

Interpolation polynomiale

Exemple

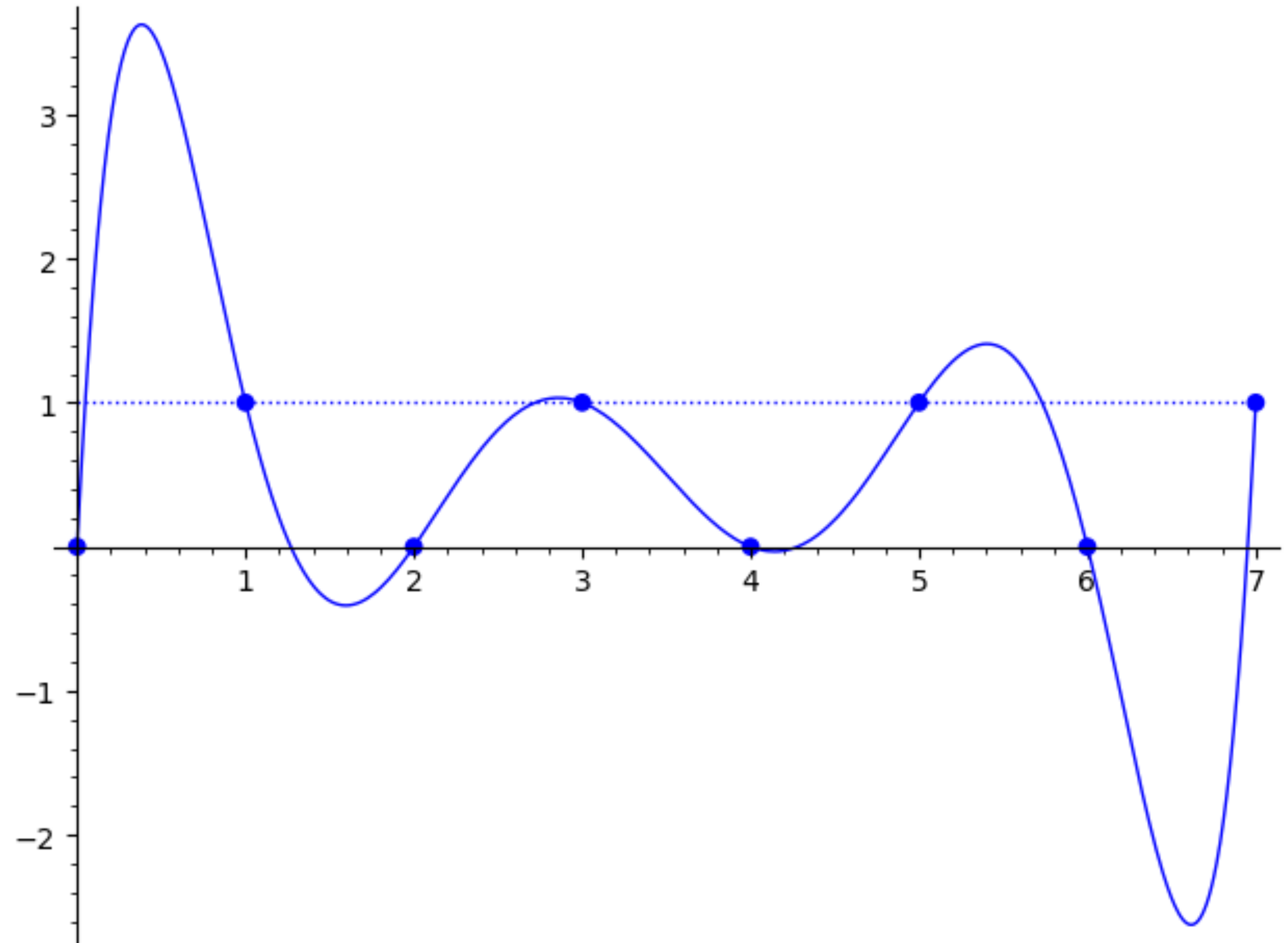
- Étant donné une fonction

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$$

x	0	1	2	3	4	5	6	7
f(x)	0	1	0	1	0	1	0	1

- Trouver un polynôme $P(x)$ équivalent

$$P(x) = \frac{4}{315}x^7 - \frac{14}{45}x^6 + \frac{136}{45}x^5 - \frac{133}{9}x^4 + \frac{1708}{45}x^3 - \frac{2156}{45}x^2 + \frac{2416}{105}x$$



Interpolation polynomiale

Par algèbre linéaire

- Étant donné $n + 1$ points $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ de f tels que $y_i = f(x_i)$
- Trouver $P(x) = a_0 + a_1x + a_2x^2 + \dots a_nx^n$ tel que $P(x_i) = f(x_i)$

$$\begin{cases} a_0 + a_1x_0 + a_2x_0^2 + \dots + a_nx_0^n = y_0 \\ a_0 + a_1x_1 + a_2x_1^2 + \dots + a_nx_1^n = y_1 \\ \vdots \\ a_0 + a_1x_n + a_2x_n^2 + \dots + a_nx_n^n = y_n \end{cases} \quad \overset{\text{Vandermonde}(x_0, \dots, x_n)}{\begin{pmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{pmatrix}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$Va = y \iff a = V^{-1}y$$