Évaluation de polynôme Paterson-Stockmeyer

Exemple pour un polynôme à 25 coefficients

$$
\begin{aligned}
P(x) = {} & a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 \\
& +a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 \\
& +a_{10}x^{10} + a_{11}x^{11} + a_{12}x^{12} + a_{13}x^{13} + a_{14}x^{14} \\
& +a_{15}x^{15} + a_{16}x^{16} + a_{17}x^{17} + a_{18}x^{18} + a_{19}x^{19} \\
& +a_{20}x^{20} + a_{21}x^{21} + a_{22}x^{22} + a_{23}x^{23} + a_{24}x^{24}
\end{aligned}
$$

$$
\begin{array}{lrcl}
= & (a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4) & & \\
+ & (a_5x^0 + a_6x^1 + a_7x^2 + a_8x^3 + a_9x^4) & \times & x^5 \\
+ & (a_{10}x^0 + a_{11}x^1 + a_{12}x^2 + a_{13}x^3 + a_{14}x^4) & \times & x^{10} \\
+ & (a_{15}x^0 + a_{16}x^1 + a_{17}x^2 + a_{18}x^3 + a_{19}x^4) & \times & x^{15} \\
+ & (a_{20}x^0 + a_{21}x^1 + a_{22}x^2 + a_{23}x^3 + a_{24}x^4) & \times & x^{20}
\end{array}
$$

$$
\begin{aligned}
x^2 &= x \times x \\
x^3 &= x^2 \times x \\
x^4 &= x^3 \times x \\
x^5 &= x^4 \times x
\end{aligned}
$$

$$
\begin{aligned}
x^{10} &= x^5 \times x^5 \\
x^{15} &= x^{10} \times x^5 \\
x^{20} &= x^{15} \times x^{10}
\end{aligned}
$$

Évaluation de polynôme Paterson-Stockmeyer

Évaluation de $P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$

- On le sépare en $m = \sqrt{n}$ blocs $Q_i(x)$ de m coefficients successifs $P(x) = \sum_{i=0}^{m} Q_i(x) \times x^{im}$
- On calcule $x^2, x^3, \ldots, x^m$ ($\approx m$ multiplication)
- Combinaisons linéaires “gratuites” $Q_i(x) = \sum_{j=0}^{m} a_{i+j} \cdot x^j$
- On calcule $x^{2m}, x^{3m}, \ldots, x^{mm}$ ($\approx m$ multiplications)
- Somme des produits $Q_i(x) \times x^{im}$ ($\approx m$ multiplication)

$\Longrightarrow \approx 3\sqrt{n}$ multiplications non-scalaires