{1}{\sigma-1}} - 1}{1} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{(1 + \epsilon\Delta\lambda)^{\frac{1}{\sigma-1}} - 1}{1}$$

Taking the limit as $\epsilon \to 0$ yields the expression in the paper:

$$g = \frac{\Delta\lambda}{\sigma - 1}$$

To understand where the term $g(\sigma-1)\partial N(\hat{z})/\partial\hat{z}$ in the value functions comes from, I work with a simplified discrete-time version of the model without innovation. This can be written as:

$$N(Z_t, \hat{z}_t) = \epsilon\pi(Z_t, \hat{z}_t) + e^{-r\epsilon} N(Z_{t+\epsilon}, \hat{z}_{t+\epsilon})$$

Because π scales multiplicatively in Z and $\hat{z}$, this can be rewritten with a constant term $\tilde{N}$:

$$\tilde{N} Z_t \hat{z}_t = \epsilon\pi Z_t \hat{z}_t + e^{-r\epsilon} \tilde{N} Z_{t+\epsilon} \hat{z}_{t+\epsilon}$$

Dividing both sides by Z_t and using the fact that $Z_{t+\epsilon} = Z_t e^{\epsilon g}$:

$$\tilde{N}\hat{z}_t = \epsilon\pi\hat{z}_t + e^{(g-r)\epsilon} \tilde{N}\hat{z}_{t+\epsilon}$$

Note that a standard Euler condition gives us that $r = \rho + g$, simplifying terms a bit. Subtracting $\tilde{N}\hat{z}_t e^{-\rho\epsilon}$ from both sides and dividing by ϵ we get:

$$\frac{\tilde{N}\hat{z}_t \left(1 - e^{-\rho\epsilon}\right)}{\epsilon} = \pi\hat{z}_t + \frac{e^{-\rho\epsilon}\tilde{N}(\hat{z}_{t+\epsilon} - \hat{z}_t)}{\epsilon}$$

Taking the limit of both sides as $\epsilon \to 0$ we get:

$$\rho\tilde{N}\hat{z}_t = \pi\hat{z}_t + \lim_{\epsilon \to 0} \frac{e^{-\rho\epsilon}\tilde{N}(\hat{z}_{t+\epsilon} - \hat{z}_t)}{\epsilon}$$

To simplify the right hand side, recall $\hat{z}_t = (z_t/Z_t)^{\sigma-1}$, meaning $\hat{z}_{t+\epsilon} = (z_t/(Z_t e^{g\epsilon}))^{\sigma-1} = \hat{z}_t e^{(1-\sigma)g\epsilon}$. This lets us rearrange and get:

$$\rho \tilde{N} = \pi + \lim_{\epsilon \to 0} \frac{e^{-\rho\epsilon}\tilde{N}(e^{(1-\sigma)g\epsilon} - 1)}{\epsilon}$$

This simplifies to:

$$\rho \tilde{N} = \pi - g(\sigma - 1)N$$

And recall that $N = \tilde{N}\hat{z}$, so $N' = \tilde{N}$.