Angle Modulation - Basic Concept :-

→ The sinusoidal carrier wave has basically three characteristics, viz. amplitude, frequency and phase.

→ In amplitude modulation, the amplitude of the sinusoidal carrier is slowly varied in accordance with the baseband signal required to be transmitted.

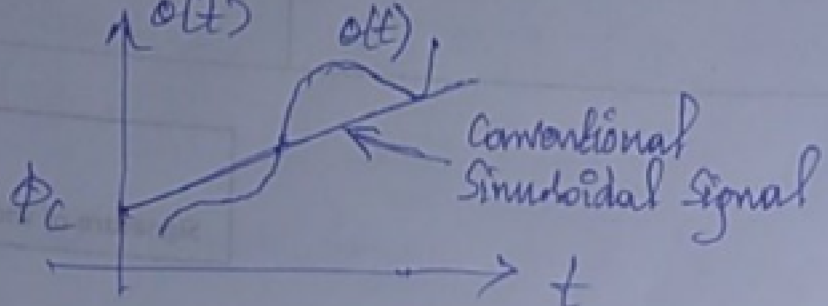
⇒ Instead of amplitude, either frequency (or) phase of the sinusoidal carrier can be changed according to the message signal, keeping the amplitude constant. Since it is termed as method of modulation termed as are Angle Modulation.

Let us denote the carrier signal

$$s(t) = A_c \cos\theta(t)$$

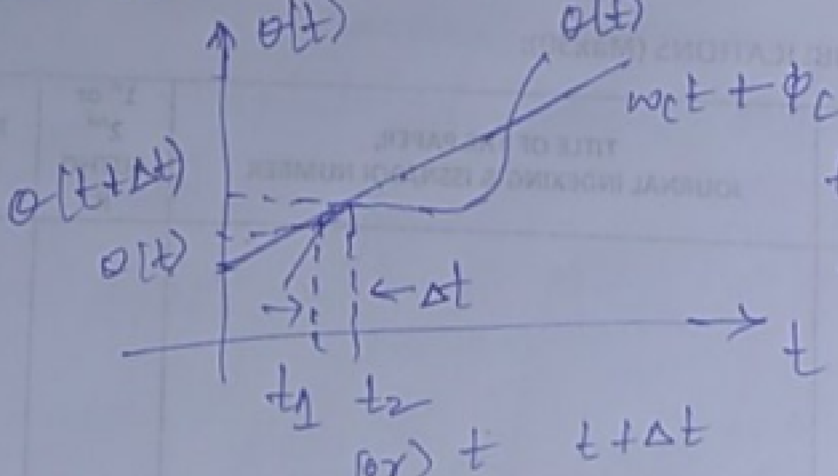
where A_c is the carrier amplitude, which is held constant in angle modulation, while the phase angle is varied by the message signal $m(t)$, which is modulating signal.

Fig:- The generalized angle $\theta(t)$ as a function of time, t.



⇒ The conventional sinusoidal signal represented by $s(t) = A_c \cos(w_c t + \phi_c)$

Fig:- The plot of $\theta(t)$ for a hypothetical case happens to be tangential to the angle $[w_c t + \phi]$ at some instant "t"



$t_1 = t$

$t_2 = t + \Delta t$

⇒ For which the angle is given by $(w_c t + \phi_c)$, where ϕ_c is the value of $\theta(t)$ at $t=0$.

⇒ The angle $[w_c t + \phi_c]$ represents a straight line with a slope w_c and interrupt w_c.

⇒ Because $[w_c t + \phi_c]$ is tangential to $\theta(t)$, the frequency of $s(t)$ is the slope of the angle $\theta(t)$ over the small interval Δt of time.

→ We can generalize this idea and say that if $\theta(t)$ increased linearly with time, the average frequency over an interval from t_1 and t_2 i.e from (t) to $(t+\Delta t)$, is given by

$$f_{\Delta t}(t) = \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t}$$

⇒ The instantaneous frequency of the angle-modulated wave $s(t)$ can now be defined as

$$f_1(t) = \lim_{\Delta t \to 0} \left[f_{\Delta t}(t)\right] = \lim_{\Delta t \to 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t}\right]$$

$$= \frac{1}{2\pi} \lim_{\Delta t \to 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t}\right]$$

$$f_1(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

→ For an unmodulated sinusoidal carrier,
the angle $\theta(t)$ is given by

$$\theta(t) = 2\pi f_c t + \phi_c$$

→ The angular frequency of the carrier w_c,
where $w_c = 2\pi f_c$, and ϕ_c is the value of
$\theta(t)$ at $t = 0$. For convenience, we assume
ϕ_c to be equal to zero.

→ Hence, we can defined frequency modulation
and phase modulation, employed to vary the
angle $\theta(t)$ with the message signal $m(t)$.

1) Phase Modulation (PM)

→ It is defined as the method of angular
modulation in which the angular argument
$\theta(t)$ is varied linearly with the message
signal $m(t)$, as given by

$$\theta(t) = w_c t + k_p m(t)$$

→ Here k_p is the constant term of the
phase sensitivity of the modulator. It is
expressed in radians per volt.

* Here it is assumed that message
signal $m(t)$ is voltage waveform

⇒ The phase modulated wave $s(t)$ can now be written in time-domain as

$$s(t) = A_c \cos[w_c t + k_p m(t)]$$

⇒ The instantaneous angular frequency w_c is given by $w_i(t) = \frac{d\theta}{dt} = \frac{d}{dt}[w_c t + k_p m(t)]$

$$w_i(t) = w_c t + k_p m(t)$$

$$w_i(t) = w_c + k_p \frac{d}{dt} m(t)$$

⇒ Thus in PM (phase modulation) the instantaneous frequency w_i varies linearly with the derivative of the message signal $m(t)$

2) Frequency Modulation :-

⇒ It is defined as that method of angular modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$, is given by

$$f_i(t) = f_c + k_f m(t)$$

⇒ Here, k_f is the constant term of the frequency sensitivity of the modulator. units for k_f is Hz/volt

⇒ Here, f_c is the frequency of the unmodulated carrier, also termed as center frequency. Assuming that $m(t)$ is a voltage waveform.

Since $f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$

$$2\pi f_i(t) = \frac{d\theta(t)}{dt}$$

$$\theta(t) = \int_0^t 2\pi f_i(t)\,dt$$

$$\theta(t) = \int_0^t 2\pi\left[f_c + k_f m(t)\right]dt$$

$$\theta(t) = 2\pi f_c \int_0^t dt + 2\pi \int_0^t k_f m(t)\,dt$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt$$

⇒ In the time domain, the frequency modulate wave can be written as

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$\because \omega_c = 2\pi f_c$$

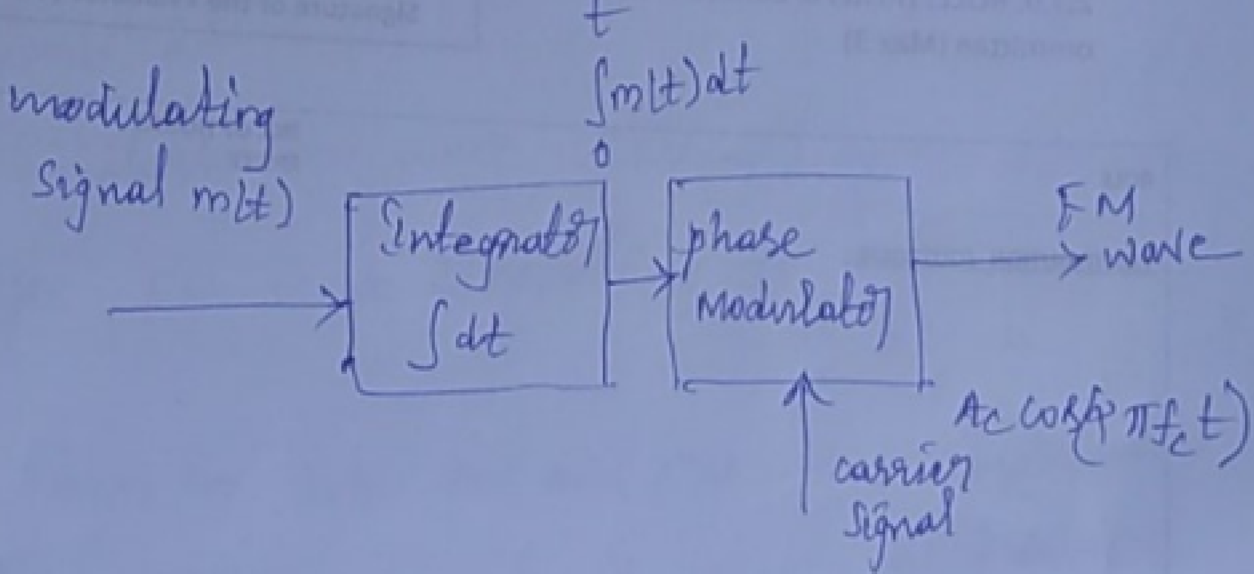
$$s(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

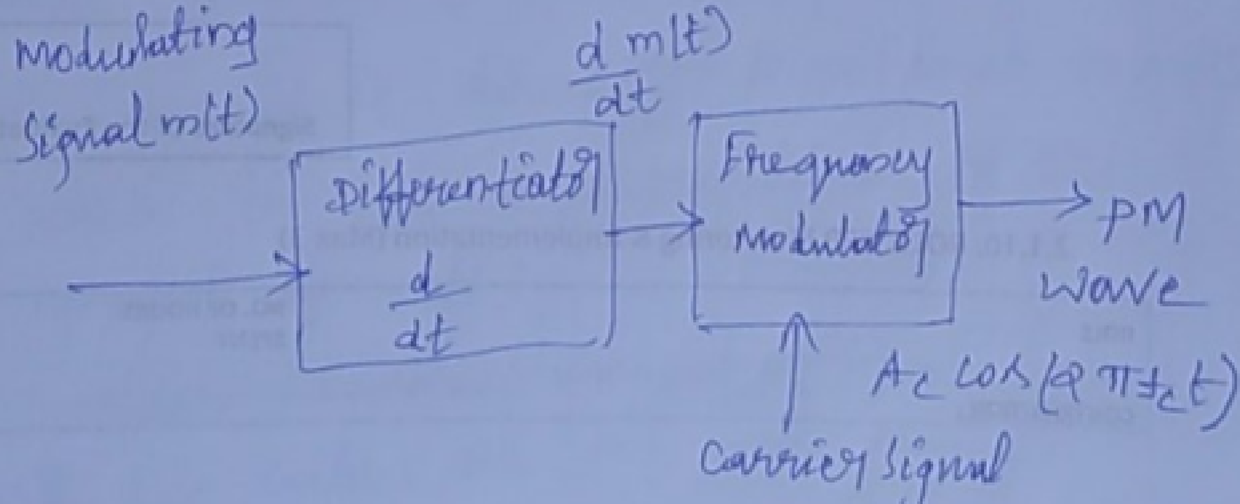
Similarly for PM

$$s(t) = A_c \cos\left[\omega_c t + k_p \frac{d}{dt} m(t)\right]$$

Relation ship Between phase and Frequency Modulation

Fig:- Block diagrams for PM and FM





⇒ The two equations for PM and FM are

$$s(t) = A_c\left[w_c t + 2\pi k_f \int_0^t m(t)\,dt\right) \text{ for FM}$$

$$s(t) = A_c\left[w_c t + k_p \frac{d}{dt} m(t)\right] \text{ for PM}$$

similar. But are insepperable.

→ Replacing $m(t)$ in equation of PM with $\int m(t)$ changes PM to FM.

→ Thus, a signal which is an FM wave corresponding to $m(t)$ is also the PM wave corresponding to $\int m(t)\,dt$.

→ Similarly PM wave corresponding to $m(t)$ is the FM wave corresponding to $\frac{dm(t)}{dt}$.

→ Thus in both PM and FM, the angle of the carrier signal is varied according to some measure of modulating signal $m(t)$.

→ In PM it is directly proportional to $m(t)$, while in FM it is proportional to the integral of $m(t)$.

→ Consider sinusoidal modulating signal $m(t)$ completely with two cycles. The FM wave produced by the integral of $m(t)$. whereas PM wave produced by differentiation of $m(t)$.

fig:- (a) modulating signal m(t) completly with two cycles.

fig:- (b) FM wave produced by m(t)

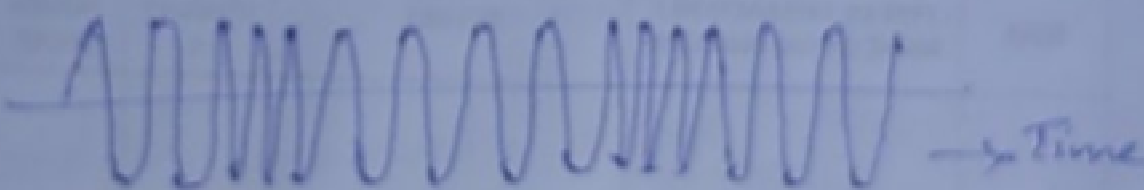


fig:- (c) modulating signal m(t)

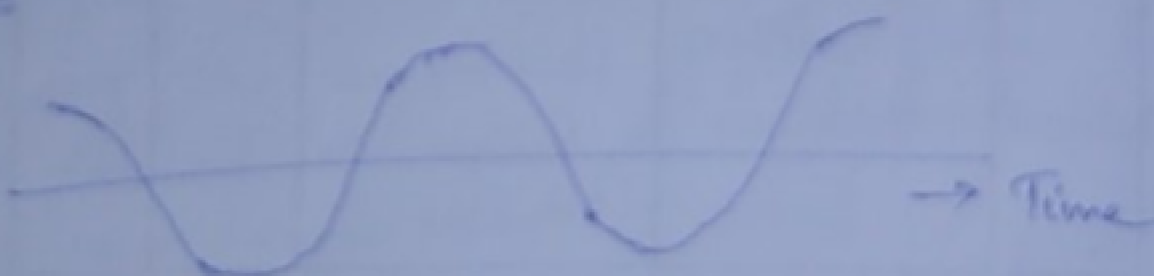


fig:- (d) PM wave produced by m(t)