

# **ANALOG COMMUNICATIONS**

**Prepared**

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# Overview

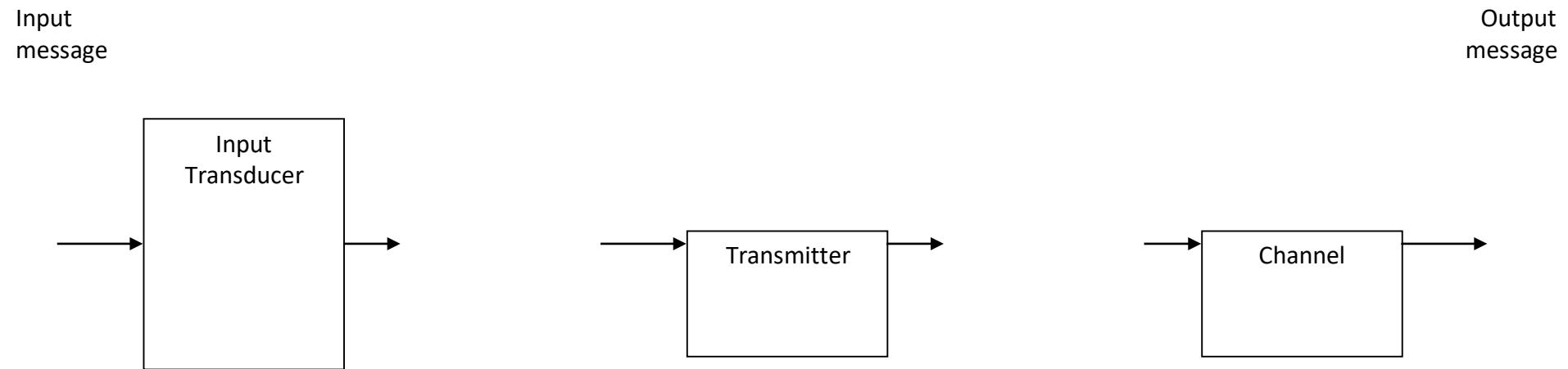
**Communication** is the transfer of information from one place to another.

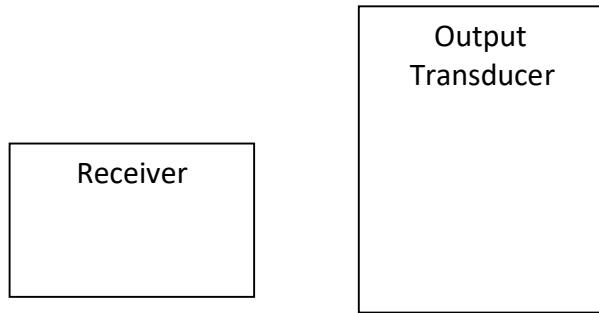
*This should be done*

- as **efficiently** as possible
- with as much **fidelity/reliability** as possible
- as **securely** as possible

**Communication System:** Components/subsystems act together to accomplish information transfer/exchange.

# Elements of a Communication System





**Input Transducer:** The message produced by a source must be converted by a transducer to a form suitable for the particular type of communication system.

*Example: In electrical communications, speech waves are converted by a microphone to voltage variation.*

**Transmitter:** The transmitter processes the input signal to produce a signal suits to the characteristics of the transmission channel.

*Signal processing for transmission almost always involves modulation and may also include coding. In addition to modulation, other functions performed by the transmitter are amplification, filtering and coupling the modulated signal to the channel.*

**Channel:** The channel can have different forms: The atmosphere (or free space), coaxial cable, fiber optic, waveguide, etc.

*The signal undergoes some amount of degradation from noise, interference and distortion*

**Receiver:** The receiver's function is to extract the desired signal from the received signal at the channel output and to convert it to a form suitable for the output transducer.

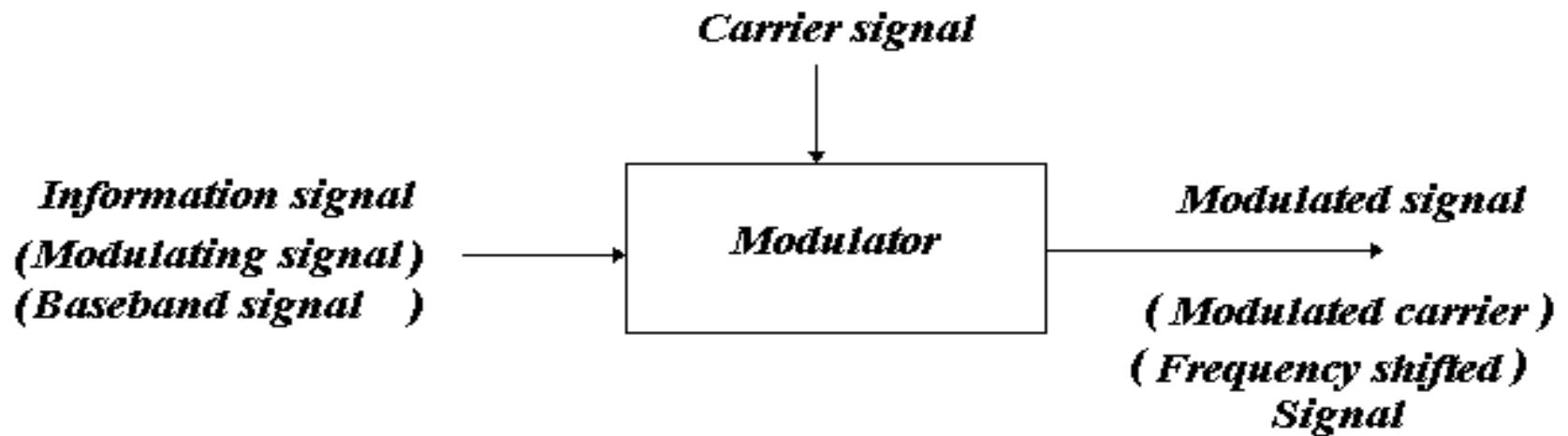
*Other functions performed by the receiver: amplification (the received signal may be extremely weak), demodulation and filtering.*

**Output Transducer:** Converts the electric signal at its input into the form desired by the system user.

*Example: Loudspeaker, personal computer (PC), tape recorders.*

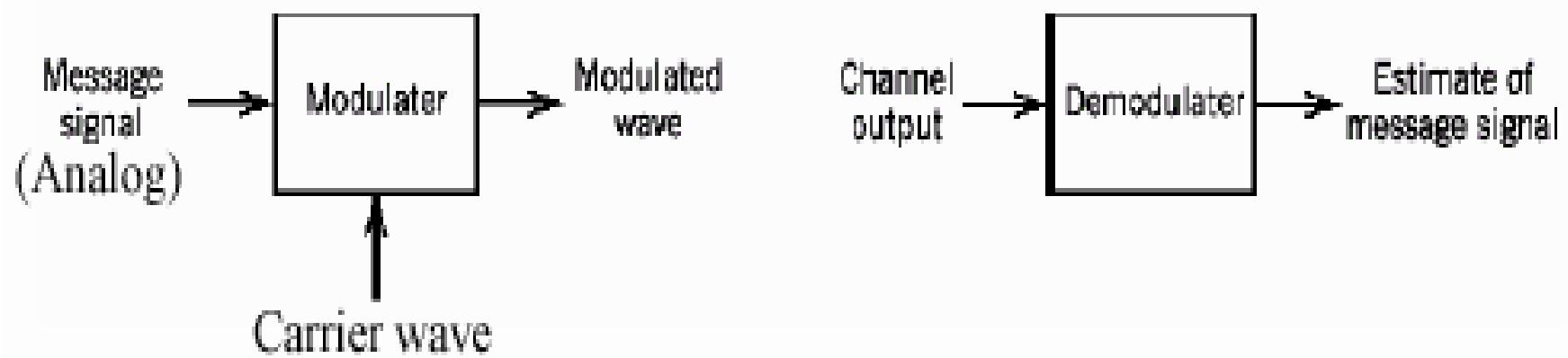
## WHAT IS MODULATION

- MODULATION is the basic requirement for transmitting the message signal through free space
- It is the process of transmission of information signal (low frequency audio signal) using a high frequency carrier signal



*Fig. Process of Modulation*

- Once this information is received, the low frequency information must be removed from the high frequency carrier. This process is known as “***Demodulation***”.



## WHY MODULATION?

- Carrying one signal to another : uses carrier (having high frequency , smaller wavelength)
- Modulated signal is transmitted
- Problems with transmitting baseband signal/  
Need of modulation
  - ❖ Height of transmitting and receiving antenna
  - ❖ Noise and interference from other sources at low frequencies: Multiplexing
  - ❖ Narrow banding

## PRACTICABILITY OF ANTENNAS

$h = \lambda/4$ , for efficient transmission.

For  $f = 30 \text{ Hz} \Rightarrow h = 2500 \text{ km}$

$f = 3 \text{ kHz} \Rightarrow h = 25 \text{ km}$

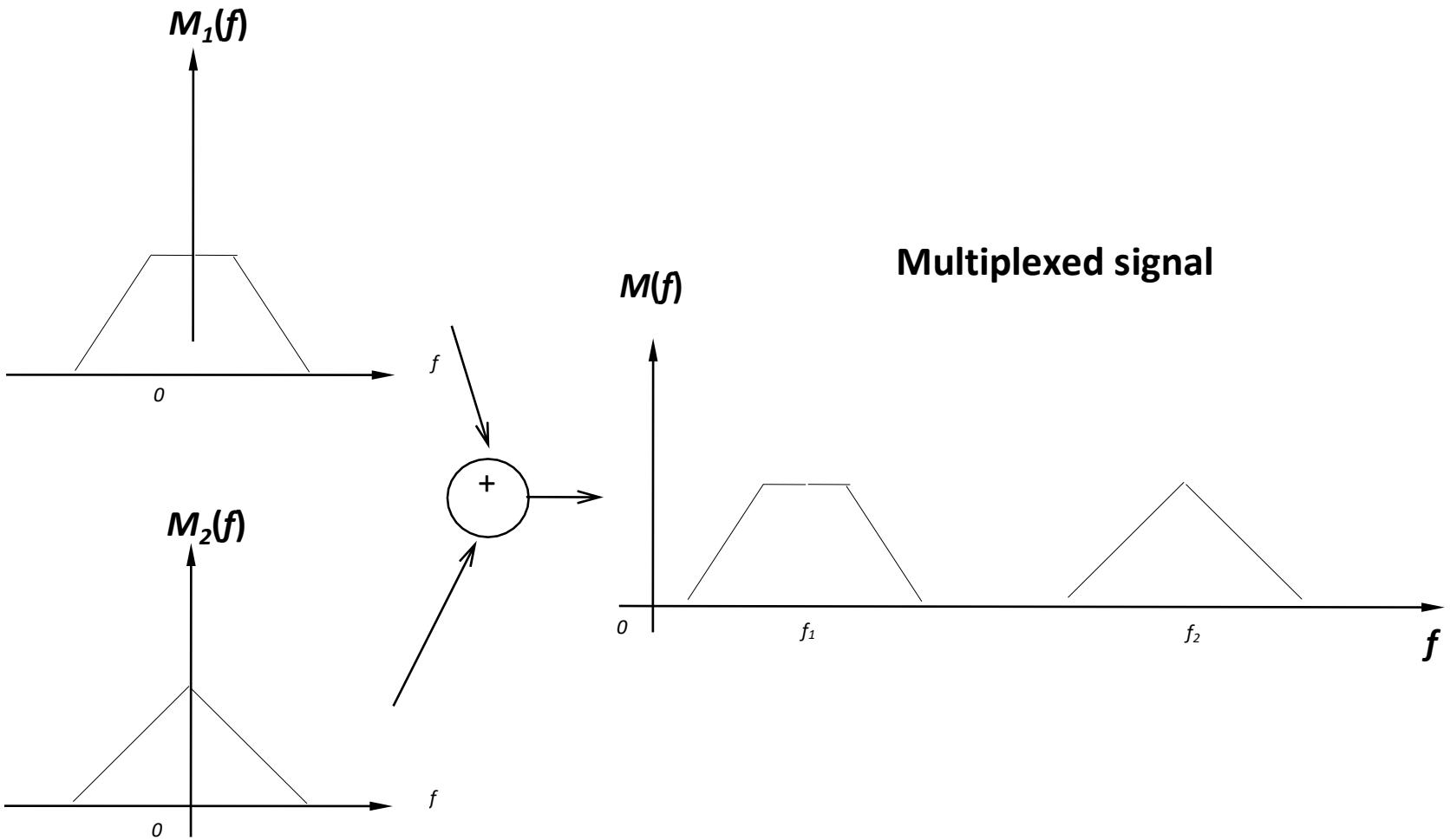
$f = 3 \text{ MHz} \Rightarrow h = 25 \text{ m}$

Thus as

Frequency increases height of the antenna  
decreases

# What are the reasons for modulation?

1. Frequency division multiplexing (To support multiple transmissions via a single channel) *To avoid interference*



## TYPES OF MODULATION

- Sine wave (carrier) described by 3 parameters: amplitude, frequency and phase.
- Let carrier signal be:

$$v(t) = A \sin(\omega t + \varphi)$$

So can have

- ❖ – **Amplitude modulation (AM)**
- ❖ – **Frequency modulation (FM)**
- ❖ – **Phase modulation (PM)**

Frequency and phase combined are known as Angle Modulation

## AMPLITUDE MODULATION

- The amplitude of the carrier is changed in accordance with the instantaneous value of modulating signal
- Carrier :  $c(t) = V_c \cos(2\pi f_c t + \varphi)$   
modulating signal  $v(t) = V_m \cos(2\pi f_m t)$
- Information is contained in the envelop

# What are the different Forms of Amplitude Modulation ?

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1. Conventional Amplitude Modulation (**DSB-LC**)  
*(Alternatively known as Full AM or Double Sideband with Large carrier (DSB-LC) modulation*
2. Double Side Band Suppressed Carrier (**DSB-SC**) modulation
3. Single Sideband (**SSB**) modulation
4. Vestigial Sideband (**VSB**) modulation

## AMPLITUDE MODULATION

➤ *Modulated signal:*

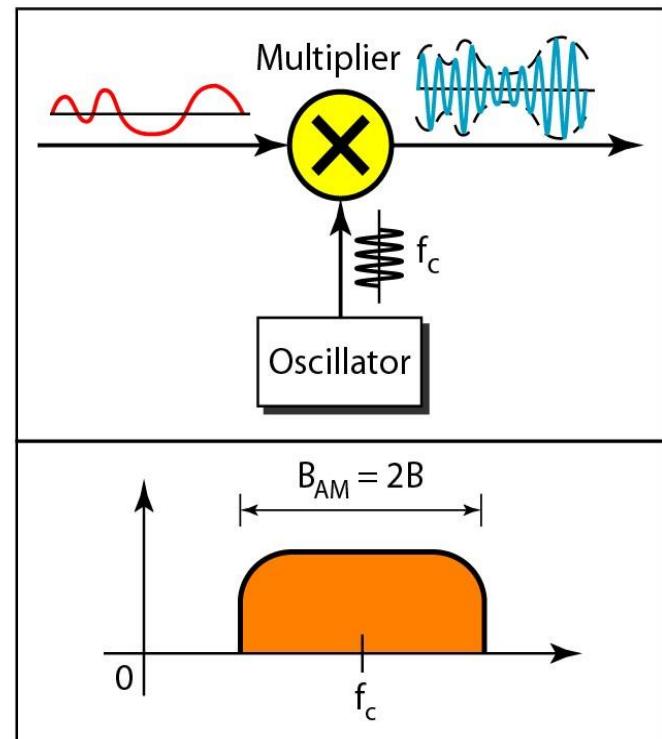
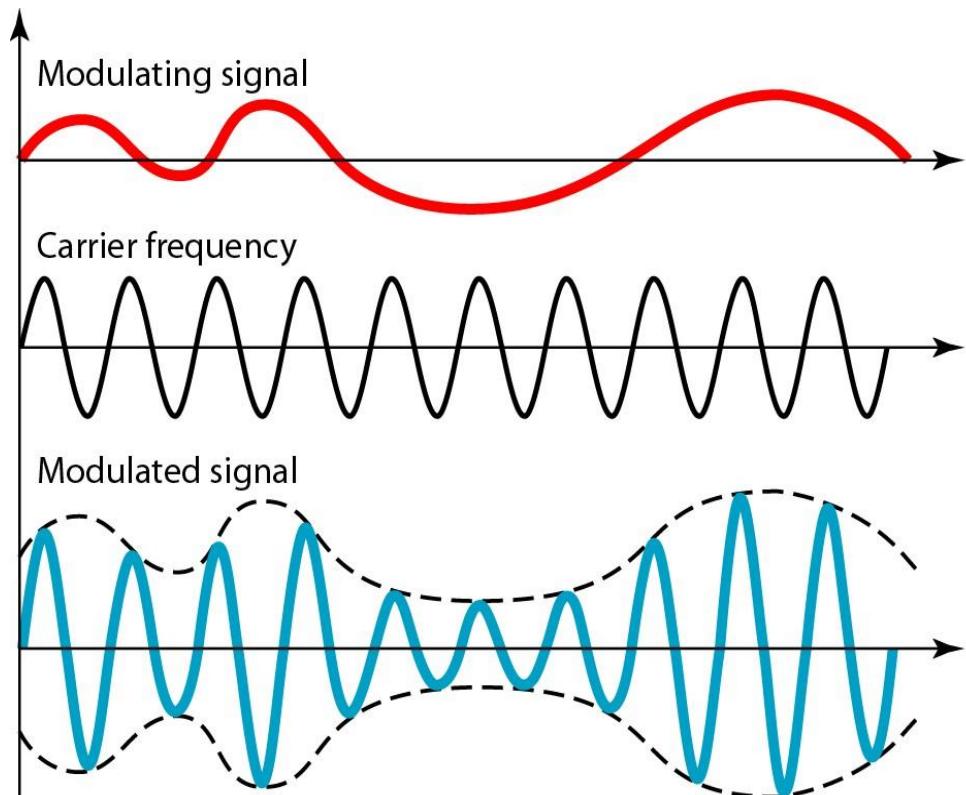
$$v(t) = V_c \cos(2\pi f_c t) \{1 + m \cos(2\pi f_m t)\}$$

- ❖  $V_c$  = unmodulated peak carrier amplitude
- ❖  $f_c$  = carrier frequency
- ❖  $f_m$  = modulation frequency
- ❖  $m$  = modulation index (“degree” of modulation)

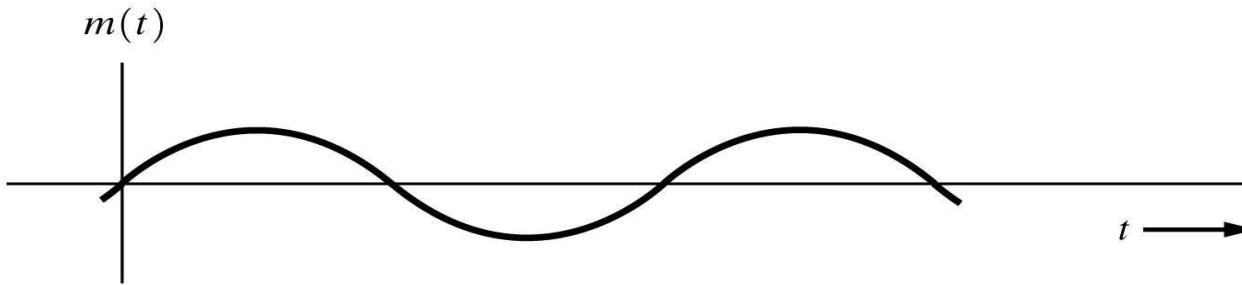
m must be between 0 and 1

If  $m > 1$  get overmodulation (bad ... distortion)

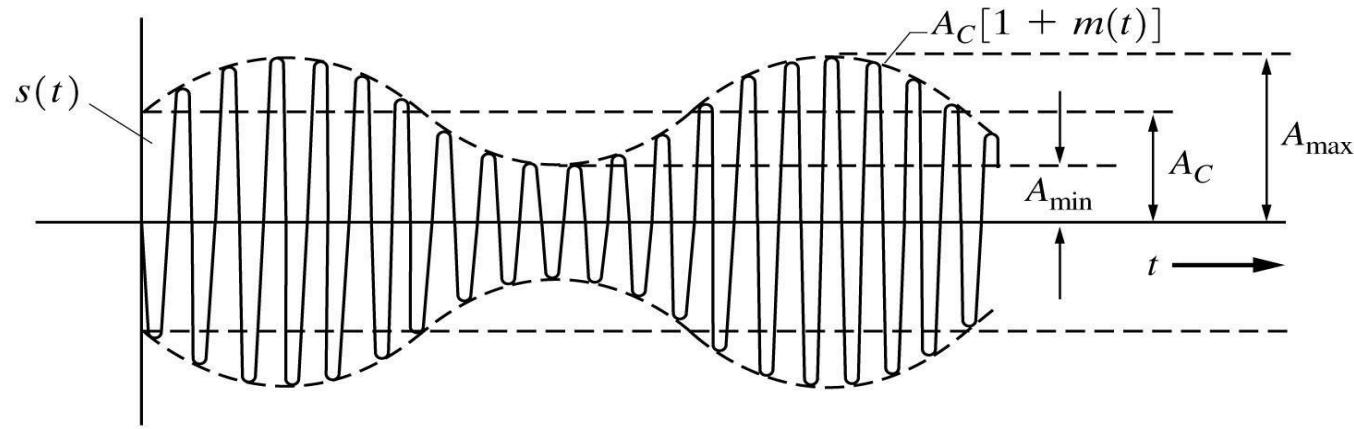
**Figure Amplitude modulation**



# AM Signal Waveform



(a) Sinusoidal Modulating Wave



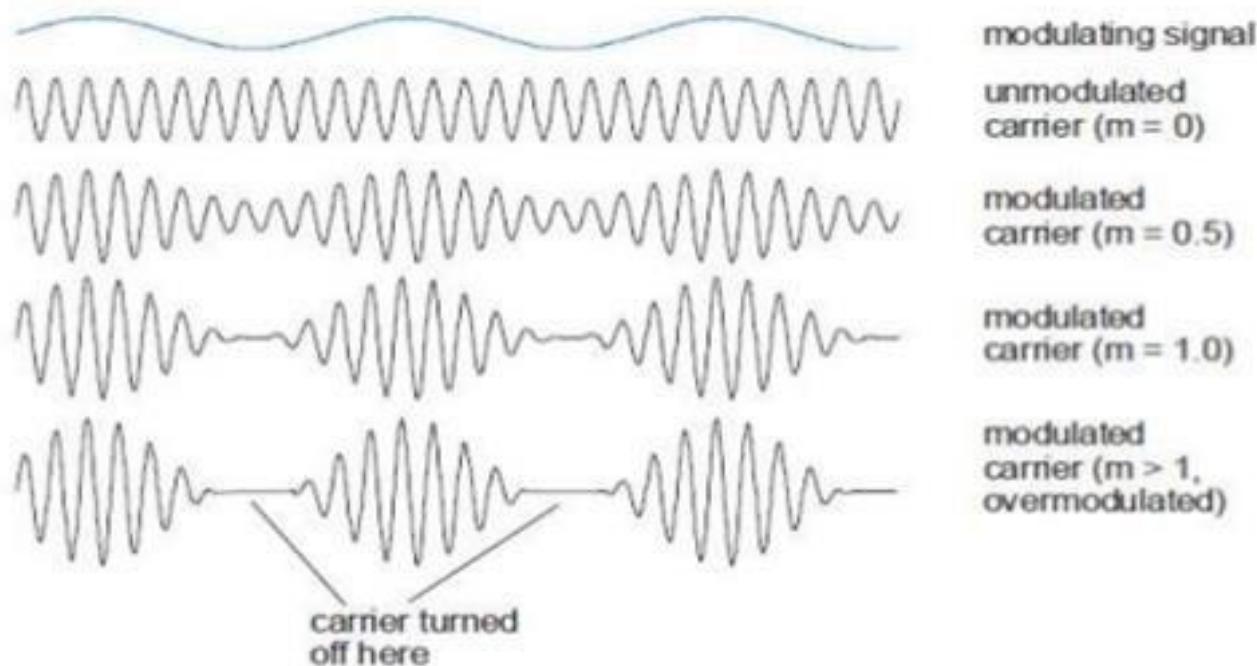
(b) Resulting AM Signal

$$A_{\max} = 1.5A_c$$

$$A_{\min} = 0.5 A_c$$

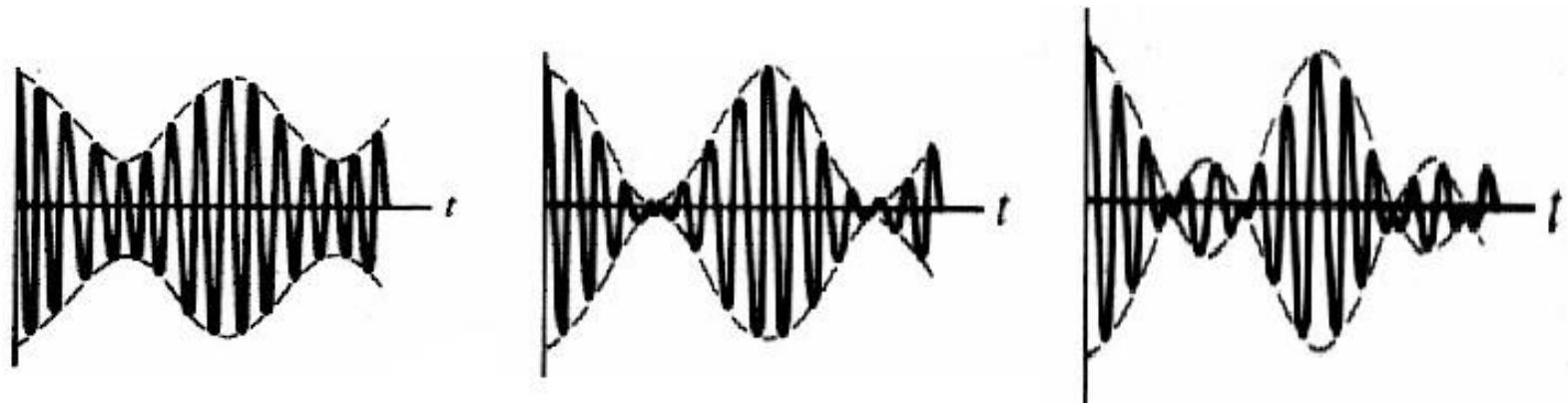
% Positive modulation= 50%  
% Negative modulation =50%  
Overall Modulation = 50%

## VARYING MODULATION INDEX



$$m = V_{\max} - V_{\min} / V_{\max} + V_{\min}$$

# AM – Percentage Modulation



**Under modulated (<100%)**

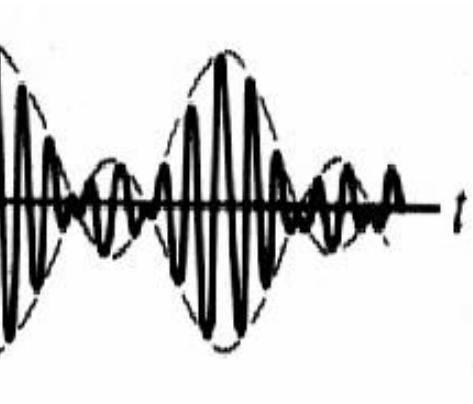


Envelope Detector

Can be used



**100% modulated**



**Over Modulated (>100%)**



Envelope Detector

Gives Distorted signal

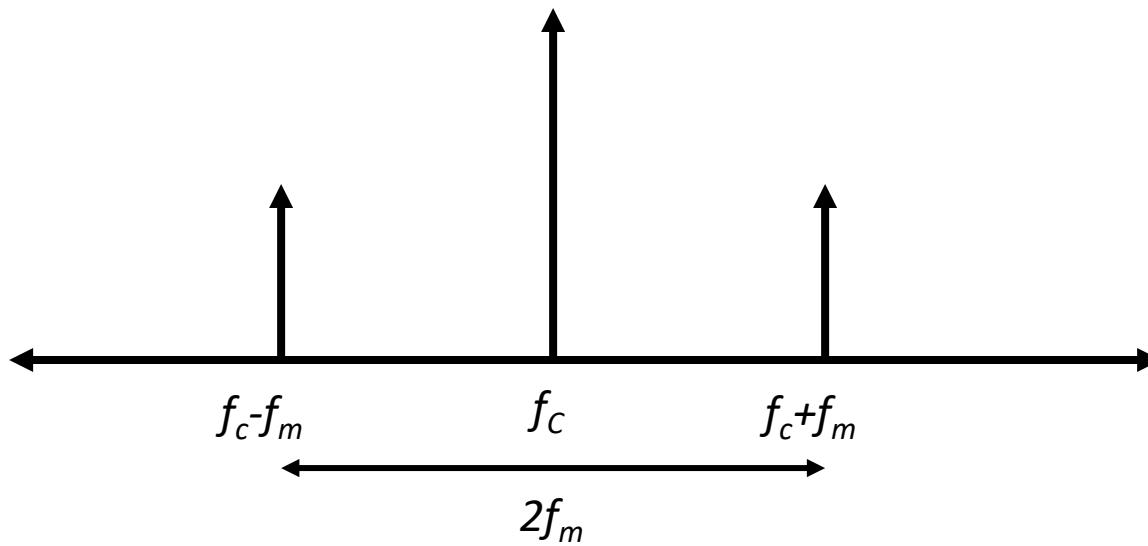
Therefore The full AM signal may be written as

$$s(t) = A_c(1 + m \cos(\omega_m t)) \cos(\omega_c t)$$

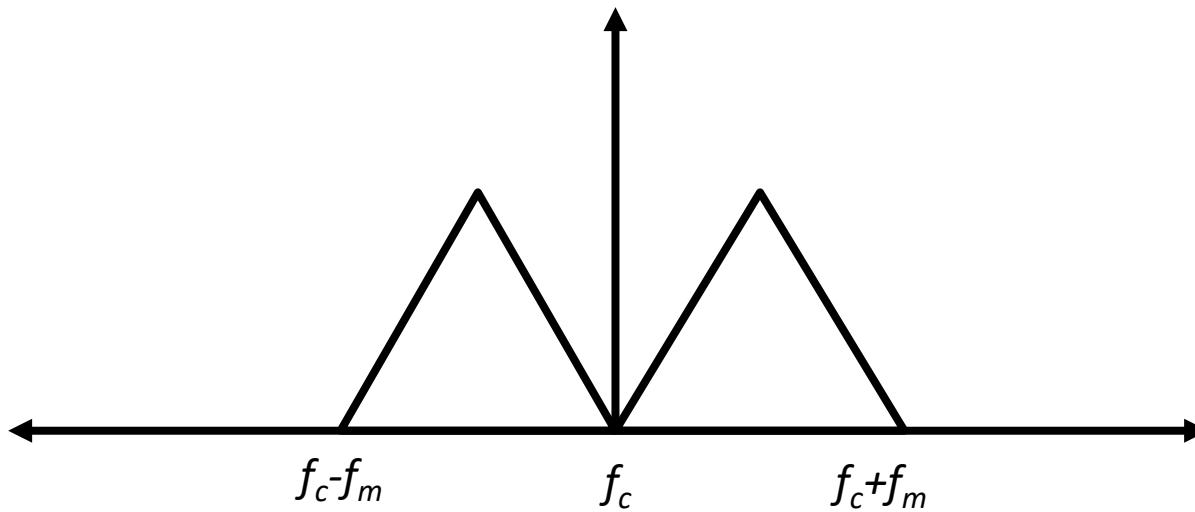
$$\cos A \cos B = 1/2[\cos(A+B) + \cos(A-B)]$$

$$s(t) = A_c(\cos \omega_c t) + \frac{mA_c}{2} \cos(\omega_c + \omega_m)t + \frac{mA_c}{2} \cos(\omega_c - \omega_m)t$$

**Draw the Frequency Spectrum of the above AM signal  
and calculate the Bandwidth**



## 8. Draw Frequency Spectrum for a complex input signal with AM



# Frequency Spectrum of an AM signal

*The frequency spectrum of AM waveform contains **three parts**:*

1. A component at the carrier frequency  $f_c$
2. An upper side band (**USB**), whose highest frequency component is at  $f_c + f_m$
3. A lower side band (**LSB**), whose highest frequency component is at  $f_c - f_m$

*The bandwidth of the modulated waveform is twice the information signal bandwidth.*

- Because of the two side bands in the frequency spectrum its often called Double Sideband with Large Carrier.(DSB-LC)
- The information in the base band (information) signal is duplicated in the **LSB** and **USB** and the **carrier** conveys **no** information.

## Modulation Index ( $m$ )

- $m$  is merely defined as a parameter, which determines the amount of modulation.
- What is the degree of modulation required to establish a desirable AM communication link?

**Answer is to maintain  $m < 1.0$  ( $m < 100\%$ ).**

- This is important for successful retrieval of the original transmitted information at the receiver end.

# AM – Normalized Average Power

The **normalized average power** of the AM signal is

$$\begin{aligned}\langle s^2(t) \rangle &= \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{1}{2} A_c^2 \langle [1 + m(t)]^2 \rangle \\ &= \frac{1}{2} A_c^2 [1 + 2m(t) + m^2(t)] \\ &= \frac{1}{2} A_c^2 + A_c^2 \langle m(t) \rangle + \frac{1}{2} A_c^2 \langle m^2(t) \rangle\end{aligned}$$

If the modulation contains no dc level, then  $\langle m(t) \rangle = 0$

The **normalized power** of the AM signal is

$$\boxed{\langle s^2(t) \rangle = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \langle m^2(t) \rangle}$$

Discrete Carrier Power      Sideband power

# AM - Modulation Efficiency

➤ **Definition :** The **Modulation Efficiency** is the percentage of the total power of the modulated signal that conveys information.

Only “**Sideband Components**” – Convey information

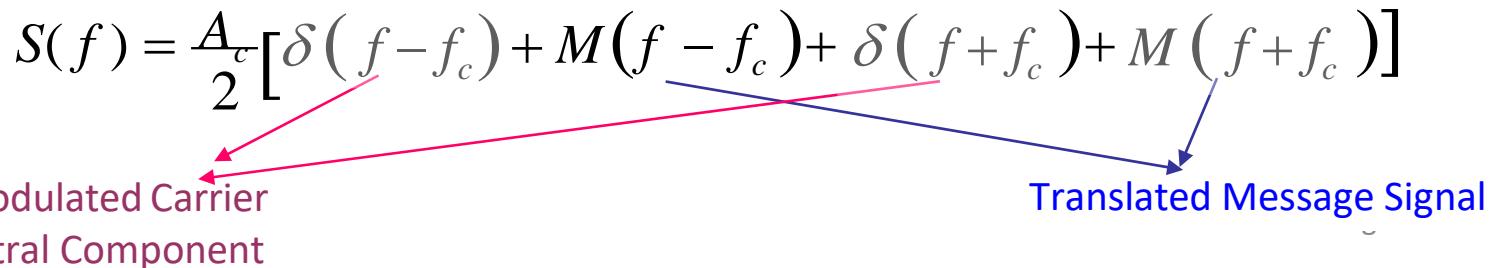
**Modulation Efficiency:** 
$$E = \frac{\langle m^2(t) \rangle}{1 + \langle m^2(t) \rangle} \times 100$$

Highest efficiency for a 100% AM signal : **50%** - square wave modulation

**Normalized Peak Envelope Power (PEP)** of the AM signal:

$$P_{PEP} = \frac{A_c^2}{2} \left\{ 1 + \max_m [m(t)] \right\}^2$$

**Voltage Spectrum** of the AM signal:



# Power of an AM signal

Suppose that a **5000-W** AM transmitter is connected to a **50 ohm** load;

Then the constant  $A_c$  is given by  $\frac{1}{2} \frac{A^2}{50} = 5,000 \Rightarrow A_c = 707 \text{ V}$

Without  
Modulation

If the transmitter is then **100% modulated** by a **1000-Hz** test tone ,  
the **total** (carrier + sideband) **average power** will be

$$1.5 \left[ \frac{1}{2} \left| \frac{A^2}{50} \right| \right] = (1.5) \times (5000) = 7,500 \text{ W}$$

$$\left[ \langle m^2(t) \rangle = \frac{1}{2} \quad \text{for 100\% modulation} \right]$$

The **peak voltage** (100% modulation) is  $(2)(707) = 1414 \text{ V}$  across the 50 ohm load.

The **peak envelope power** (PEP) is

$$4 \left[ \frac{1}{2} \left| \frac{A^2}{50} \right| \right] = (4) \times (5000) = 20,000 \text{ W}$$

The **modulation efficiency** would be 33% since  $\langle m^2(t) \rangle = 1/2$

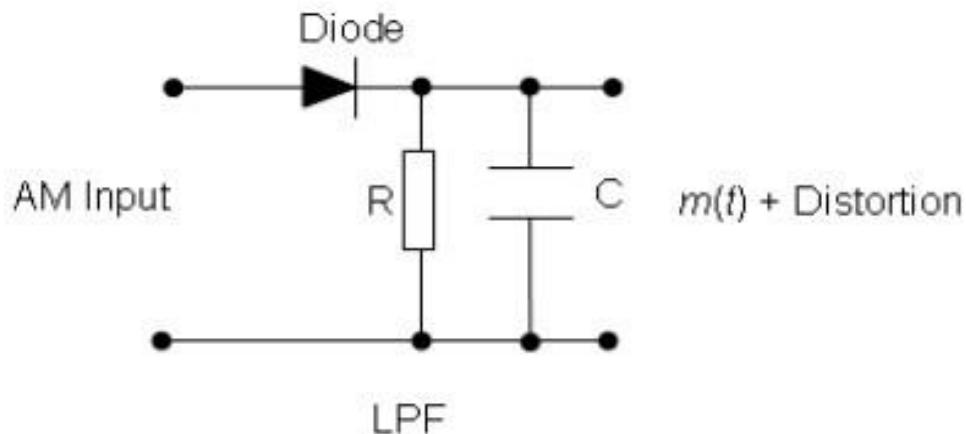
# **Demodulation of Amplitude Modulated Signals**

**There are 2 main methods of AM Demodulation:**

- Envelope or non-coherent Detection/Demodulation.**
- Synchronised or coherent Demodulation.**

# Envelope or Non-Coherent Detection

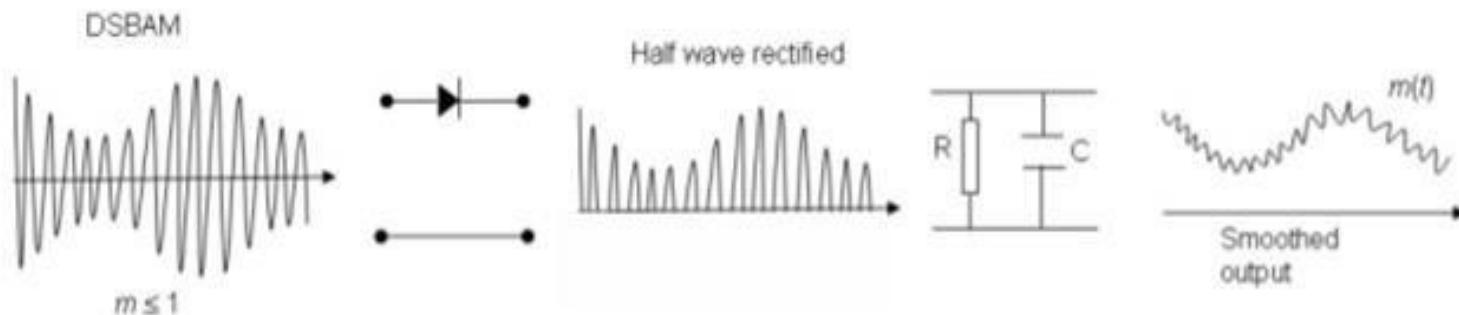
An envelope detector for AM is shown below:



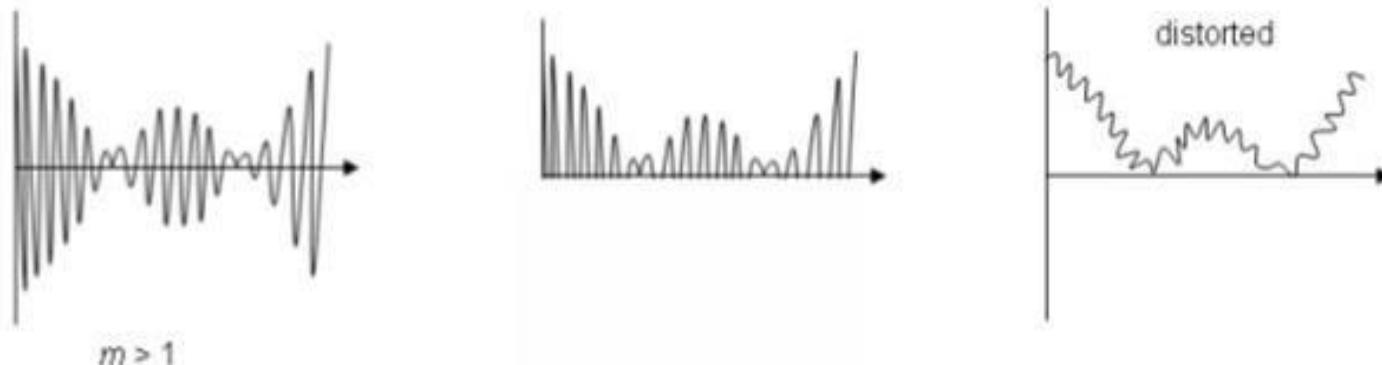
This is obviously simple, low cost. But the AM input must be DSBAM with  $m \ll 1$ , i.e. it does not demodulate DSBDimC, DSBSC or SSBxx.

# Large Signal Operation

For large signal inputs, ( $\approx$  Volts) the diode is switched i.e. forward biased  $\equiv$  ON, reverse biased  $\equiv$  OFF, and acts as a half wave rectifier. The 'RC' combination acts as a 'smoothing circuit' and the output is  $m(t)$  plus 'distortion'.

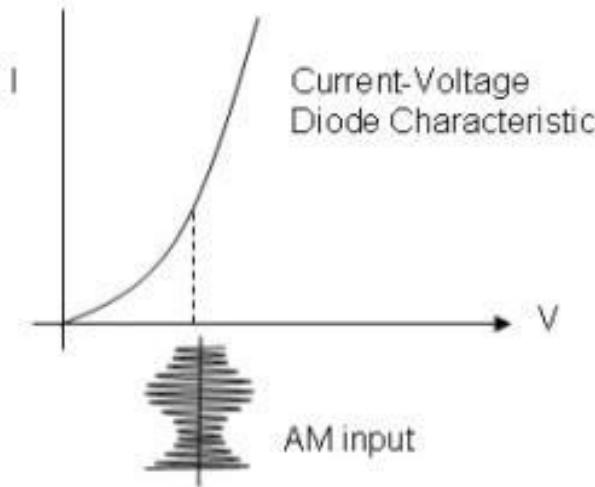
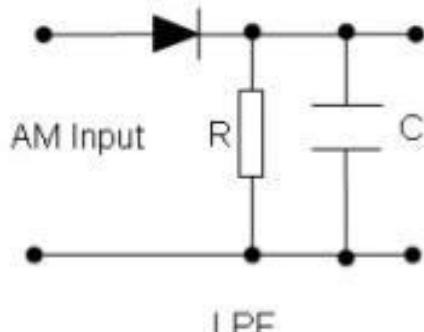


If the modulation depth is  $> 1$ , the distortion below occurs



## Small Signal Operation – Square Law Detector

For small AM signals (~ millivolts) demodulation depends on the diode square law characteristic.



The diode characteristic is of the form  $i(t) = av + bv^2 + cv^3 + \dots$ , where

$$v = (V_{DC} + m(t))\cos(\omega_c t) \quad \text{i.e. DSBAM signal.}$$

## **Small Signal Operation – Square Law Detector**

$$\text{i.e. } a(V_{DC} + m(t))\cos(\omega_c t) + b((V_{DC} + m(t))\cos(\omega_c t))^2 + \dots$$

$$= aV_{DC} + am(t)\cos(\omega_c t) + b[V_{DC}^2 + 2V_{DC}m(t) + m(t)^2]\cos^2(\omega_c t) + \dots$$

$$= aV_{DC} + am(t)\cos(\omega_c t) + \left(bV_{DC}^2 + 2bV_{DC}m(t) + bm(t)^2\right) \left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)$$

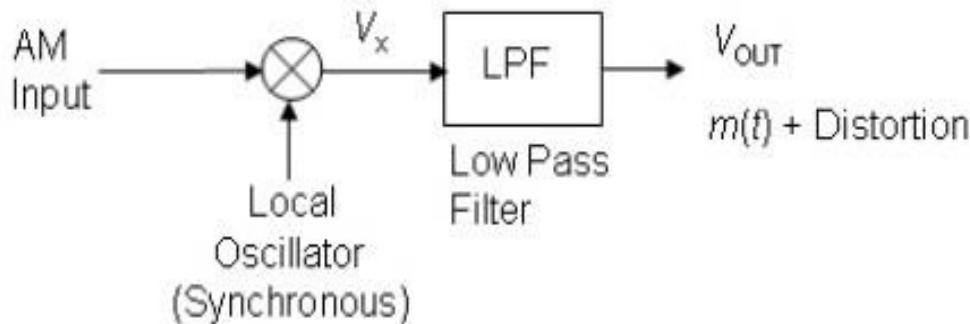
$$= aV_{DC} + am(t)\cos(\omega_c t) + \frac{bV_{DC}^2}{2} + \frac{2bV_{DC}m(t)}{2} + \frac{bm(t)^2}{2} + b\frac{V_{DC}^2}{2}\cos(2\omega_c t) + \dots$$

'LPF' removes components.

$$\text{Signal out} = aV_{DC} + \frac{bV_{DC}^2}{2} + bV_{DC}m(t) \text{ i.e. the output contains } m(t)$$

## Synchronous or Coherent Demodulation

A synchronous demodulator is shown below



This is relatively more complex and more expensive. The Local Oscillator (LO) must be synchronised or coherent, *i.e.* at the same frequency and in phase with the carrier in the AM input signal. This additional requirement adds to the complexity and the cost.

However, the AM input may be any form of AM, *i.e.* DSBAM, DSBDimC, DSBSC or SSBAM, SSBDimC, SSBSC. (Note – this is a 'universal' AM demodulator and the process is similar to correlation – the LPF is similar to an integrator).

# Double Side Band Suppressed Carrier (DSB-SC) Modulation

- The carrier component in full AM or DSB-LC does not convey any information. Hence it may be removed or suppressed during the modulation process to attain higher power efficiency.
- The trade off of achieving a higher power efficiency using DSB-SC is at the expense of requiring a complex and expensive receiver due to the absence of carrier in order to maintain transmitter/receiver synchronization.

# Derive the Frequency Spectrum for Double Sideband Suppressed Carrier Modulation (DSB-SC)

1 Consider the carrier

$$s_c(t) = A_c \cos(\omega_c t) \quad \text{where } \omega_c = 2\pi f_c$$

2 modulated by a single sinusoidal signal

$$s_m(t) = A_m \cos \omega_m t \quad \text{where } \omega_m = 2\pi f_m$$

3 The modulated signal is simply the product of these two

$$s(t) = A_c \cos(\omega_c t) A_m \cos(\omega_m t)$$

$$= A_c A_m \cos(\omega_c t) \cos(\omega_m t)$$

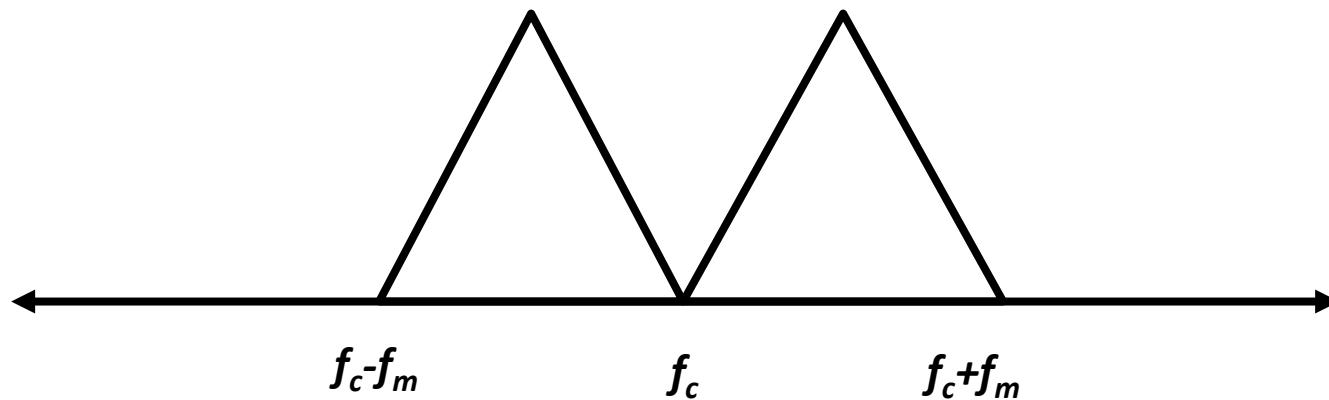
$$\text{since } \cos A \cos B = \frac{1}{2} (\cos(A + B) + \cos(A - B))$$

$$= \underbrace{\frac{A_c A_m}{2} \cos(\omega_c + \omega_m)t}_{USB} + \underbrace{\frac{A_c A_m}{2} \cos(\omega_c - \omega_m)t}_{LSB}$$

$$s_c(t) = A_c \cos \omega_c t$$

$$s_m(t) = A_m \cos \omega_m t \quad s(t) = A_c \cos(\omega_c t) A_m \cos(\omega_m t)$$

## Frequency Spectrum of a DSB-SC AM Signal

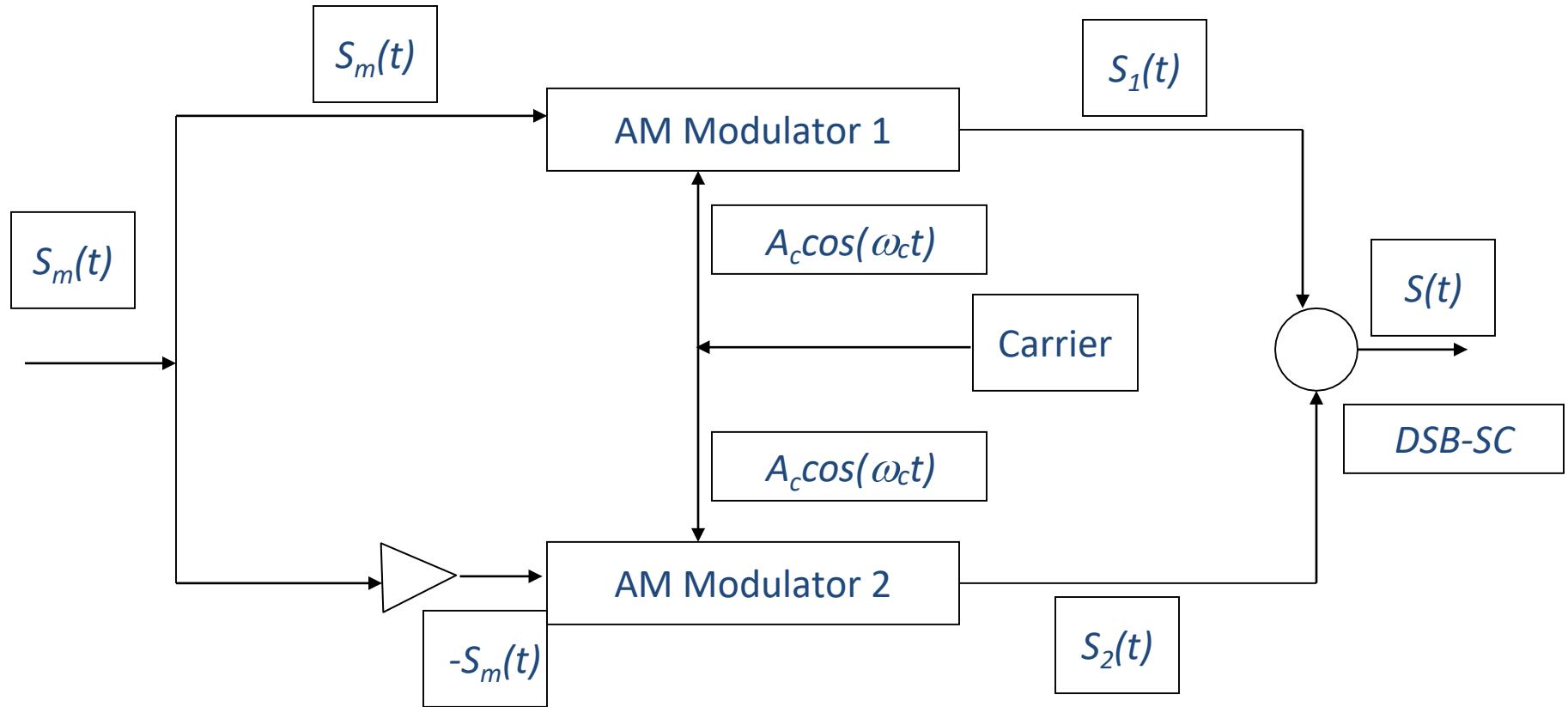


- All the transmitted power is contained in the two sidebands (no carrier present).
- The bandwidth is twice the modulating signal bandwidth.
- USB displays the positive components of  $s_m(t)$  and LSB displays the negative components of  $s_m(t)$ .

## Generation and Detection of DSB-SC

- The simplest method of generating a DSB-SC signal is merely to filter out the carrier portion of a full AM (or DSB-LC) waveform.
- Given carrier reference, modulation and demodulation (detection) can be implemented using product devices or balanced modulators.

# **BALANCED MODULATOR**



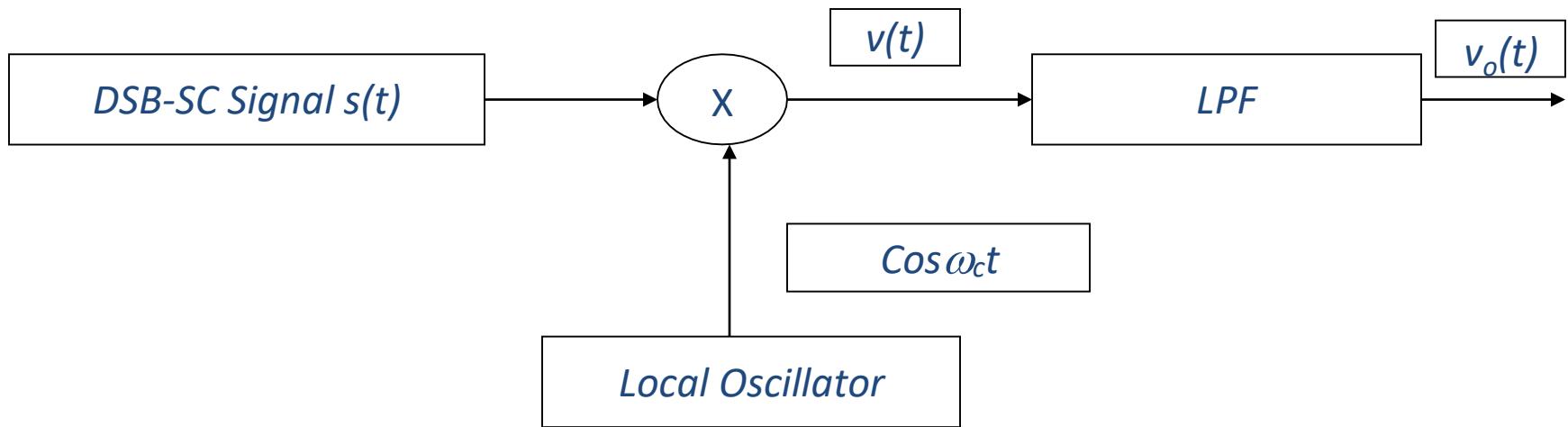
- The two modulators are identical except for the sign reversal of the input to one of them. Thus,

$$s_1(t) = A_c(1 + m \cos(\omega_m t)) \cos(\omega_c t)$$

$$s_2(t) = A_c(1 - m \cos(\omega_m t)) \cos(\omega_c t)$$

$$\begin{aligned} s(t) &= s_1(t) - s_2(t) \\ &= 2m A_c \cos(\omega_m t) \cos(\omega_c t) \end{aligned}$$

## COHERENT (SYNCHRONOUS) DETECTOR OR DSB-SC (PRODUCT DETECTOR)



- Since the carrier is suppressed the envelope no longer represents the modulating signal and hence envelope detector which is of the non-coherent type cannot be used.

$$\begin{aligned}
v(t) &= s(t) \cos(\omega_c t) = [2mA_c \cos(\omega_m t) \cos(\omega_c t)] \cos(\omega_c t) \\
&= 2 \frac{A_m}{A_c} A_c \cos(\omega_m t) \cos^2(\omega_c t) \\
&= 2A_m \cos(\omega_m t) \left( 1 + \cos 2\omega_c t \right) \\
&\quad m \qquad \qquad \qquad m t) \left| \begin{array}{c} \overline{\phantom{\omega_m t)}} \\ 2 \end{array} \right. \\
&= A_m \cos(\omega_m t) + A_m \cos(\omega_m t) \cos(2\omega_c t)
\end{aligned}$$

since  $s_m(t) = A_m \cos(\omega_m t)$

$$\begin{aligned}
&= s_m(t) + \underbrace{s_m(t) \cos(2\omega_c t)}_{\text{Unwanted term (removed by LPF)}}
\end{aligned}$$

- It is necessary to have synchronization in both frequency and phase between the transmitter (modulator) & receiver (demodulator), when DSB-SC modulation ,which is of the coherent type, is used. Both phase and frequency must be known to demodulate DSB-SC waveforms.

## **LACK OF PHASE SYNCHRONISATION**

Let the received DSB-SC signal be

$$s_{DSB-SC}(t) = s_m(t) \cos(\omega_c t + \theta) A_c$$

if  $\theta$  is unknown,

$$\begin{aligned} v(t) &= s_{DSB-SC}(t) \cos \omega_c t \\ &= A_c s_m(t) \cos(\omega_c t + \theta) \cos \omega_c t \\ &= \frac{A_c}{2} s_m(t) [\cos \theta + \cos(2\omega_c t + \theta)] \end{aligned}$$

Output of LPF

$$v_o(t) = \frac{A_c}{2} s_m(t) \cos \theta$$

But we want just

$$v_o(t) = \frac{A_e}{2} s_m(t)$$

Due to lack of phase synchronization, we will see that the wanted signal at the output of LPF will be attenuated by an amount of  $\cos\theta$ .

In other words, phase error causes an attenuation of the output signal proportional to the cosine of the phase error.

The worst scenario is when  $\theta=\pi/2$ , which will give rise to zero or no output at the output of the LPF.

## LACK OF FREQUENCY SYNCHRONISATION

Suppose that the local oscillator is not stable at  $f_c$  but at

$f_c + \Delta f$ , then

$$\begin{aligned} v(t) &= s_{DSB-SC}(t) \cos(\omega_c + \Delta\omega)t \\ &= A_c s_m(t) \cos \omega_c t \cos(\omega_c + \Delta\omega)t \\ &= \frac{A_c}{2} s_m(t) [\cos \Delta\omega t + \cos(2\omega_c t + \Delta\omega)] \end{aligned}$$

Output of LPF

$$v_o(t) = \frac{A_c}{2} s_m(t) \cos \Delta\omega t$$

Thus, the recovered baseband information signal will vary sinusoidal according to  $\cos \Delta\omega t$

This problem can be overcome by adding an extra synchronization circuitry which is required to detect  $\theta$  and  $\Delta \omega t$  and by providing the carrier signal to the receiver.

A synchronizer is introduced to curb the synchronization problem exhibited in a coherent system.

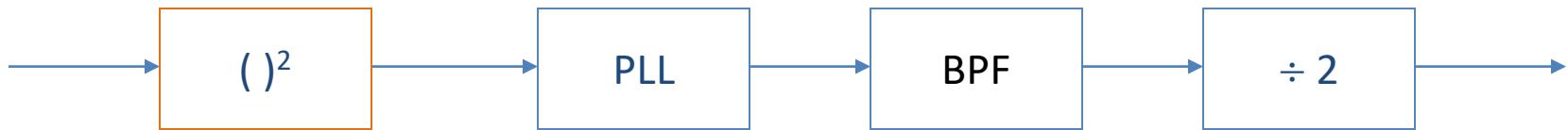
Let the baseband signal be

$$s_m(t) = A_m \cos \omega_m t$$

Received DSB-SC signal

$$s(t) = A_c s_m(t) \cos \omega_c t$$

# SYNCHRONISER



Mathematical analysis of the synchronizer is shown below:

$$\begin{aligned}
 s^2(t) &= A_c^2 A_m^2 \cos^2 \omega_m t \cos^2 \omega_c t \\
 &= \frac{A_c^2 A_m^2}{4} [1 + \cos 2\omega_m t][1 + \cos 2\omega_c t] \\
 &= \frac{A_c^2 A_m^2}{4} \left[ 1 + \cos 2\omega_m t + \cos 2\omega_c t + \cos 2\omega_m t \cos 2\omega_c t \right] \\
 &= \frac{A_c^2 A_m^2}{4} \left[ 1 + \cos 2\omega_m t + \cos 2\omega_c t + \frac{1}{2} (\omega_c - \omega_m) + \frac{1}{2} (\omega_c + \omega_m) \right]
 \end{aligned}$$

Output of BPF       $\frac{A_c^2 A_m^2}{4} \cos 2\omega_c t$

4

Output of frequency divider

$$k \cos \omega_c t$$

where  $k$  is a constant of proportionality.

## **DISADVANTAGE OF USING COHERENT SYSTEMS**

- The frequency and phase of the local oscillator signal must be very precise which is very difficult to achieve.

It requires additional circuitry such as synchronizer circuit and hence the cost is higher.

# Single Side Band Modulation (SSB)

How to generate SSB signal?

- Generate DSB-SC signal
- Band-pass filter to pass only one of the sideband and suppress the other.

For the generation of an SSB modulated signal to be possible, the message spectrum must have an *energy gap* centered at the origin.

# Double Side Band Suppressed Carrier (DSBSC)

- Power in a AM signal is given by

$$\langle s^2(t) \rangle = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \langle m^2(t) \rangle$$

Carrier Power      Sideband power

➤ DSBSC is obtained by eliminating carrier component

If  $m(t)$  is assumed to have a zero DC level, then

$$s(t) = A_c m(t) \cos \omega_c t$$

Spectrum →  $S(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$

Power →  $\langle s^2(t) \rangle = \frac{1}{2} A_c^2 \langle m^2(t) \rangle$

Modulation Efficiency →  $E = \frac{\langle m^2(t) \rangle}{\langle m^2(t) \rangle} \times 100 = 100\%$

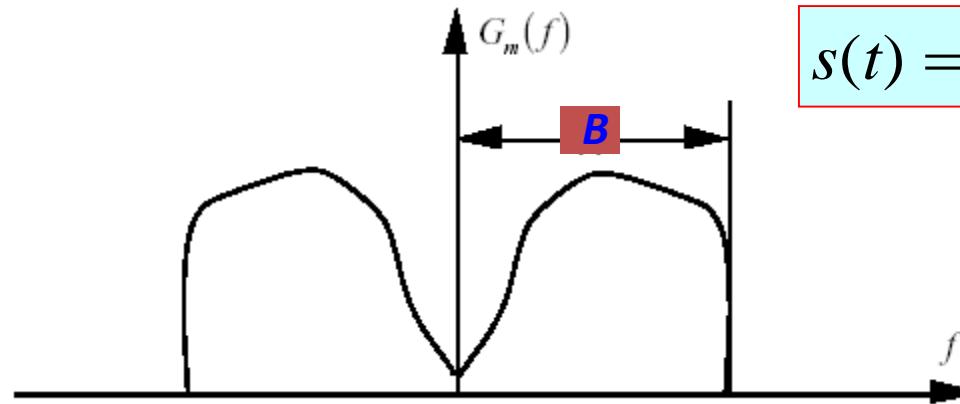
Disadvantages of DSBSC:

- Less information about the carrier will be delivered to the receiver.
- Needs a coherent carrier detector at receiver

## EFFICIENCY

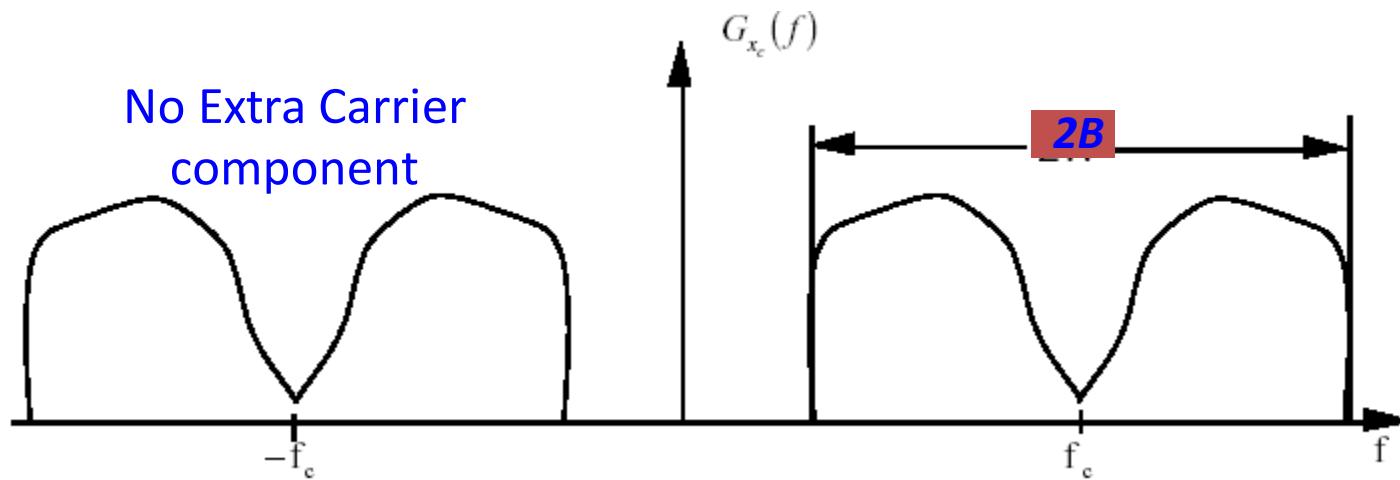
- For a fully modulated carrier ( $m=1$ ),  $2/3$  of the power is in the carrier, the rest in the sidebands (33.33% efficient )
- Total power  $P_t = P_c (1 + m^2 / 2)$ 
  - ❖ Carrier Power ( $P_c$ ) =  $V_c^2 / 2$
  - ❖ Side band Power =  $P_{lsb} = P_{usb} = m^2 P_c / 4$
- Information in side band : Power gets wasted in carrier
- AM is bandwidth inefficient (2 fm)
- Gets effected due to noise

# DSBSC Modulation



$$s(t) = A_c m(t) \cos \omega_c t$$

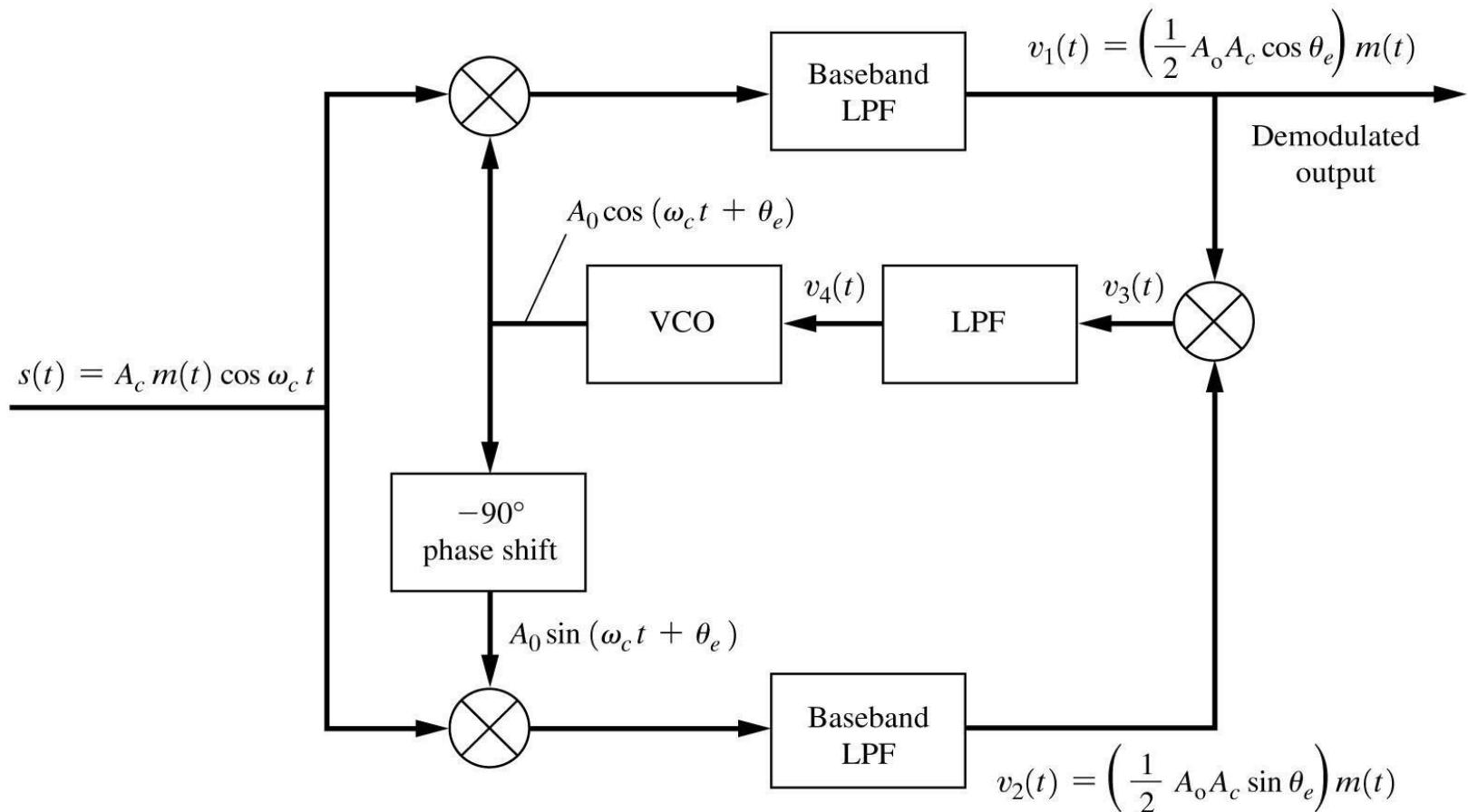
An Example of message energy spectral density.



Energy spectrum of the DSBSC modulated message signal.

# Carrier Recovery for DSBSC Demodulation

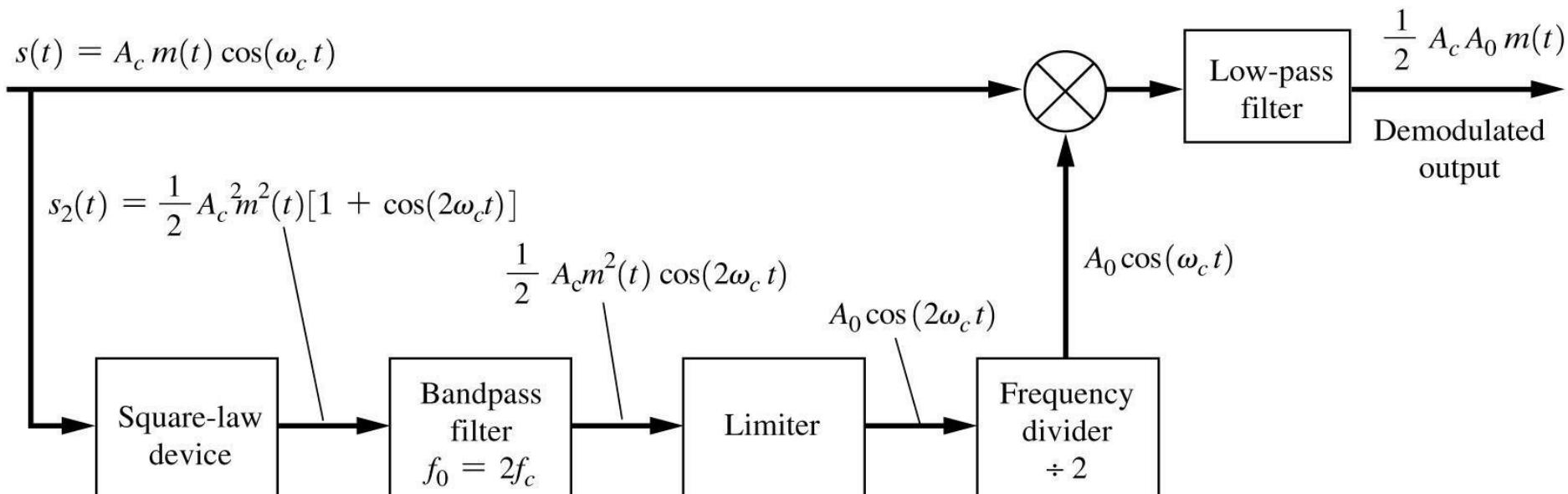
- Coherent reference for product detection of DSBSC can not be obtained by the use of ordinary PLL because there are no spectral line components at  $f_c$ .



(a) Costas Phase-Locked Loop

# Carrier Recovery for DSBSC Demodulation

- A squaring loop can also be used to obtain coherent reference carrier for product detection of DSBSC. A frequency divider is needed to bring the double carrier frequency to  $f_c$ .



(b) Squaring Loop

# Single Sideband (SSB) Modulation

❑ An **upper single sideband** (USSB) signal has a zero-valued spectrum for

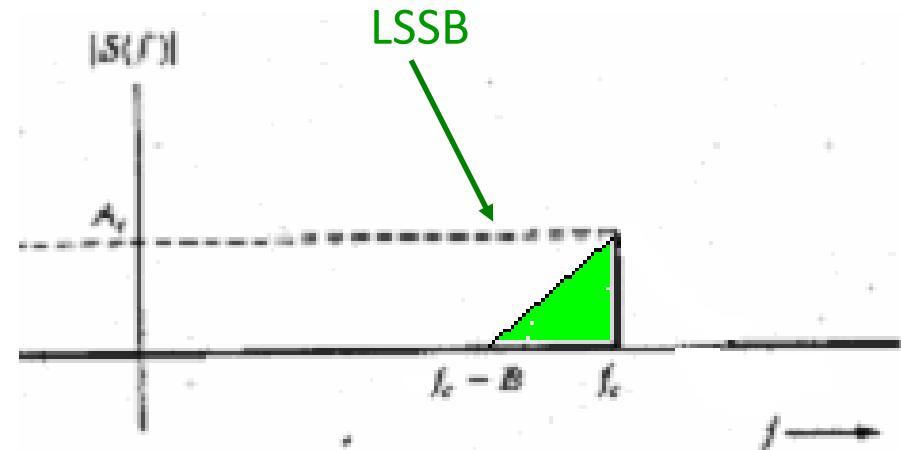
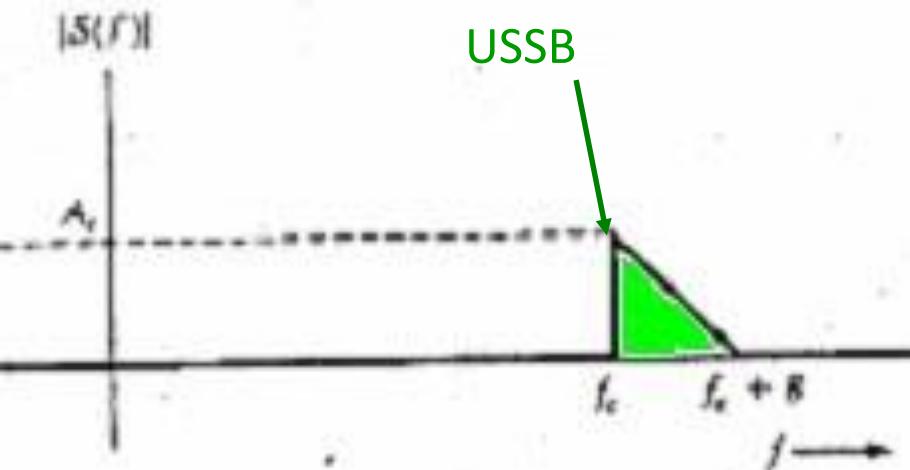
$$|f| < f_c$$

❑ A **lower single sideband** (LSSB) signal has a zero-valued spectrum for

$$|f| > f_c$$

➤ **SSB-AM** – popular method ~ **BW** is same as that of the modulating signal.

Note: Normally SSB refers to SSB-AM type of signal



# Single Sideband Signal

¶ **Theorem :** A SSB signal has **Complex Envelope** and bandpass form as:

$$g(t) = A_c [m(t) \pm j\hat{m}(t)]$$

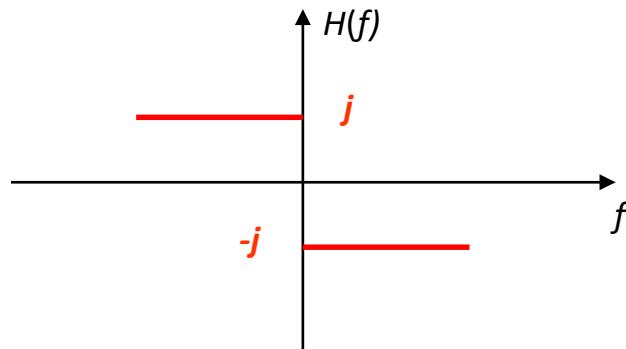
$$s(t) = A_c [m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t]$$

Upper sign (-) → USSB  
Lower sign (+) → LSSB

$$\hat{m}(t) - \text{Hilbert transform of } m(t) \rightarrow \hat{m}(t) \equiv m(t) * h(t) \quad \text{Where} \quad h(t) = \frac{1}{\pi t}$$

$$H(f) = \Im[h(t)] \quad \text{and} \quad H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$

Hilbert Transform corresponds to a  $-90^\circ$  phase shift



Chapter 3

## FREQUENCY MODULATION

# INTRODUCTION

3 properties of an analog signal can be modulated by information signal:

- o Amplitude - - -> produce AM
- o Frequency - - -> produce FM
- o Phase - - -> produce PM

FM & PM are forms of angle modulation and often referred as frequency modulation.

## **FM VS AM**

FM is considered to be superior to AM.

Transmission efficiency:

- AM use linear amplifier to produced the final RF signal.
- FM has constant carrier amplitude so it is not necessary to use linear amplifier.

Fidelity (capture effect):

- The stronger signal will be capture and eliminate the weaker.
- In AM, the weaker signal can be heard in the background.

Noise immunity (noise reduction):

- Constant carrier amplitude.
- FM receiver have limiter circuit

## Disadvantages of FM

Use too much spectrum space.

Requiring a wider bandwidth

- ✓ Reduce modulation index to minimize BW  
but in FM although we reduced the  
modulation index, BW is still larger.
- ✓ typically used at high frequencies (VHF,UHF  
& microwave frequencies

More complex circuitry

## ANGLE MODULATION

Amplitude of the modulated carrier is held constant and either the phase or the time derivative of the phase of the carrier is varied linearly with the message signal  $m(t)$ .

General angle-modulated signal is given by

$$m(t) = V_c \cos[\omega_c t + \theta(t)]$$

In angle modulation,  $\theta(t)$  is prescribed as being a function of the modulating signal  $\theta(t) = F[v_m(t)]$

If  $v_m(t)$  is the modulating signal, angle modulation is expressed as

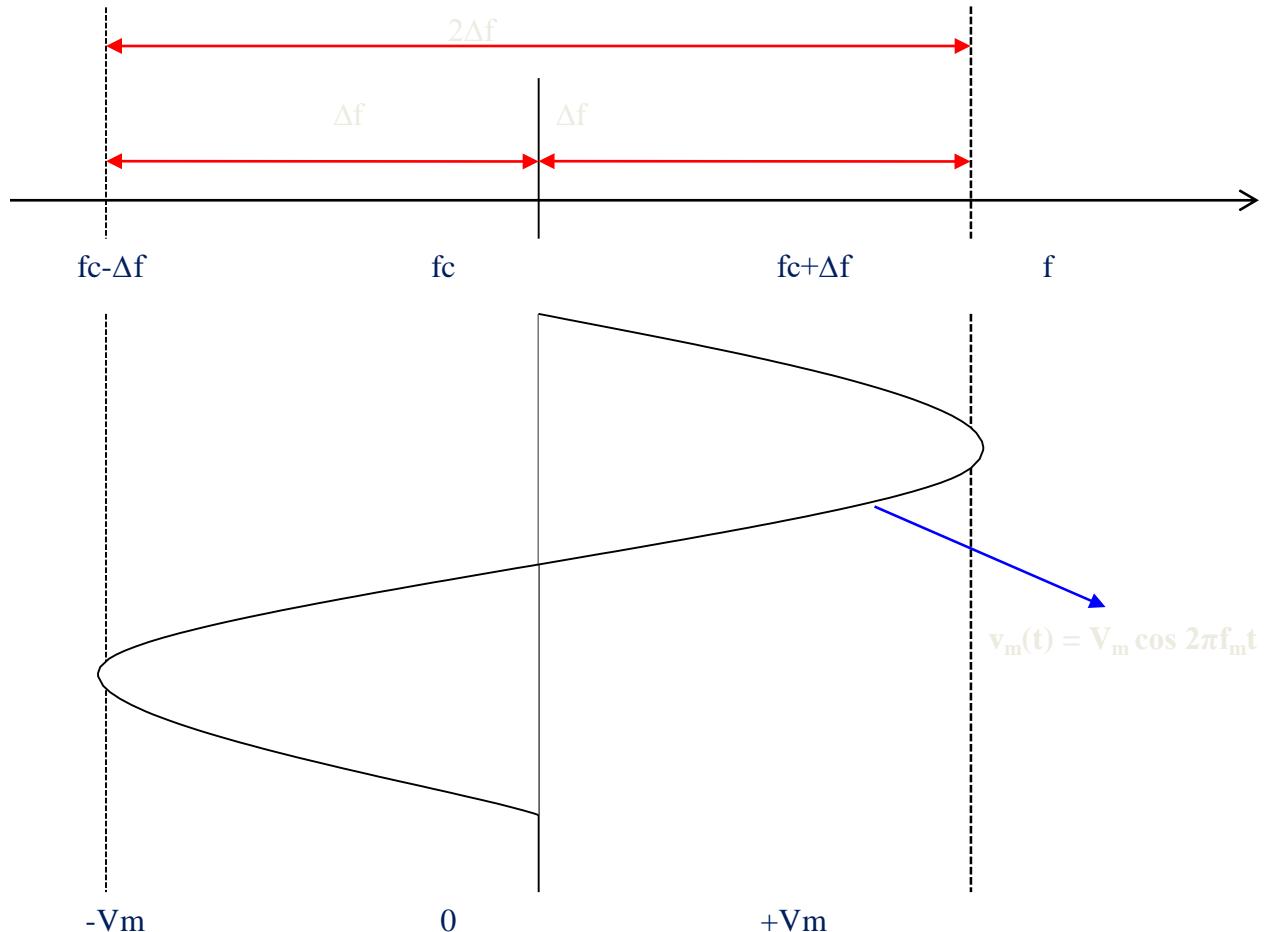
$$v_m(t) = V_m \sin(\omega_m t)$$

$$\omega_m = 2\pi f_m$$

where

## FM OR PM ?

FM	PM
<p><i>Instantaneous frequency</i> of the carrier is varied from its reference value by an amount proportional to the modulating signal amplitude</p> <p>Freq. carrier - - - &gt; directly varied Phase carrier - - -&gt; indirectly varied</p>	<p><i>Phase angle</i> of the carrier is varied from its reference value by an amount proportional to the modulating signal amplitude</p> <p>Phase carrier - - - &gt; directly varied Freq. carrier - - -&gt; indirectly varied</p>
Both must occur whenever either form of angle modulation is performed	



# MATHEMATICAL ANALYSIS

- Instantaneous frequency deviation
  - Instantaneous change in the frequency of the carrier and is defined as the first time derivative of the instantaneous phase deviation

instantaneous frequency deviation =  $\theta'(t)$  rad/s

or  $-\frac{\theta'(t) \text{ rad/s}}{2\pi \text{ rad/cycle}} = \frac{\text{cycle}}{\text{s}} = \text{Hz}$

- Instantaneous frequency
  - the precise frequency of the carrier at any given instant of time and is defined as the first time derivative of the instantaneous

$$\begin{aligned}\text{instantaneous frequency} &= \omega(t) = \frac{d}{dt} [\omega_c t_c + \theta(t)] \\ &= \omega_c + \theta'(t) \text{ rad/s}\end{aligned}$$



- Substituting  $2\pi f_c$  for  $\omega_c$  gives

instantaneous frequency =  $f_i(t)$

$$\text{and } \omega_i(t) = \left(2\pi \frac{\text{rad}}{\text{cycle}}\right) \left(f_c \frac{\text{cycles}}{\text{s}}\right) + \theta'(t) = 2\pi f_c + \theta'(t) \text{ rad/s}$$

- Frequency modulation is angle modulation in which the instantaneous frequency deviation,  $\theta'(t)$ , is proportional to the amplitude of the modulating signal, and the instantaneous phase deviation is proportional to the integral of the modulating signal voltage.

# DEVIATION SENSITIVITY

- For modulating signal  $v_m(t)$ , the frequency modulation are  
frequency modulation =  $\theta'(t) = k_f v_m(t)$  rad/s

where  $k_f$  are constant and are the deviation sensitivities of the frequency modulator.

- Deviation sensitivities are the output-versus-input transfer function for the modulators, which gave the relationship between what output parameter changes in respect to specified changes in the input signal.
- frequency modulator,

$$k_f = \frac{\text{rad/s}}{\text{V}} \left( \frac{\Delta\omega}{\Delta V} \right)$$

# FREQUENCY MODULATION (FM)

- Variation of  $d\theta/dt$  produces Frequency Modulation
- Frequency modulation implies that  $d\theta/dt$  is proportional to the modulating signal.
- This yields

$$\begin{aligned}v_{FM}(t) &= V_c \sin [\omega_c t + \theta(t)] \\&= V_c \sin \left[ \omega_c t + \int \theta'(t) dt \right] \\&= V_c \sin \left[ \omega_c t + \int k_f v_m(t) dt \right] \\&= V_c \sin \left[ \omega_c t + k_f V_m \int \sin \omega_m(t) dt \right] \\&= V_c \sin \left[ \omega_c t - \frac{k_f V_m}{\omega_m} \cos \omega_m(t) \right]\end{aligned}$$

# Derive the FM signal using both cosine wave signal.

$$v(t) = V_c \cos(\omega_c t + \theta(t))$$

$$v_m(t) = V_m \cos(\omega_m t)$$

for PM

$$v_{PM}(t) = V_c \cos(\omega_c t + k_p v_m(t))$$

$$= V_c \cos(\omega_c t + k_p V_m \cos(\omega_m t))$$

for FM

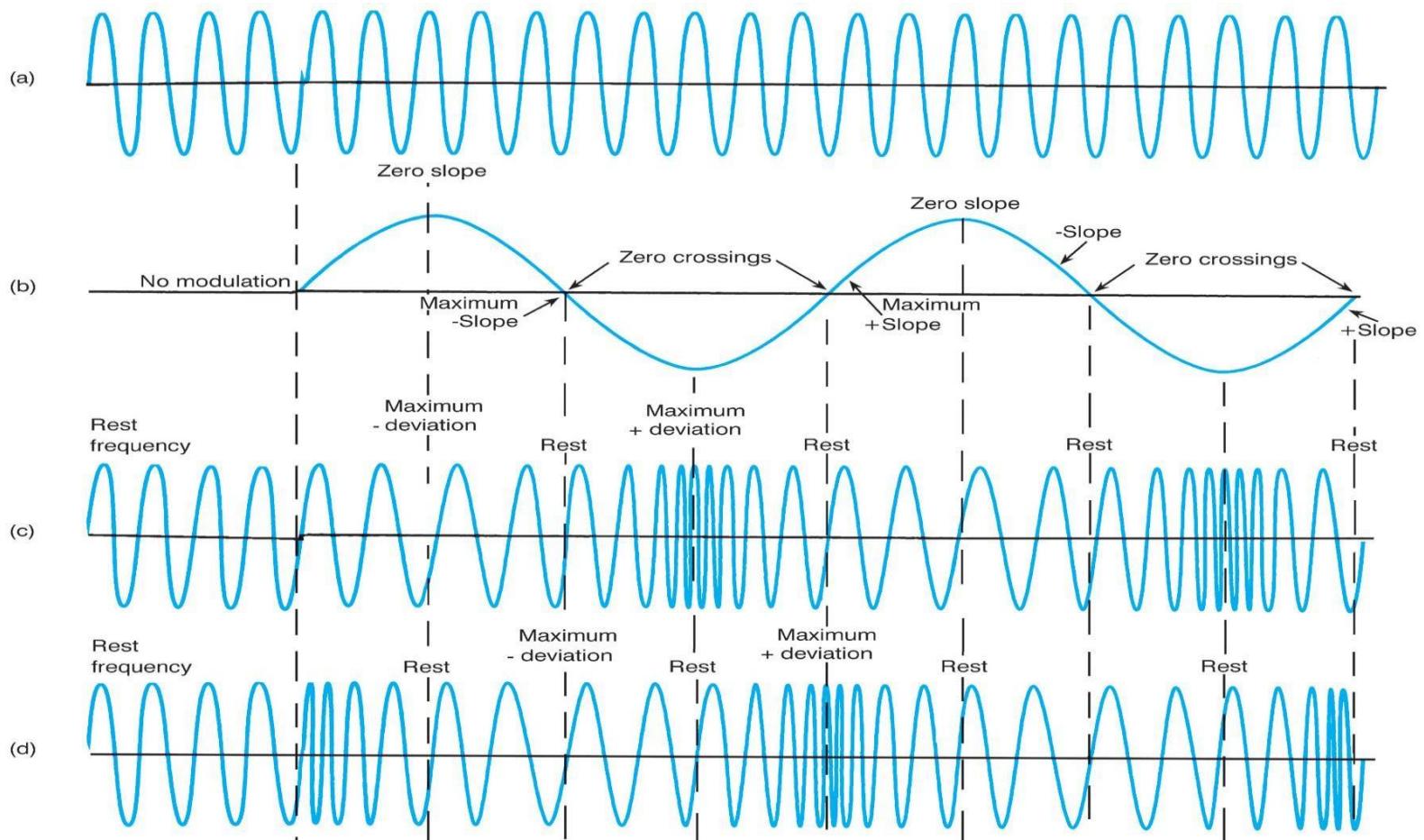
$$v_{FM}(t) = V_c \cos\left(\omega_c t + \int k_f v_m(t) dt\right)$$

$$= V_c \cos\left(\omega_c t + \int k_f V_m \cos(\omega_m t) dt\right)$$

$$= V_c \cos\left(\omega_c t + k_f V_m \int \cos(\omega_m t) dt\right)$$

$$= V_c \cos\left(\omega_c t + \frac{k_f V_m}{\omega_m} \sin(\omega_m t)\right)$$

# FM WAVEFORM



Phase and Frequency modulation ; (a) carrier signal (b) modulating signal (c) frequency modulated wave (d) phase modulated wave

- Carrier amplitude remains constant
- Carrier frequency is changed by the modulating signal.
  - ❑ amplitude of the information signal varies, the carrier frequency shift proportionately.
  - ❑ modulating signal amplitude increases, the carrier frequency increases.
  - ❑ modulating signal amplitude varies, the carrier frequency varies below and above its normal center or resting, frequency with no modulation.
- The amount of the change in carrier frequency produced by the modulating signal known as frequency deviation  $f_d$ .
- Maximum frequency deviation occurs at the maximum amplitude of the modulating signal.
- The frequency of the modulating signal determines the frequency deviation rate

# MODULATION INDEX

- Directly proportional to the amplitude of the modulating signal and inversely proportional to the frequency of the modulating signal
- Ratio of the frequency deviation and the modulating frequency
- FM equation :  $v_{FM}(t) = V_c \sin[\omega_c t - \beta \cos \omega_m(t)]$
- $\beta$  as modulation index :

$$\beta = \frac{k_f V_m}{\omega_m} = \frac{\Delta f_c}{f_m}$$

## Example:

- Determine the modulation index for FM signal with modulating frequency is 10KHz deviated by  $\pm 10\text{kHz}$ .
  - ✓ Answer :  $(20\text{kHz}/10\text{kHz}) = 2.0$  (unitless)
- The total frequency change,  $10\text{kHz} \times 2$  is called the **carrier swing**

## Example:

- a simple transmitter with an assigned rest frequency of 100MHz deviated by a  $\pm 25\text{kHz}$ , the carrier changes frequency with modulation between the limits of 99.975MHz and 100.025MHz
- The total frequency change,  $25\text{kHz} \times 2$  is called the **carrier swing**
- Table 1 display the transmission band that use FM and the legal frequency deviation limit for each category
- Deviation limits are based on the quality of the intended transmissions, wider deviation results in higher fidelity
- The frequency deviation is a useful parameter for determining the bandwidth of the FM-signals

# Display the transmission band that use FM and the legal frequency deviation limit for each category

Service Type	Frequency Assignment	Channel Bandwidth	Maximum Deviation	Highest Audio
Commercial FM radio broadcast	88.0 to 108.0 MHz	200 kHz	±75 kHz	15 kHz
Television sound	4.5 MHz above the picture carrier frequency	100 kHz	±25 kHz monaural; ± 50 kHz stereo	15 kHz
Public safety: police, fire, ambulance, taxi, forestry, utilities, transportation, government, etc.	50 MHz and 122 to 174 MHz	20 kHz	±5 kHz	3 kHz
Amateur and CE class A and business band radio	216 to 470 MHz	15 kHz	±3 kHz	3 kHz
Wireless mics., wireless telephones	The same as commercial FM broadcast, but limited in power to less than 1 W			
Videotape recorders	All functions are within a closed system and are not restricted, except to radiation into the air. System specs may vary with each manufacturer. Typically: the carrier is at 3.4 MHz, sync tips cause a frequency change to 3.0 MHz, the white level to 4.0 MHz, with a typical bandwidth of 4.0 MHz.			
Satellites	See FSK and data communications (special)			
Military	Intermingled with public safety, and extending to microwave frequencies			

Specifications for transmission of FM signal

## PERCENT MODULATION

- Simply the ratio of the frequency deviation actually produced to the maximum frequency deviation allowed by law stated in percent form

$$\% \text{ modulation} = \frac{\Delta f_{actual}}{\Delta f_{max}}$$

- For example if a given modulating signal produces  $\pm 50\text{kHz}$  frequency deviation, and the law stated that maximum frequency deviation allowed is  $\pm 75\text{kHz}$ , then

$$\% \text{ modulation} = \frac{50\text{kHz}}{75\text{kHz}} \times 100 = 67\%$$

A 1 MHz carrier freq with a measured sensitivity of 3 kHz/V is modulated with a 2 V, 4 kHz sinusoid.  
Determine

1. the max freq deviation of the carrier
2. the modulation index
3. the modulation index if the modulation voltage is doubled
4. the modulation index for  $v_m(t)=2\cos[2\pi(8\text{kHz})t]\text{V}$
5. express the FM signal mathematically for a cosine carrier & the cosine-modulating signal of part 4.  
Carrier amplitude is 10V

	FM	PM
Modulated wave	$m(t) = V_c \cos \left[ \omega_c t + \frac{K_1 V_m}{f_m} \sin(\omega_m t) \right]$ <p>or</p> $m(t) = V_c \cos[\omega_c t + m \sin(\omega_m t)]$ <p>or</p> $m(t) = V_c \cos \left[ \omega_c t + \frac{\Delta f}{f_m} \sin(\omega_m t) \right]$	$m(t) = V_c \cos[\omega_c t + KV_m \cos(\omega_m t)]$ $m(t) = V_c \cos[\omega_c t + \Delta\theta \cos(\omega_m t)]$
Deviation sensitivity	$K_1$ (Hz/V)	$K$ (rad/V)
Deviation	$\Delta f = K_1 V_m$ (Hz)	$\Delta\theta = KV_m$ (rad)
Modulation index	$m = \frac{K_1 V_m}{f_m}$ (unitless) <p>or</p> $m = \frac{\Delta f}{f_m}$ (unitless)	$m = KV_m$ (rad) <p><math>m = \Delta\theta</math> (rad)</p>
Modulating signal	$v_m(t) = V_m \sin(\omega_m t)$	$v_m(t) = V_m \cos(\omega_m t)$
Modulating frequency	$\omega_m = 2\pi f_m$ rad/s <p>or</p> $\omega_m/2\pi = f_m$ (Hz)	$\omega_m = 2\pi f_m$ rad/s <p><math>\omega_m/2\pi = f_m</math> (Hz)</p>
Carrier signal	$V_c \cos(\omega_c t)$	$V_c \cos(\omega_c t)$

# FM RADIO FREQUENCY

- Commercial radio FM band, 88MHz – 108MHz
- Each station allotted to a frequency deviation of  $\pm 75\text{kHz}$  (150 carrier swing) and 25kHz of guard band added above and below the carrier frequency swing
- Total bandwidth is 200kHz
- Therefore, maximum of 100 stations can be made available

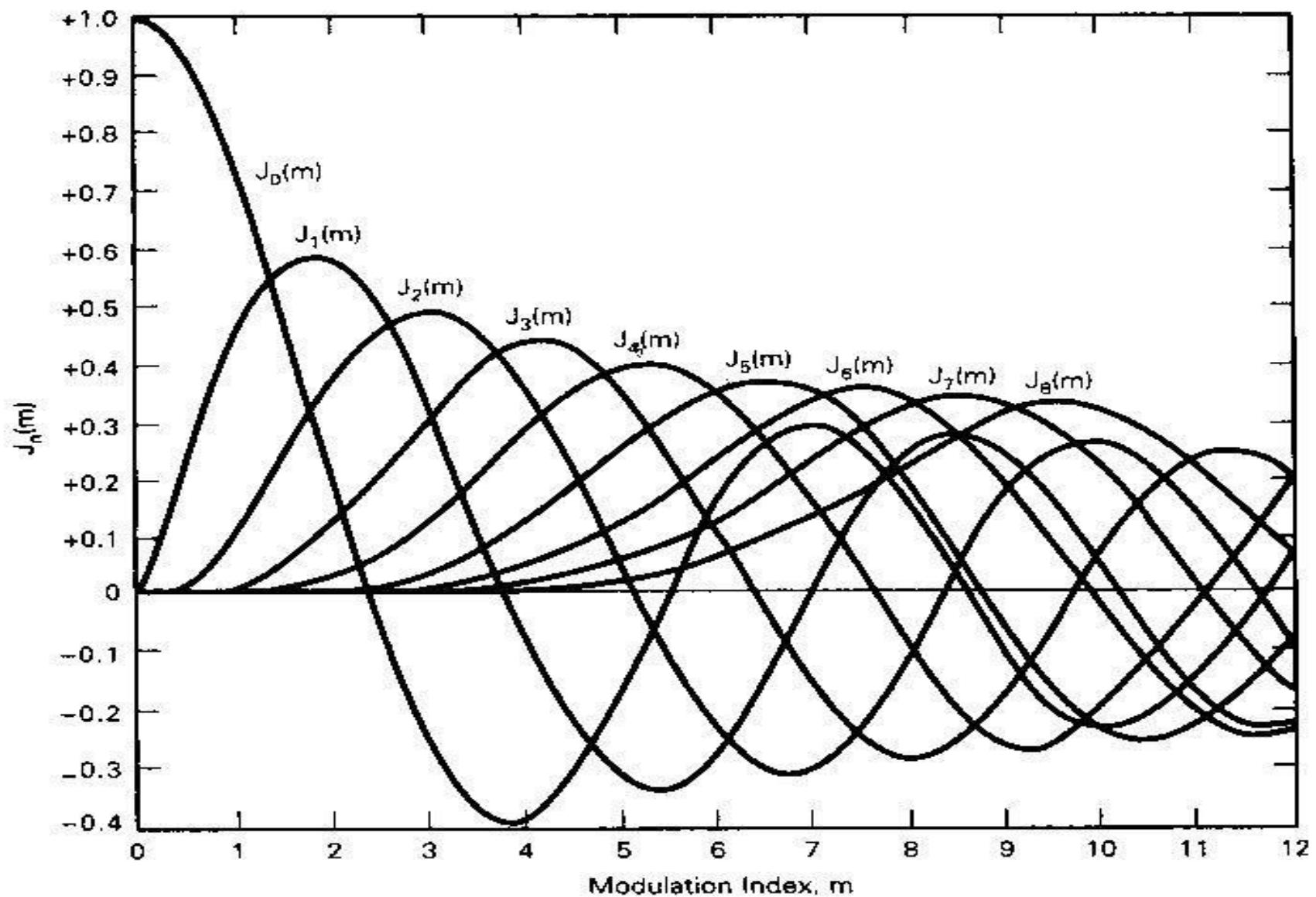
# FREQUENCY ANALYSIS OF FM WAVES

# BESSEL TABLE

Modulation index $\beta$	Carrier	Sidebands										
		$J_0$	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$	$J_7$	$J_8$	$J_9$	$J_{10}$
0.0	1.00	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.06	0.02	—	—	—
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—
7.0	0.30	0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	0.02
8.0	0.17	0.23	-0.11	-0.29	0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.06

Tabulated value for Bessel Function for the first kind of the  $n^{\text{th}}$  order

- The first column gives the modulation , while the first row gives the Bessel function.
- The remaining columns indicate the amplitudes of the carrier and the various pairs of sidebands.
- Sidebands with relative magnitude of less than 0.001 have been eliminated.
- Some of the carrier and sideband amplitudes have negative signs. This means that the signal represented by that amplitude is simply shifted in phase  $180^\circ$  (phase inversion).
- The spectrum of a FM signal varies considerably in bandwidth depending upon the value of the modulation index. The higher the modulation index, the wider the bandwidth of the FM signal.
- With the increase in the modulation index, the carrier amplitude decreases while the amplitude of the various sidebands increases. With some values of modulation index, the carrier can disappear completely.



Bessel Function,  $J_n(m)$  vs  $m$

# PROPERTIES OF BESSSEL FUNCTION

- **Property - 1:**

For  $n$  even,

$$\text{we have } J_n(\beta) = J_{-n}(\beta)$$

For  $n$  odd,

$$\text{we have } J_n(\beta) = (-1) J_{-n}(\beta)$$

Thus,

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

- **Property - 3:**

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

- **Property - 2:**

For small values of the modulation index  $\beta$ , we have

$$J_0(\beta) \cong 1$$

$$J_1(\beta) \cong \beta/2$$

$$J_3(\beta) \cong 0 \quad \text{for } n > 2$$

## FM BANDWIDTH

- The total BW of an FM signal can be determined by knowing the modulation index and Bessel function.

$$BW = 2f_m N$$

N = number of significant sidebands

$f_m$  = modulating signal frequency (Hz)

- Another way to determine the BW is use Carson's rule
- This rule recognizes only the power in the most significant sidebands with amplitude greater than 2% of the carrier.

Calculate the bandwidth occupied by a FM signal with a modulation index of 2 and a highest modulating frequency of 2.5 kHz. Determine bandwidth with table of Bessel functions.

Referring to the table, this produces 4 significant pairs of sidebands.

$$\begin{aligned}BW &= 2 \times 4 \times 2.5 \\&= 20 \text{ kHz}\end{aligned}$$

## CARSON'S RULE

$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$f_{d(\max)}$  = max. frequency deviation

$f_{m(\max)}$  = max. modulating frequency

- Carson's rule always give a lower BW calculated with the formula  $BW = 2f_m N$ .
- Consider only the power in the most significant sidebands whose amplitudes are greater than 1% of the carrier.
- Rule for the transmission bandwidth of an FM signal generated by a single oscillator of frequency  $f_m$  as follows:

$$B_T = BW \cong 2\Delta f + 2f_m = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

$$\text{or} \qquad \qquad = 2f_m (1 + \beta)$$

For an FM modulator with a modulation index  $\beta = 1$ , a modulating signal

$v_m(t) = V_m \sin(2\pi 1000t)$  and unmodulated carrier

$$v_c(t) = 10 \sin(2\pi 500kt), \text{ determine}$$

- a) Number of sets of significant sideband
- b) Their amplitude
- c) Then draw the frequency spectrum showing their relative amplitudes

For an FM modulator with a peak freq deviation  $\Delta f = 10\text{kHz}$ , a modulating signal freq  $f_m = 10\text{kHz}$ ,  $V_c = 10\text{V}$  and 500kHz carrier, determine

- a) Actual minimum bandwidth from the Bessel function table
- b) Approximate minimum bandwidth using Carson's rule
- c) Plot the output freq spectrum for the Bessel approximation

## DEVIATION RATIO (DR)

- Minimum bandwidth is greatest when maximum freq deviation is obtained with the maximum modulating signal frequency
- Worst case modulation index and is equal to the maximum peak frequency deviation divided by the maximum modulating signal frequency
- Worst case modulation index produces the widest output frequency spectrum
- Mathematically,

$$DR = \frac{\text{max peak freq deviation}}{\text{max mod signal freq}} = \frac{\Delta f_{\max}}{f_{m(\max)}}$$

- Determine the deviation ratio and bandwidth for the worst case (widest bandwidth) modulation index for an FM broadcast band transmitter with a maximum frequency deviation of 75kHz and a maximum modulating signal frequency of 15kHz
- Determine the deviation ratio and maximum bandwidth for an equal modulation index with only half the peak frequency deviation and modulating signal frequency

# POWER IN ANGLE-MODULATED SIGNAL

- The power in an angle-modulated signal is easily computed

$$P = V_c^2 / 2R \text{ W}$$

- Thus the power contained in the FM signal is independent of the message signal. This is an important difference between FM and AM.
- The time-average power of an FM signal may also be obtained from

$$v_{FM}(t) = V_c \cos(2\pi f_c t + \theta(t))$$

An FM signal is given as  $v_{FM}(t)=12\cos[(6\pi \times 10^6 t) + 5\sin(2\pi \times 1250t)]$  V. Determine

- a. freq of the carrier signal
- b. freq of the modulating signal
- c. modulation index
- d. freq deviation
- e. power dissipated in 10 ohm resistor.

Determine the unmodulated carrier power for the FM modulator given that  $\beta = 1$ ,  $V_c = 10 \text{ V}$ ,  $R = 50 \Omega$ . Then, determine the total power in the angle-modulated wave.

Solution:

→ not exactly equal because values in Bessel table have been rounded off.

An FM signal expressed as  
is measured in a 50 ohm antenna. Determine the  
following :-

$$v_{FM}(t) = 1000 \cos(2\pi 10^7 t + 0.5 \sin 2\pi 10^4 t)$$

- a. total power
- b. modulation index
- c. peak freq deviation
- d. modulation sensitivity if 200 mV is required to achieve part c
- e. amplitude spectrum
- f. bandwidth (99%) and approximate bandwidth by Carson's rule
- g. power in the smallest sideband of the 99% BW
- h. total information power

An FM signal with 5W carrier power is fluctuating at the rate of 10000 times per second from 99.96 MHz to 100.04 MHz. Find

- a. carrier freq
- b. carrier swing
- c. freq deviation
- d. modulation index
- e. power spectrum

In an FM transmitter, the freq is changing between 100 MHz to 99.98 MHz, 400 times per seconds. The amplitude of the FM signal is 5 V, determine :-

1. carrier and modulating freq
2. carrier freq swing
3. amplitude spectrum
4. bandwidth by using Bessel Table and Carson's rule
5. average power at the transmitter if the modulator carrier power is 5 W.

# FM SIGNAL GENERATION

- They are two basic methods of generating frequency-Modulated signals:
  - Direct Method
  - Indirect Method

## DIRECT FM

- In a direct FM system the instantaneous frequency is directly varied with the information signal. To vary the frequency of the carrier is to use an Oscillator whose resonant frequency is determined by components that can be varied. The oscillator frequency is thus changed by the modulating signal amplitude.

$$f_i = f_c + k_f v_m(t)$$

- For example, an electronic Oscillator has an output frequency that depends on energy-storage devices. There are a wide variety of oscillators whose frequencies depend on a particular capacitor value. By varying the capacitor value, the frequency of oscillation varies. If the capacitor variations are controlled by  $v_m(t)$ , the result is an FM waveform

## INDIRECT FM

- Angle modulation includes frequency modulation FM and phase modulation PM.
- FM and PM are interrelated; one cannot change without the other changing. The information signal frequency also deviates the carrier frequency in PM.
- Phase modulation produces frequency modulation. Since the amount of phase shift is varying, the effect is that, as if the frequency is changed.
- Since FM is produced by PM , the later is referred to as indirect FM.
- The information signal is first integrated and then used to phase modulate a crystal-controlled oscillator, which provides frequency stability.

# NOISE AND PHASE SHIFT

- The noise amplitude added to an FM signal introduces a small frequency variation or phase shift, which changes or distorts the signal.
- Noise to signal ratio N/S

$$\frac{N}{S} = \frac{\text{Frequency deviation produced by noise}}{\text{Maximum allowed deviation}}$$

- Signal to noise ration S/N

$$\frac{S}{N} = \frac{1}{N/S}$$

# INTERFERENCE

- A major benefit of FM is that interfering signals on the same frequency will be effectively rejected.
- If the signal of one is more than twice the amplitude of the other, the stronger signal will "capture" the channel and will totally eliminate the weaker, interfering signal.
- This is known as the *capture effect* in FM.
- In FM, the capture effect allows the stronger signal to dominate while the weaker signal is eliminated.
- However, when the strengths of the two FM signals begin to be nearly the same, the capture effect may cause the signals to alternate in their domination of the frequency.

- Despite the fact that FM has superior noise rejection qualities, noise still interferes with an FM signal. This is particularly true for the high-frequency components in the modulating signal.
- Since noise is primarily sharp spikes of energy, it contains a considerable number of harmonics and other high-frequency components.
- These high frequencies can at times be larger in amplitude than the high-frequency content of the modulating signal.
- This causes a form of frequency distortion that can make the signal unintelligible.
- To overcome this problem Most FM system use a technique known as Pre-emphasis and De-emphasis.

- ❑ Noise in electrical terms may be defined as any unwanted introduction of energy tending to interfere with the proper reception and reproduction of transmitted signals.
  - Noise is mainly of concern in receiving system, where it sets a lower limit on the size of signal that can be usefully received. Even when precautions are taken to eliminate noise from faulty connections or that arising from external sources, it is found that certain fundamental sources of noise are present within electronic equipment that limit the receivers sensitivity.

### Classification of noise

NOISE



NOISE WHOSE SOURCES ARE  
EXTERNAL TO THE RECEIVER

NOISE CREATED WITHIN  
THE RECEIVER ITSELF

## EXTERNAL NOISE

- Noise created outside the receiver
- External noise can be further classified as:
  1. Atmospheric
  2. Extraterrestrial
  3. Industrial

## ATMOSPHERIC NOISE

- Atmospheric noise or static is generally caused by lightning discharges in thunderstorms and other natural electrical disturbances occurring in the atmosphere.
- Since these processes are random in nature, it is spread over most of the RF spectrum normally used for broadcasting.

- Atmospheric Noise consists of spurious radio signals with components distributed over a wide range of frequencies. It is propagated over the earth in the same way as ordinary radio waves of same frequencies, so that at any point on the ground, static will be received from all thunderstorms, local and distant.
- Atmospheric Noise becomes less at frequencies above 30 MHz Because of two factors:-
  1. Higher frequencies are limited to line of sight propagation i.e. less than 80 km or so.
  2. Nature of mechanism generating this noise is such that very little of it is created in VHF range and above.

## EXTRATERRESTRIAL NOISE



**COSMIC NOISE**



**SOLAR NOISE**

# Solar Noise

- Under normal conditions there is a constant noise radiation from sun, simply because it is a large body at a very high temperature ( over 6000°C on the surface, it therefore radiates over a very broad frequency spectrum which includes frequencies we use for communication.
- Due to constant changing nature of the sun, it undergoes cycles of peak activity from which electrical disturbances erupt, such as corona flares and sunspots. This additional noise produced from a limited portion of the sun, may be of higher magnitude than noise received during periods of quite sun.

# Cosmic Noise

- Sources of cosmic noise are distant stars ( as they have high

- ¶ temperatures), they radiate RF noise in a similar manner as our Sun, and their lack in nearness is nearly compensated by their significant number.
- ¶ The noise received is called Black Body noise and is distributed fairly uniformly over the entire sky.

## **INDUSTRIAL NOISE**

- ¶ This noise ranges between 1 to 600 MHz ( in urban, suburban and other industrial areas) and is most prominent.
- ¶ Sources of such Noise : Automobiles and aircraft ignition, electric motors, switching equipment, leakage from high voltage lines and a multitude of other heavy electrical machines.

- ❑ The Noise is produced by the arc discharge present in all these operations. ( this noise is most intense industrial and densely populated areas)

## **INTERNAL NOISE**

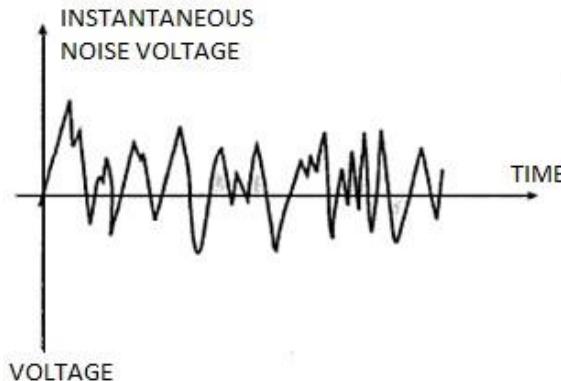
- ❑ Noise created by any of the active or passive devices found in receivers.
- ❑ Such noise is generally random, impossible to treat on individual voltage basis, but easy to observe and describe statistically. Because the noise is randomly distributed over the entire radio spectrum therefore it is proportional to bandwidth over which it is measured.
- ❑ Internal noise can be further classified as:
  1. Thermal Noise
  2. Shot Noise
  3. Low frequency or flicker Noise

#### 4. Burst Noise

## Thermal Noise

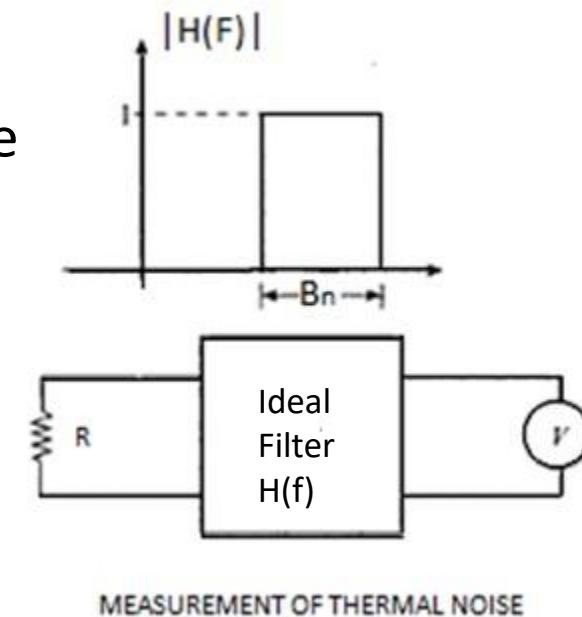
- ❑ The noise generated in a resistance or a resistive component is random and is referred to as thermal, agitation, white or Johnson noise.
- ❑ CAUSE :
  - The free electrons within an electrical conductor possess kinetic energy as a result of heat exchange between the conductor and its surroundings.
  - Due to this kinetic energy the electrons are in motion, this motion is randomized through collisions with imperfections in the structure of the conductor. This process occurs in all real conductors and gives rise to conductors resistance.
  - As a result, the electron density throughout the conductor varies

randomly, giving rise to randomly varying voltage across the ends of conductor. Such voltage can be observed as flickering on a very sensitive voltmeter.



- The average or mean noise voltage across the conductor is zero, but the root-mean-square value is finite and can be measured.
- The mean square value of the noise voltage is proportional to the resistance of the conductor, to its absolute temperature, to the frequency bandwidth of the device measuring the noise.
- The mean-square voltage measured on the meter is found to be

$$E_n^2 = 4RkTB_n$$



Where  $E_n$  = root-mean-square noise voltage, volts

$R$  = resistance of the conductor, ohms

$T$  = conductor temperature, kelvins

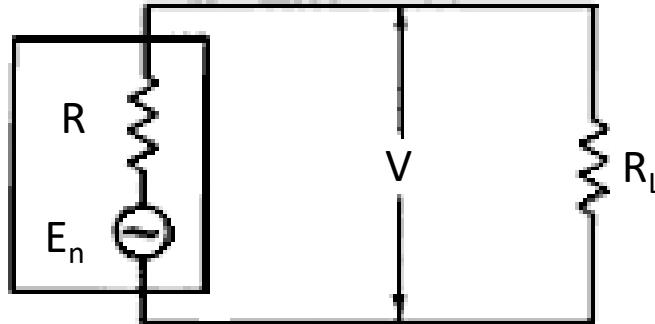
$B_n$  = noise bandwidth, hertz

$k$  = Boltzmann's constant ( $1.38 \times 10^{-23} \text{ J/K}$ )

And the rms noise voltage is given by :

$$E_n = \sqrt{4RkTB_n}$$

NOTE: Thermal Noise is not a free source of energy. To abstract the noise power, the resistance  $R$  is to be connected to a resistive load, and in thermal equilibrium the load will supply as much energy to  $R$  as it receives.



- ◻ In analogy with any electrical source, the available average power is defined as the maximum average power the source can deliver. Consider a generator of EMF  $E_n$  volts and internal resistance  $R$ .
- ◻ Assuming that  $R_L$  is noiseless and receiving the maximum noise power generated by  $R$ ; under these conditions of maximum power transfer,  $R_L$  must be equal to  $R$ . Then

$$P_n = V^2/R_L = V^2/R = (E_n/2)^2 / R = E_n^2 / 4R$$

Using Equation ①,

$$P_n = kTB_n$$

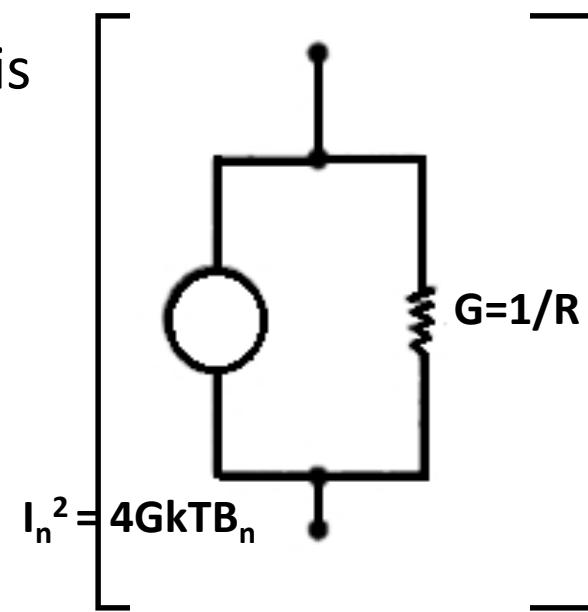
❖ Example:

Calculate the thermal noise power available from any resistor at room temperature (290 K) for a bandwidth of 1MHz. Calculate also the corresponding noise voltage, given that  $R = 50 \Omega$

Solution For a 1MHz bandwidth, the noise power is

$$\begin{aligned} P_n &= 1.38 \times 10^{-23} \times 290 \times 10^6 \\ &= 4 \times 10^{-15} \text{ W} \end{aligned}$$

$$\begin{aligned} E_n^2 &= 4 \times 50 \times 1.38 \times 10^{-23} \times 290 \\ &= 810^{-13} \\ &= 0.895 \mu\text{V} \end{aligned}$$



❓ The thermal noise properties of a resistor  $R$  may be  
represented by the equivalent voltage generator .

a.) Equivalent Voltage  
Source

➤ Equivalent current generator is found using the Norton's Theorem.  
Using conductance  $G = (1/R)$ , the rms noise current is given by :

$$I_n^2 = 4GkTB_n$$

## Resistors in Series

➤ let  $R_{ser}$  represent the total resistance of the series chain, where  $R_{ser} = R_1 + R_2 + R_3 + \dots$ ; then the noise voltage of equivalent series resistance is

$$\begin{aligned}E_n^2 &= 4R_{ser}kTB_n \\&= 4(R_1 + R_2 + R_3 + \dots)kTB_n \\&= E_{n1}^2 + E_{n2}^2 + E_{n3}^2 + \dots\end{aligned}$$

Hence the noise voltage of the series chain is given by:

$$E_n = \sqrt{(E_{n1}^2 + E_{n2}^2 + E_{n3}^2 + \dots)}$$

## Resistors in Parallel

- ☒ With resistors in parallel it is best to work in terms of conductance.
- ☒ Let  $G_{par}$  represent the parallel combination where  $G_{par} = G_1 + G_2 + G_3 + \dots$ ; then

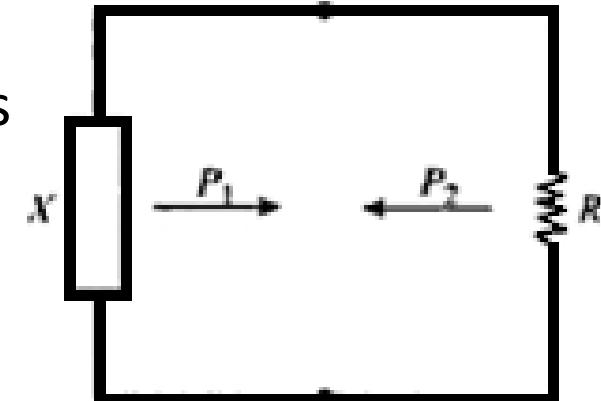
$$\begin{aligned}I_n^2 &= 4G_{par}kTB_n \\&= 4(G_{n1} + G_{n2} + G_{n3} + \dots)kTB_n \\&= I_{n1}^2 + I_{n2}^2 + I_{n3}^2 + \dots\end{aligned}$$

# REACTANCE

❑ Reactances do not generate thermal noise. This follows from the fact that reactances cannot Dissipate power.

❑ Consider an inductive or capacitive reactance connected in parallel with a resistor R.

❑ In thermal equilibrium, equal amounts of power must be exchanged; that is,  $P_1 = P_2$ . But since the reactance cannot dissipate power, the power  $P_2$  must be zero, and hence  $P_1$  must also be zero.

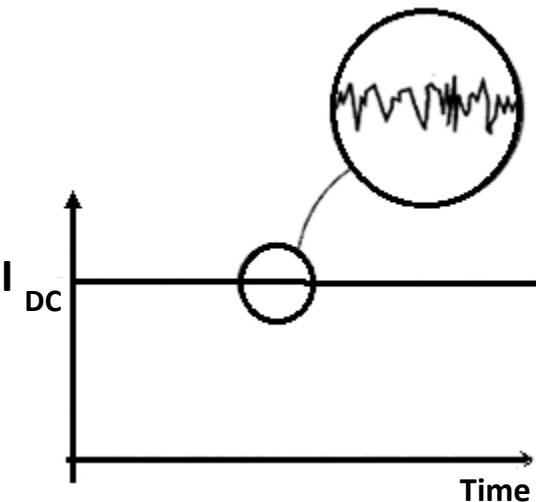


## Shot Noise

❑ Shot noise is random fluctuation that accompanies any direct current crossing potential barrier. The effect occurs because the carriers (electrons and holes in semiconductors) do not cross the barrier simultaneously but rather with random distribution in the timing of each carrier, which gives rise to random component of current superimpose on the steady current.

- Q In case of bipolar junction transistors , the bias current crossing the forward biased emitter base junction carries the shot noise.
- Q When amplified, this noise sounds as though a shower of lead shots were falling on a metal sheet. Hence the name shot noise.
- Q Although it is always present, shot noise is not normally observed during measurement of direct current because it is small compared to the DC value; however it does contribute significantly to the noise in amplifier circuits.
- Q The mean square noise component is proportion to the DC flowing, and for most devices the mean-Square, shot-noise is given by:

$$I_n^2 = 2I_{dc} q_e B_n \text{ ampere}^2$$



Where  $I_{dc}$  is the direct current in ampere's,  $q_e$  is the magnitude of electronic charge and  $B_n$  is the equivalent noise bandwidth in hertz.

## ❖ Example

Calculate the shot noise component of the current present on the direct current of 1mA flowing across a semiconductor junction, given that the effective noise bandwidth is 1 MHz.

SOLUTION

$$\begin{aligned}I_n^2 &= 2 \times 10^{-3} \times 1.6 \times 10^{-19} \times 10^6 \\&= 3.2 \times 10^{-16} \text{ A}^2 \\&= 18 \text{ nA}\end{aligned}$$

## Flicker Noise ( or $1/f$ noise )

- ❑ This noise is observed below frequencies of few kilohertz and its spectral density increases with decrease in frequency. For this reason it is sometimes referred to as  $1/f$  noise.
  
- ❑ The cause of flicker noise are not well understood and is recognizable by its frequency dependence. Flicker noise becomes significant at frequency lower than about 100 Hz. Flicker noise can be reduced

significantly by using wire-wound or metallic film resistors rather than the more common carbon composition type.

In semiconductors, flicker noise arises from fluctuations in the carrier densities (holes and electrons), which in turn give rise to fluctuations in the conductivity of the material. I.e the noise voltage will be developed whenever direct current flows through the semiconductor, and the mean square voltage will be proportional to the square of the direct current.

- ❑ In Electronic devices, it shows up as a low frequency phenomenon, as the higher frequencies overshadowed by white noise from other sources.

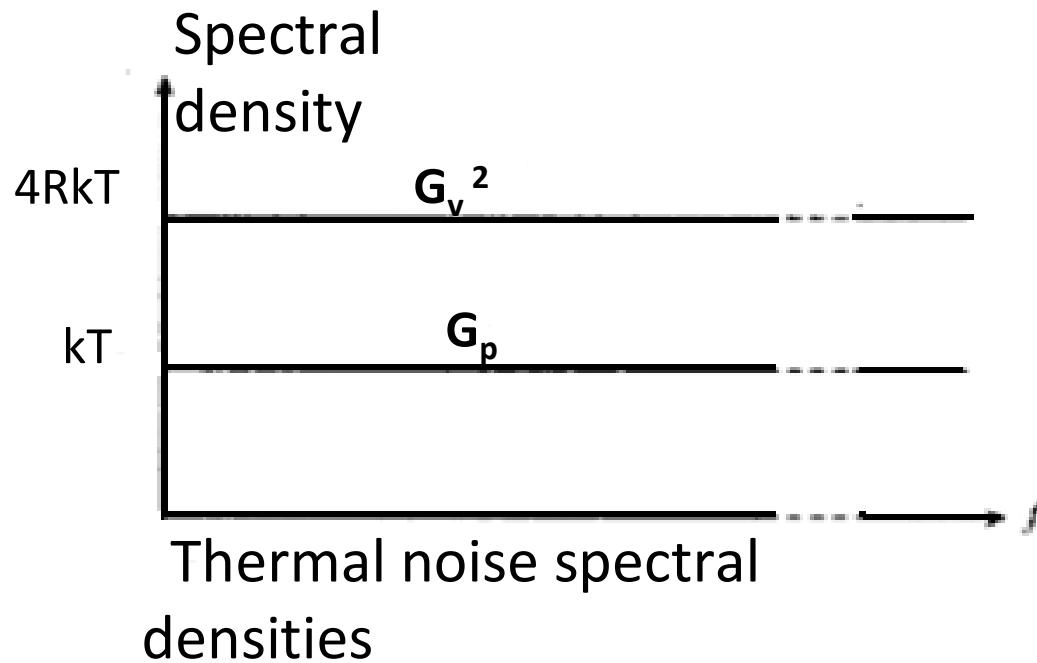
## Burst Noise

- ❑ It consists of sudden step-like transitions between two or more discrete voltage or current levels, as high as several hundred microvolts, at random and unpredictable times. Each shift in offset voltage or

# SPECTRAL DENSITY

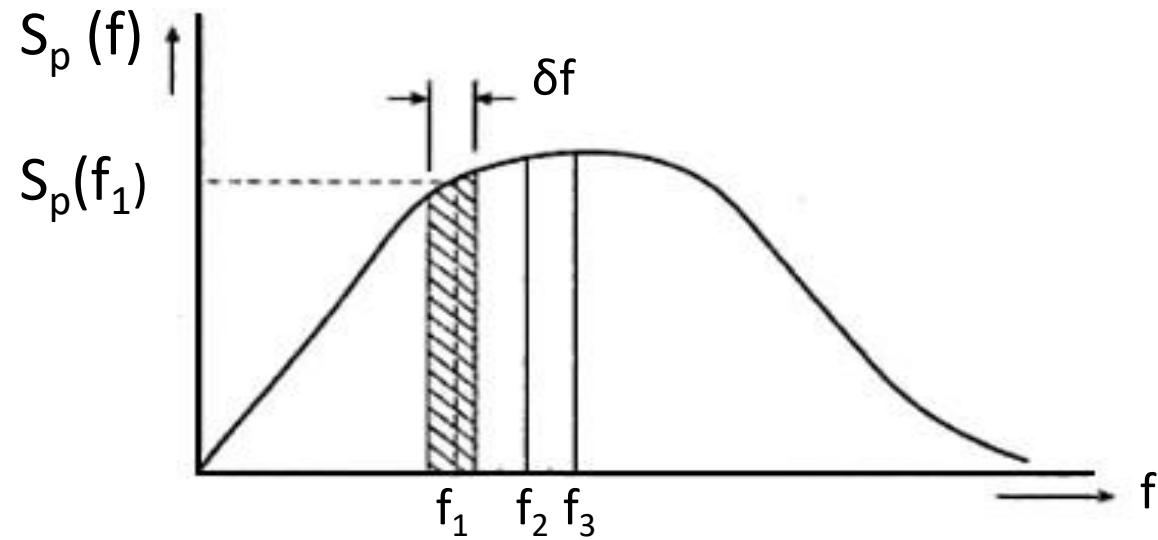
- ② spectral density is power distribution in frequency spectrum  
it use to distinguish type of noise. plot of light intensity/power as a function of frequency or wavelength.
- ② Thermal noise lies in category of power signals has spectral densities.  $B_n$  is property of external receiving system and  $B_n$  assume flat.
- ② from eq.  $P_n = E_n^2 / 4R = 4RkTB_n / 4R = kTB_n$ , since  $E_n^2 = 4RkTB_n$  where  
 $E_n$ =rms noise voltage(volt)  
 $R$ =resistance of conductor( $\Omega$ )  
 $B_n$ =noise bandwidth(hertz)  
 $K$ =Boltzmaan constant= $1.38 \times 10^{-23}$ (J/k)  
 $T$ =conductor temperature(kelvin)  
available power spectral density in Watts/Hz or Joule is  
 $G_a(f) = P_n / B_n = kT$

- ☐ spectral density for mean square voltage is given by  $G_v(f) = E^2 n / B_n = 4RkT(v^2/\text{hz})$  since  $E^2 n = 4RkTB_n$ . Spectral densities are flat that is independent of frequency in fig. below



- ❑ So thermal noise is also called white noise because of analogy to white light that has flat spectrum means signal contain equal power within a fixed bandwidth at any center frequency.
- ❑ when white noise is passed through a network then spectral density is changed by shape of network frequency response.
- ❑ total noise power at output is sum of the noise contribution over complete frequency range. taking into account of frequency response shape.

➤ Consider a power spectral density response in figure below

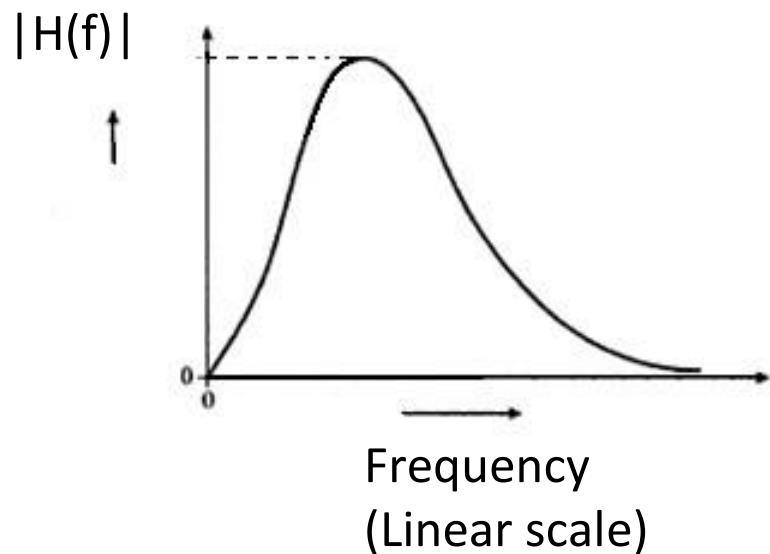
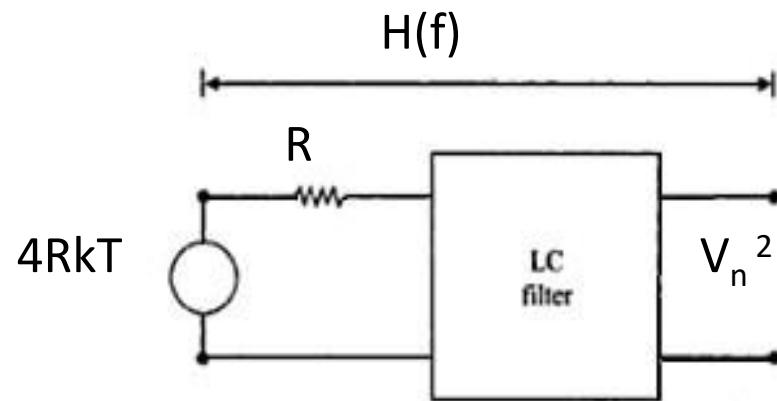


- at frequency  $f_1$  the noise power for small band width  $\delta f$  about  $f_1$  is  $\delta P_{n1} = S_p(f_1) \cdot \delta f$  Here  $\delta f$  assume flat about  $f_1$
- $\delta P_{n1}$  = spectral density (watts/hertz). Bandwidth (Hz) or  $\delta P_{n1} = S_p(f_1) \cdot \delta f$
- So noise power is equal to area of shaded strip about  $f_1$  similarly for  $f_2, f_3, \dots$  so that power is sum of all these small areas equal to total area under the curve. Area of curves gives total mean square voltage
- area of curve =  $\int G_a(f) ;$  for  $f=0$  to  $f=\infty$

- for mean square voltage  $S_p(f_1) \times \delta f = S_p(f_1) \delta f (V^2)$  = area of curves gives total mean square voltage.

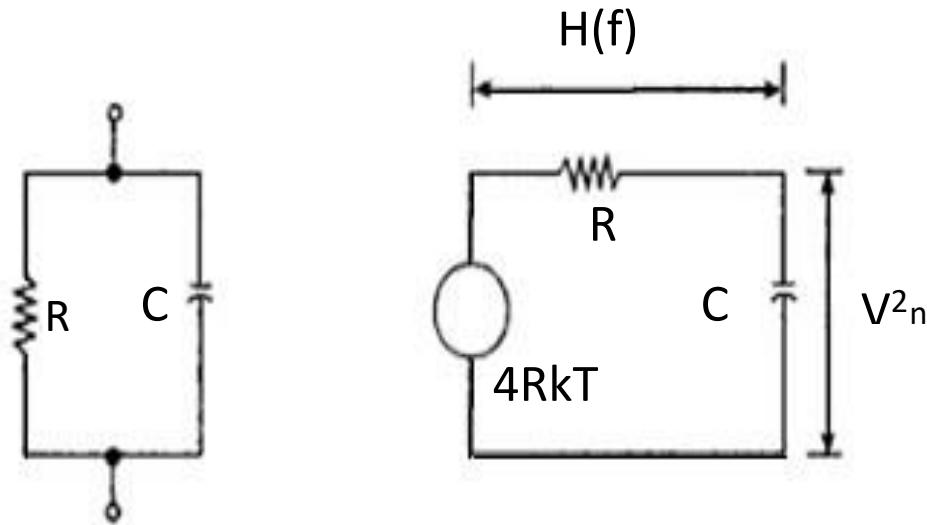
## Equivalent noise bandwidth

- ② it is the frequency span of a noise power curve with amplitude equal to actual peak value and with same integrated area
- ② If R is connected to input of LC filter as in figure (a) this represent an input generator of mean square voltage spectral density  $4RkT$  feeding a network of R and LC filter
- ② Let transfer function of network including R be  $H(f)$ , so spectral density for mean square voltage is  $= 4RkT |H(f)|^2$ ;  $H(f)$  is ratio of output to input voltage for mean square voltage.



- Total mean square output voltage is given by  
$$V_n^2 = \int 4RkT \cdot |H(f)|^2 \delta f \text{ for } f=0 \text{ to } \infty$$
$$= 4RkT \times (\text{area under } |H(f)|^2 \text{ curve})$$
- or total mean square voltage at the output can be stated as  
$$V_n^2 = 4RkTB_n$$
- By above two equation gets equivalent noise bandwidth of network
- $B_n = \int (|H(f)|^2 \delta f) = \text{area under curve } |H(f)|^2 \text{ for } f=0 \text{ to } \infty$

Example considered in fig. below input capacitance of the voltmeter used measure the noise voltage across R Circuit diagram



*RC* network and its transfer function used in determining noise bandwidth.

- transfer function of RC network  $|H(f)| = 1/[1+(\omega CR)]^{1/2}$
- Equivalence noise bandwidth of the RC network  
is  $B_n = \int |H(f)|^2 df = 1/4RC$ ;  $f=0$  to  $\infty$
- $V_n^2 = 4RkT \times 1/4RC = kT/C$ , so mean square voltage originates from R even though it is independent of R and inversely proportional to C even though C does not generate noise.
- transfer function  $|H(f)| = |X_c/Z_s|$ ;  $Z_s = r(1+j\omega Q)$  is impedance of the series tuned circuit and,  $X_c = 1/j\omega C$  i.e. reactance of C.

- let circuit is resonant at  $f_o$  and noise restricted to bandwidth  $\Delta f \ll f_o$  about resonant frequency  $f_o$  so transfer function given by  $|H(f)| = 1/\omega_o Cr = Q$ ; area under  $|H(f)|^2$  curve of small bandwidth  $\delta f$  is  $Q^2 \delta f$  so
  
  
  
- mean square noise voltage is  

$$V_n^2 = 4r k T B_n = 4r Q^2 k T \delta f = 4R_D k T \Delta f$$
 use the relation  

$$Q^2 r = R_D ; f=0 \text{ to } \infty \text{ and } B_n = 1/4R_D C$$
  
  
  
- $R_D$  is dynamic resistance of the tuned circuit . Noise bandwidth expressed as a function of 3-dB bandwidth of circuit
  
  
  
- From equ.  $B_{3dB} = f_o/Q$  and  $R_D = Q/\omega_o C$  combine these equ. Gets  

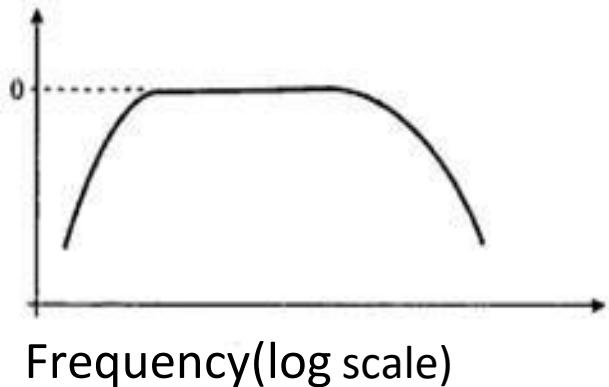
$$B_n = \pi/2 B_{3dB}$$

- ④ the mean square voltage at the output as  
 $V_n^2 = 4R_D kT x 1/4R_D C = kT/C$  So noise from resistor  $R_D$  is limited by bandwidth  $B_n$  is  $V^2 n = 4R_D kT x 1/4R_D C = kT/C$
- ④ here assume that Q factor is constant and independent of frequency for end result gives good indication of noise expected in practice but not true for range zero to infinity
- ④ In radio receivers noise is generated at antenna receiver input and output noise bandwidth is determined by the audio part of the receiver
- ④ Equivalent noise bandwidth=area of normalized frequency curve for low frequency section. normalized means curve max. value is unity

- ❑ Response curve show output in decibels relative to maximum as in next slide fig.
- ❑ Equivalent noise bandwidth is area of curve for a single sideband type, then noise bandwidth appears on both side of the carrier and gets doubled.

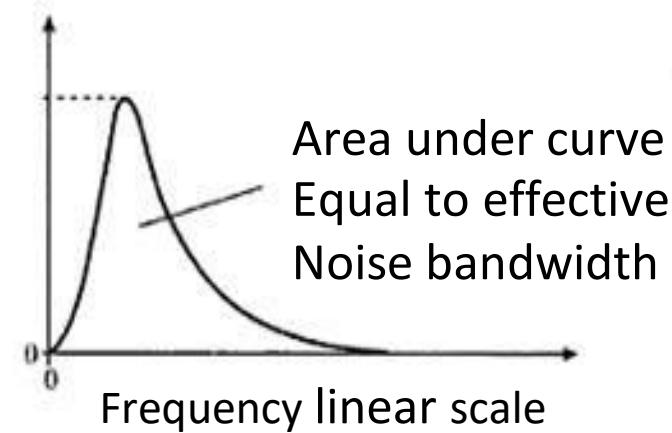
# Response curve

Relative  
Response  
dB

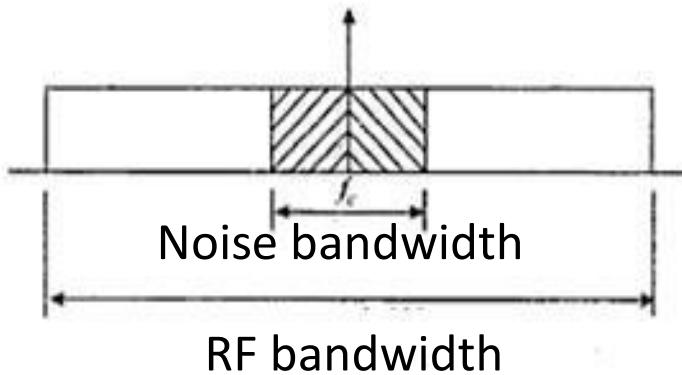


(a)

Relative response  
Power ratio



(b)



(c)

(a)Amplifier frequency response curve (b) curve of (a) using linear scales (c)  
noise bandwidth Of a double sideband receiver

## Signal to noise ratio

- ❑ in communication it is the signal to noise ratio rather than absolute value of noise.
- ❑ It is defined as a power ratio  $S/N = P_s/P_n = V_s^2/V_n^2$
- ❑ Repeater /amplifier insert to make up for the loss in analog telephone cable

If power loss of a line section is L then repeater amplifier power gain G is chosen so  $LG=1$ , long line divided into identical section

If input signal power= $P_s$  to first section as signal passes along the link power output at

- each repeater is  $P_s$  since  $LG=1$  for each link but noise power are additive and the total noise at the output of  $m^{\text{th}}$  link is  
$$P_n = P_{n1} + P_{n2} + \dots + P_{nm}$$

- ☒ If each links are identical and contribute  $P_n$  then total noise power is  $P_{nm} = M P_n$  then output signal to noise ratio is  
$$(N/S)_o \text{dB} = 10 \log P_s / M P_n = (S/N)_1 \text{dB} - (M) \text{dB}$$
- ☒ Where  $(S/N)_1$  is ratio for one link and  $(M) \text{dB}$  is no of links expressed as power ratio in decibels

**Ques:** the equivalent noise resistance for an amplifier is  $300\Omega$  and equivalent shot noise current  $5\mu A$ . the amplifier is fed from  $150\Omega$   $10\mu V$  rms sinusoidal signal source. Calculate the individual noise voltage at the input signal to noise ratio in decibels. the noise bandwidth is  $10MHz$ .

**Solution:** let room temperature so that  $kT=4\times 10^{-21}J$

And  $q_e=1.6\times 10^{-19}C$  shot noise current is  $I_{na}=[2q_e I_{EQ} B_n]^{1/2}=4nA$

So noise voltage across source resistance is

$I_{na} R_s = 0.6\mu V$  shot noise current does not develop a voltage across  $R_n$ . The noise voltage generated by  $R_n$  is  $V_{na}=[4R_n kT_o B_n]^{1/2}=6.93\mu V$

Thermal noise voltage from source is  $V_{ns}=[4R_s kT_o B_n]^{1/2}=4.9\mu V$

total noise voltage at input to the amplifier is  $V_n=[4.9^2+6.93^2+.6^2]^{1/2}=8.51\mu V$   
so signal to noise ratio in decibels is

$$S/N=20\log V_s/V_n=1.4dB$$

## NOISE FACTOR

- Noise factor is the ratio of available S/N ratio at the input to the available S/N ratio at the output .
  
- Consider a signal source at room temperature  $T_o = 290K$  providing an input to an amplifier . The available noise power from this would be

$$P_{ni} = kT_oB_n .$$

where , k = boltzmann constant =  $1.38 \times 10^{-23} J/K$

$B_n$  = equivalent noise bandwidth in Hz

- Let the available signal power be  $P_{si}$ , then available signal to noise ratio from the source is

$$P_n = kT_o B_n \longrightarrow$$

$$(S/N)_{ni} = P_{si} / kT_o B_n$$

"Ideal"  
noiseless  
amplifier  
Gain, G

$$P_{so} = GP_{si}$$

$$P_{no} = GkT_o B_n$$

- The source connected to the amplifier represents available signal to noise ratio.
- If amplifier has the available power gain denoted by G , the available output signal power  $P_{so} = GP_{si}$  and if the amplifier was entirely noiseless , the available noise power would be

$$P_{no} = FGkT_o B_n$$

Noisy  
amplifier

Gain, G  
Noise factor, F

$$\longrightarrow P_{no} = FGkT_o B_n$$

However , it is known that all real amplifiers contribute noise and the available output signal to noise ratio will be less than that at the input.

➤ The noise factor F is defined as

$$F = (\text{available S/N power ratio at the input}) / (\text{available S/N power ratio at the output})$$

$$F = (P_{si} / kT_o B_n) \times (P_{no} / GP_{si})$$

$$F = P_{no} / GkT_o B_n$$

?] It follows from this that the available output noise power is given by

$$P_{no} = FGkT_o B_n$$

F can be interpreted as the factor by which the amplifier increases the output noise , for ,if amplifier were noiseless the output noise would be  $GkT_n B_n$  .

The available output power depends on the actual input power delivered to the amplifier .

¶ Noise factor is a measured quantity and will be specified for given amplifier or network. It is usually specified in decibels , when it is referred to as the noise figure. Thus

$$\text{noise figure} = (F) \text{ dB} = 10 \log F$$

### ❖ Example

The noise figure of an amplifier is 7dB. Calculate the output signal to noise ratio when the input signal to noise ratio is 35 dB.

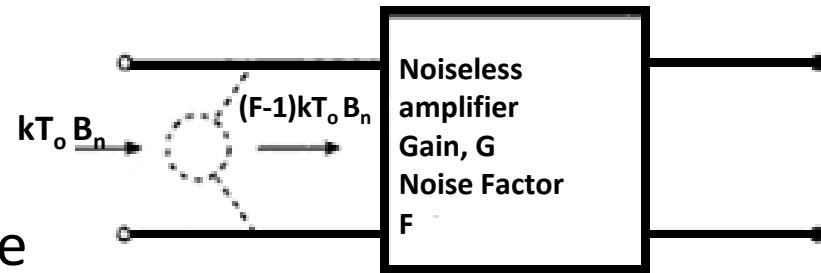
Sol . From the definition of noise factor ,

$$\begin{aligned}(S/N)_o &= (S/N)_{in} - (F) \text{ dB} \\ &= (35 - 7) \text{ dB} \\ &= 28 \text{ db}\end{aligned}$$

## Amplifier Input Noise in terms of F

➤ Amplifier noise is generated in many components throughout the amplifier , but it proves convenient to imagine it to originate from some equivalent power source at the input of the amplifier . Then the total available power input noise is

$$\begin{aligned} P_{ni} &= P_{no} / G \\ &= FkT_o B_n \end{aligned}$$



➤ The source contributes an available power  $kT_o B_n$  and hence the amplifier must contribute  $P_{na}$  , where

$$\begin{aligned} P_{na} &= FkT_o B_n - kT_o B_n \\ &= (F - 1)kT_o B_n \end{aligned}$$

## ❖ Example

An amplifier has a noise figure of 13dB. Calculate equivalent amplifier input noise for a bandwidth of 1 MHz.

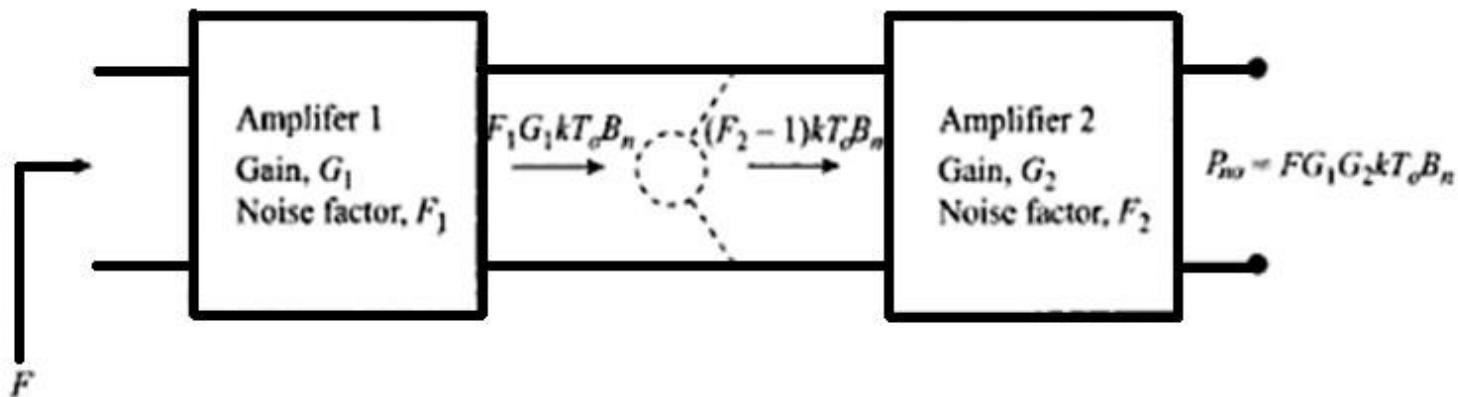
Sol. 13 dB is a power ratio of approximately 20 : 1. hence

$$\begin{aligned} P_{na} &= (20 - 1) \times 4 \times 10^{-21} \times 10^6 \\ &= 1.44 \text{pW}. \end{aligned}$$

Noise figure must be converted to a power ratio F to be used in the calculation.

## Noise factor of amplifiers in cascade

- consider first two amplifiers in cascade . The problem is to determine the overall noise factor **F** in terms of individual noise factors and available power gains .  
the available noise power at the output of the amplifier 1 is  
 $P_{no1} = F_1 G_1 kT_o B_n$  and this available to amplifier 2.



4.15.1 Noise factor of two amplifiers in cascade.

➤ Amplifier 2 has noise  $(F_2 - 1)kT_o B_n$  of its own at its input, hence total available noise power at the input of amplifier 2 is

$$P_{ni2} = F_1 G_1 kT_o B_n + (F_2 - 1)kT_o B_n$$

☒ Now since the noise of amplifier 2 is represented by its equivalent input source , the amplifier itself can be regarded as being noiseless and of available power gain  $G_2$  , so the available noise output of amplifier 2 is

$$\begin{aligned} P_{no2} &= G_2 P_{ni2} \\ &= G_2 ( F_1 G_1 kT_o B_n + (F_2 - 1)kT_o B_n ) \end{aligned} \quad (1)$$

☒ The overall available power of the two amplifiers in cascade is

$G = G_1 G_2$  and let overall noise factor be  $F$  ; then output noise power can also be expressed as

$$P_{no} = FGkT_o B_n \quad (2)$$

equating the two equations for output noise (1) and (2)

?

$$F_1 G_1 kT_o B_n + (F_2 - 1)kT_o B_n = FGkT_o B_n$$

?

$$F_1 G_1 + (F_2 - 1) = FG$$

➤

$$F = F_1 G_1 / G + (F_2 - 1) / G$$

where  $G = G_1 G_2$

➤

$$F = F_1 + (F_2 - 1) / G_1$$

- ❑ This equation shows the importance of high gain , low noise amplifier as the first stage of a cascaded system. By making  $G_1$  large, the noise contribution of the second stage can be made negligible, and  $F_1$  must also be small so that the noise contribution of the first amplifier is low.
- ❑ The argument is easily extended for additional amplifiers to give

$$F = F_1 + (F_2 - 1)/G_1 + (F_3 - 1)/G_1 G_2$$

This is known as **FRISS' FORMULA.**

- ❑ There are two particular situations where a low noise , front end amplifier is employed to reduce the noise. One of these is in satellite receiving systems.
- ❑ The other is in radio receivers used to pick up weak signals such as short wave receivers.
- ❑ In most receivers , a stage known as the *mixer stage* is employed to change the frequency of the incoming signal , and it is known that the mixer stages have notoriously high noise factors. By inserting an RF amplifier ahead of the mixer , the effect of the mixer noise can be reduced to negligible levels. This is illustrated in following example.

## ❖ Example

A mixer stage has a noise figure of 20dB and this is preceded by an amplifier that has a noise figure of 9 dB and an available power gain of 15dB. Calculate overall noise figure referred to the input .

Sol. It is necessary to convert all decibel values to the equivalent power ratios :

$$F_2 = 20\text{dB} = 100:1 \text{ power ratio}$$

$$F_1 = 9\text{dB} = 7.94:1 \text{ power ratio}$$

$$G_1 = 15\text{dB} = 31.62:1 \text{ power ratio}$$

$$F = F_1 + (F_2 - 1)/ G_1$$

$$= 7.94 + (100-1)/31.62$$

$$= \mathbf{11.07}$$

This is overall noise factor. The overall noise figure is

$$(F)\text{dB} = 10 \log 11.07$$

$$= \mathbf{10.44\text{dB}}$$

## NOISE TEMPERATURE

- ❑ The concept of noise temperature is based on available noise power equation

$$P_n = kT_a B_n$$

Here the subscript has been included to indicate the noise temperature is associated only with available noise power.

- ❑ In general,  $T_a$  will not be same as that physical temperature of the noise source. As an example, an antenna pointed at deep space will pick up a small amount of cosmic noise. The equivalent noise temperature of antenna that represents this noise power may be a few tens of kelvins, well below the physical ambient temperature of the antenna. If the antenna is directly pointed at the sun, the received noise power increases enormously and the corresponding equivalent noise temperature is well above the ambient temperature.
- ❑ When the concept is applied to an amplifier, it relates to equivalent noise of the amplifier referred to the input. If the amplifier noise referred to the input is denoted by  $P_{na}$ , the equivalent noise temperature of the amplifier referred to the input is

$$T_e = P_{na} / kB_n \quad (3)$$

□ We know equivalent input power for an amplifier is given in terms of its noise factor by

$$P_{na} = (F-1)kT_o B_n$$

putting this in equation (3)

we get equivalent noise temperature of the amplifier as

$$T_e = (F-1)/T_a$$

This shows the proportionality between  $T_e$  and F.

□ In practice it is found that noise temperature is the better measure for low noise devices , such as low noise amplifiers used in satellite receiving systems while noise factor is a better measure for the main receiving system.

➤ Friss's formula can be expressed in terms of equivalent noise temperatures. Denoting by  $T_e$  the overall noise of the cascaded system referred to the input , and by  $T_{e1}, T_{e2}$ , and so on , the noise temperatures of the individual stages , the in Friss's formula is easily rearranged to give

$$T_e = T_{e1} + T_{e2}/G_1 + T_{e3}/G_1 G_2 + \dots$$

Q. A receiver has a noise figure of 12dB and it is fed by a low noise amplifier that has gain of 50dB and a noise temperature of 90 K. calculate the noise temperature of the receiver and the overall noise temperature of the receiving system.

SOL. 12dB represents a power ratio of 15.85 : 1. Hence

$$T_{\text{em}} = (15.85 - 1) \times 290 = 4306 \text{ K}$$

The 50dB gain represents a power ratio of  $10^5$  : 1 . Hence

$$\begin{aligned} T_e &= 90 + 4306 / 10^5 \\ &= 90 \text{ K} \end{aligned}$$

This example shows the relatively high noise temperature of the receiver , which clearly cannot be its physical temperature. It also shows how the low noise amplifier controls the noise temperature of the overall receiving system.

# RECEIVERS

Radio receiver is an electronic equipment which pick ups the desired signal, reject the unwanted signal and demodulate the carrier signal to get back the original modulating signal.



# **Function of Radio Receivers**

- Intercept the incoming modulated signal
- Select desired signal and reject unwanted signals
- Amplify selected R.F signal
- Detect modulated signal to get back original modulating signal
- Amplify modulating frequency signal

# Design of Receiver

- The radio receiver has to be cost effective
- Requirements:
  - Has to work according to application as for AM or FM signals
  - Tune to and amplify desired radio station
  - Filter out all other stations
  - Demodulator has to work with all radio stations regardless of carrier frequency

# Classification of Radio Receivers

*Depending upon application*

- AM Receivers - receive broadcast of speech or music from AM transmitters which operate on long wave, medium wave or short wave bands.
- FM Receivers – receive broadcast programs from FM transmitters which operate in VHF or UHF bands.

- Communication Receivers - used for reception of telegraph and short wave telephone signals.
- Television Receivers - used to receive television broadcast in VHF or UHF bands.
- Radar Receivers – used to receive radio detection and ranging signals.

*Depending upon fundamental aspects*

- Tuned Radio Frequency (TRF) Receivers
- Super-heterodyne Receivers

# RECEIVERS

## Tuned Radio Frequency (TRF) Receiver:

- Composed of RF amplifiers and detectors.
- No frequency conversion
- It is not often used.
- Difficult to design tunable RF stages.
- Difficult to obtain high gain RF amplifiers

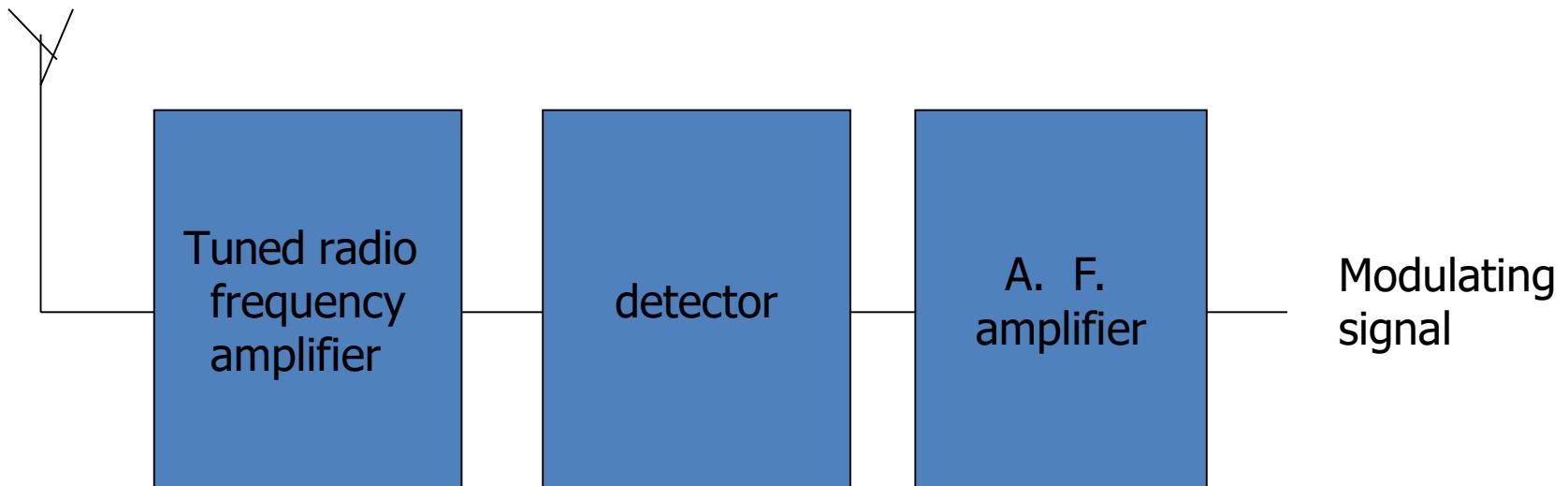
## Super-hetrodyne Receiver

- Downconvert RF signal to lower IF frequency
- Main amplification takes place at IF

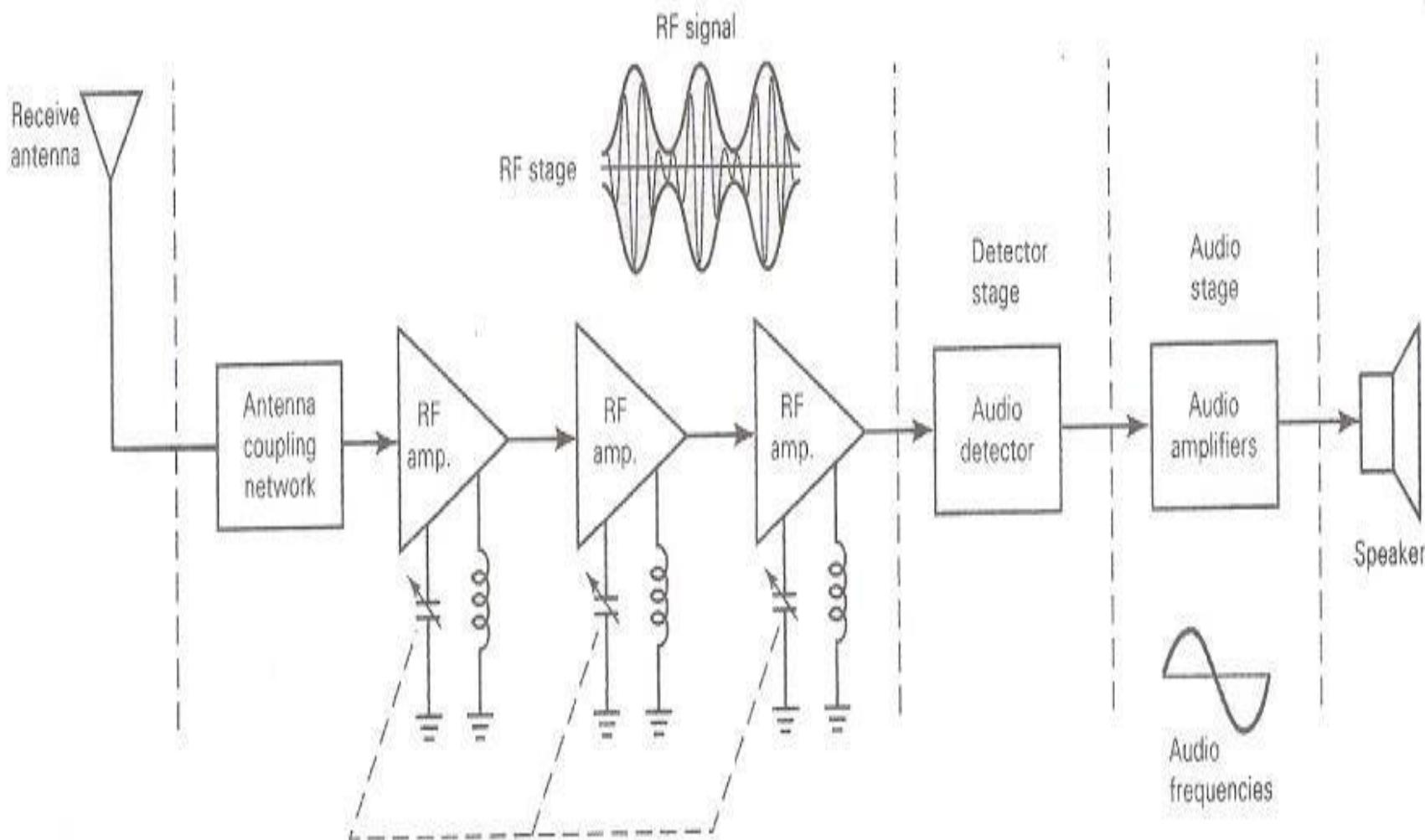
## Communication Receiver

- Downconvert RF signal to two IF frequency

# TRF (Tuned Radio frequency) RECEIVER



- TRF receiver includes an
  - RF stage
  - a detector stage
  - and an audio stage .
- Two or three RF amplifiers are required to filter and amplify the received signal to a level sufficient to drive the detector stage.



- RF section (Receiver front end)
  - used to detect the signal
  - bandlimit the received RF signal
  - and amplifying the received RF signal.
- AM detector
  - Demodulates the AM wave and converts it to the original information signal.
- Audio section
  - Used to amplify the recovered signal

# **Advantages of TRF**

- TRF receivers are simple to design and allow the broadcast frequency 535 KHz to 1640 KHz.
- High sensitivity.

# Disadvantages of TRF

- At the higher frequency, it produces difficulty in design.
- It has poor audio quality.
- Drawbacks
  - Instability
  - Variation in BW
  - Poor Selectivity

- INSTABILITY

- Due to high frequency, multi stage amplifiers are susceptible to breaking into oscillation.
- As gain of RF amplifier is very high ,a small feedback from output to input with correct phase can lead to oscillations.
- Correct phase means a positive feedback and it takes place due through stray capacitances
- As reactance of stray capacitances decreases at higher frequencies resulting in increased feedback.
- Forcing the device to work as an oscillator instead of an amplifier.

- **VARIATION IN BANDWIDTH**

- The bandwidth is inconsistent and varies with the center frequency when tuned over a wide range of input frequencies.
- As frequency increases, the bandwidth ( $f/Q$ ) increases. Thus, the selectivity of the input filter changes over any appreciable range of input frequencies.

## Example

Suppose required BW=10KHz

We have  $f_1=545\text{KHz}$ ,  $f_2=1640\text{KHz}$

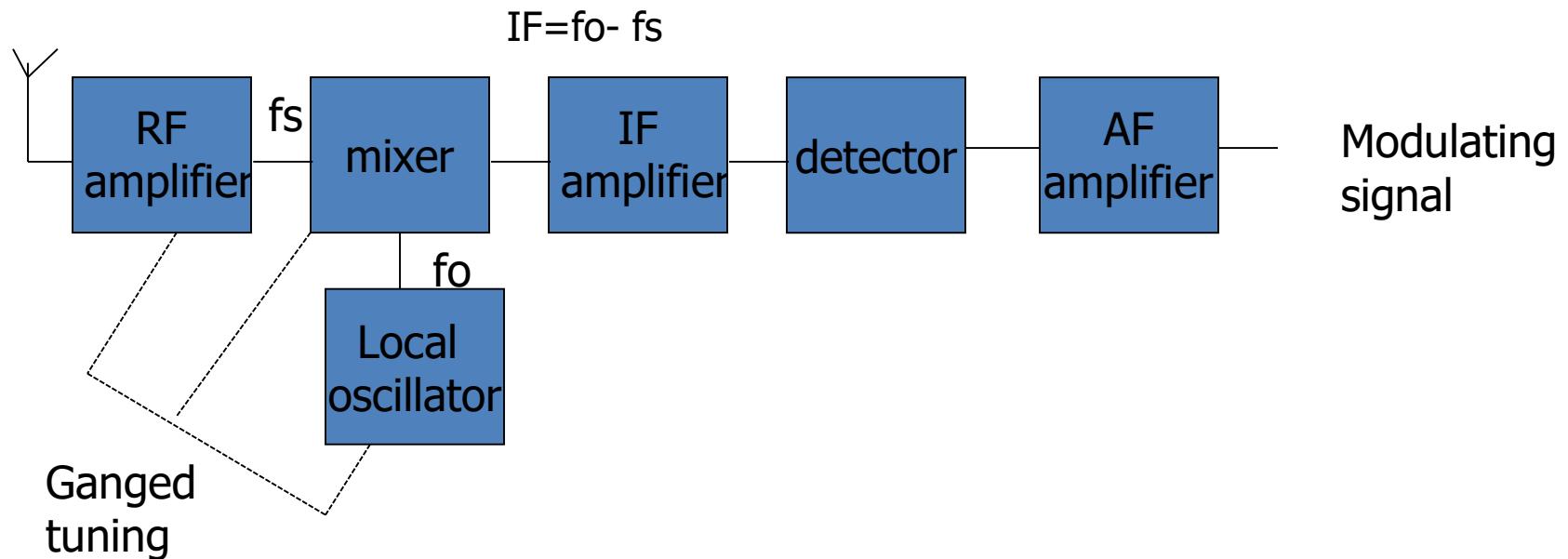
- $Q_1 = f_1/\text{BW} = 54.5$  ,
- $Q_2 = f_2/\text{BW} = 164$
- But practically Q is limited upto 120
- Considering Q limit 120 , BW changes to 13.6 KHz  
( as  $\text{BW} = f_2/Q_2 = 1640/120$ )
- So Adjacent channel is picked up resulting in variation in bandwidth.

- **POOR SELECTIVITY**
  - The gains are not uniform over a very wide frequency range.
  - Due to higher frequencies ability to select desired signal is affected.

**Due to these drawbacks TRF are rarely used.**

# SUPER HETRODYNE RECEIVER

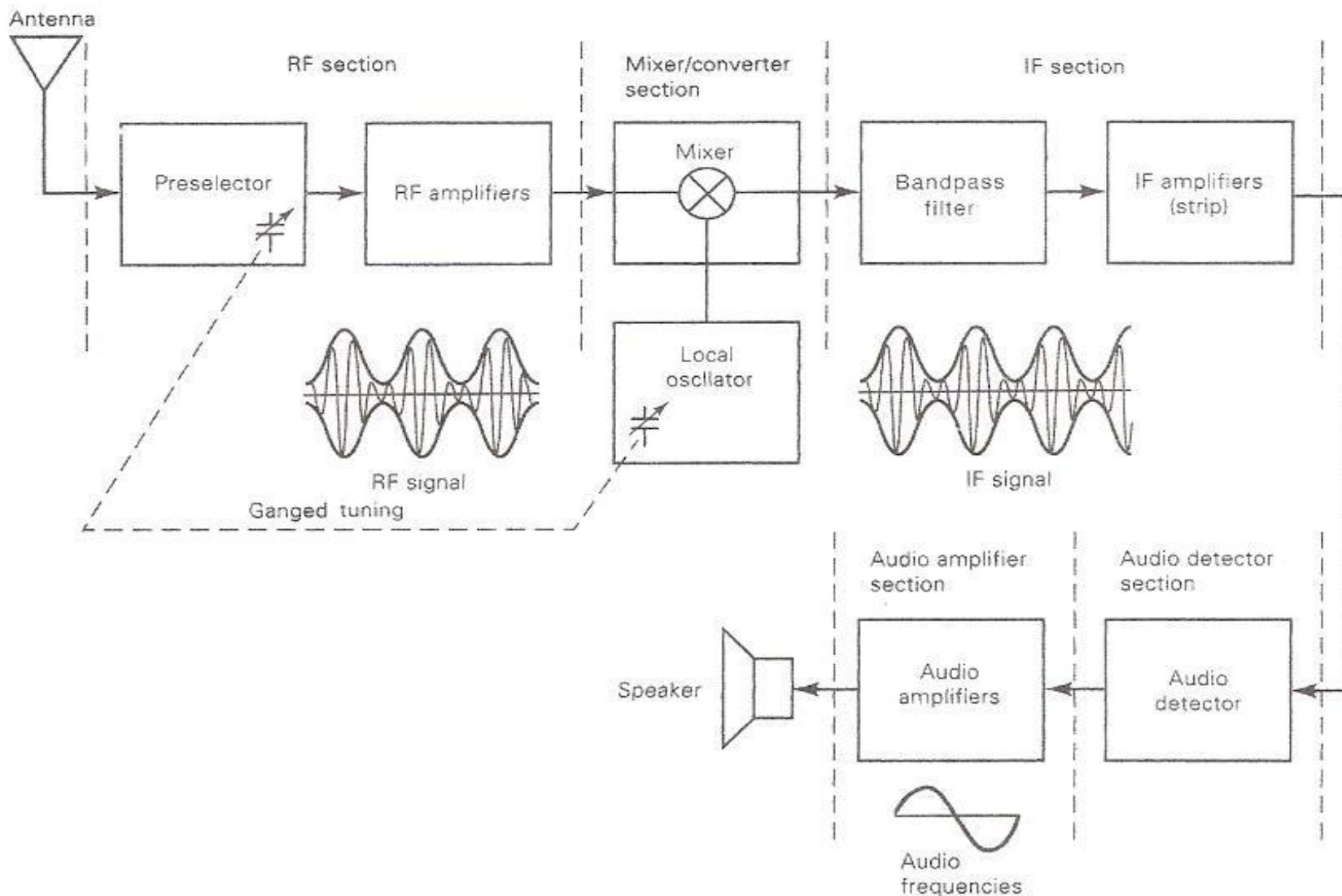
The shortcomings of the TRF receiver are overcome by the super heterodyne receiver.



- *Heterodyne – to mix two frequencies together in a nonlinear device or to transmit one frequency to another using nonlinear mixing.*
- Also known as *frequency conversion* , high frequency down converted to low frequency.(IF)
- A super heterodyne receiver converts all incoming radio frequency (RF) signals to a lower frequency known as an intermediate frequency (IF).

# DRAWBACKS OVERCOME

- Stability – as high frequency is down converted to IF the reactance of stray capacitances will not decrease as it was at higher frequencies resulting in increased feedback.
- No variation in BW- as IF range is 438 to 465 KHz (in case of AM receivers) mostly 455KHz ,appropriate for Q limit (120).
- Better selectivity- as no adjacent channels are picked due to variation in BW.



- **RF section**
  - Consists of a pre-selector and an amplifier
  - Pre-selector is a broad-tuned bandpass filter with an adjustable center frequency used to reject unwanted radio frequency and to reduce the noise bandwidth.
  - RF amplifier determines the sensitivity of the receiver and a predominant factor in determining the noise figure for the receiver.

- **Mixer/converter section**

- Consists of a radio-frequency oscillator and a mixer.
- Choice of oscillator depends on the stability and accuracy desired.
- Mixer is a nonlinear device to convert radio frequency to intermediate frequencies (i.e. heterodyning process).
- The shape of the envelope, the bandwidth and the original information contained in the envelope remains unchanged although the carrier and sideband frequencies are translated from RF to IF.

## ¶ IF section

- Consists of a series of IF amplifiers and bandpass filters to achieve most of the receiver gain and selectivity.
- The IF is always lower than the RF because it is easier and less expensive to construct high-gain, stable amplifiers for low frequency signals.
- IF amplifiers are also less likely to oscillate than their RF counterparts.

## ¶ **Detector section**

- To convert the IF signals back to the original source information (demodulation).
- Can be as simple as a single diode or as complex as a PLL or balanced demodulator.

## ¶ **Audio amplifier section**

- Comprises several cascaded audio amplifiers and one or more speakers

- **AGC ( Automatic Gain Control )**

- Adjust the IF amplifier gain according to signal level(to the average amplitude signal almost constant).
- AGC is a system by means of which the overall gain of radio receiver is varied automatically with the variations in the strength of received signals, to maintain the output constant.

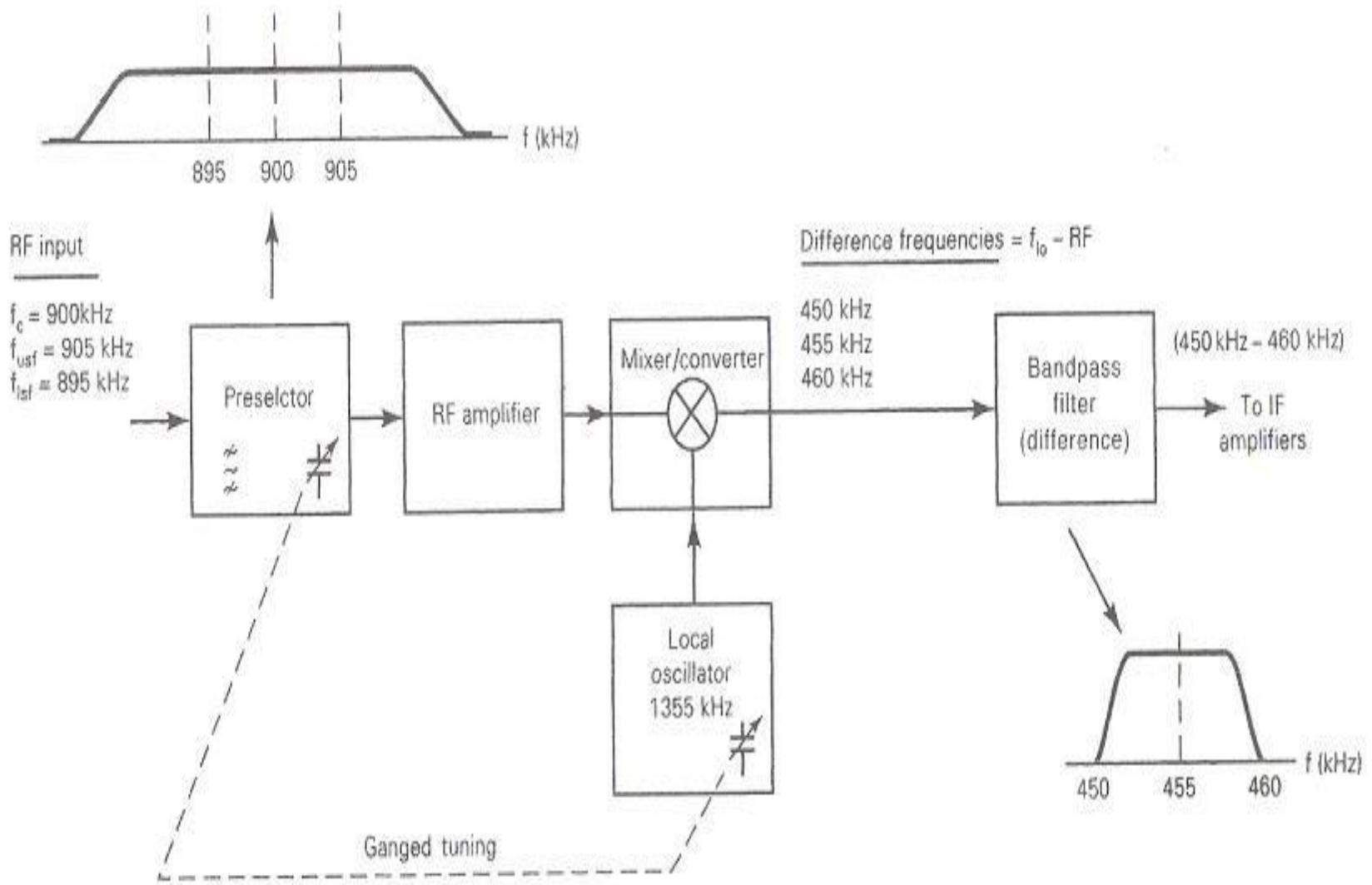
- AGC circuit is used to adjust and stabilize the frequency of local oscillator.
- Types of AGC –
  - No AGC
  - Simple AGC
  - Delayed AGC

- FREQUENCY CONVERSION in the mixer stage is identical to the frequency conversion in the modulator except that in the receiver, the frequencies are down-converted rather than up-converted.
  - In the mixer, RF signals are combined with the local oscillator frequency
  - The local oscillator is designed such that its frequency of oscillation is always above or below the desired RF carrier by an amount equal to the IF center frequency.
  - Therefore the difference of RF and oscillator frequency is always equal to the IF frequency

- The adjustment for the center frequency of the pre-selector and the local oscillator frequency are gang-tune (the two adjustments are tied together so that single adjustment will change the center frequency of the pre-selector and at the same time change the local oscillator)
- when local oscillator frequency is tuned above the RF – *high side injection*  
when local oscillator frequency is tuned below the RF – *low side injection*
- Mathematically expressed :
  - High side injection
  - Low side injection

$$f_{lo} = f_{RF} + f_{IF}$$

$$f_{lo} = f_{RF} - f_{IF}$$



# COMPARISON

## TRF Receiver

- No frequency conversion
- No IF frequency
- Instability , variation in BW and poor selectivity due to high frequencies
- Difficult to design tunable RF stages.
- Rarely used

## Super hetrodyne Receiver

- Frequency conversion
- Downconvert RF signal to lower IF frequency
- No instability, variation in BW and poor selectivity as IF introduced.
- Main amplifixcation takes place at IF
- Mostly used

# CHARACTERISTICS OF RADIO RECEIVERS

- Sensitivity
- Selectivity
- Fidelity

# Sensitivity

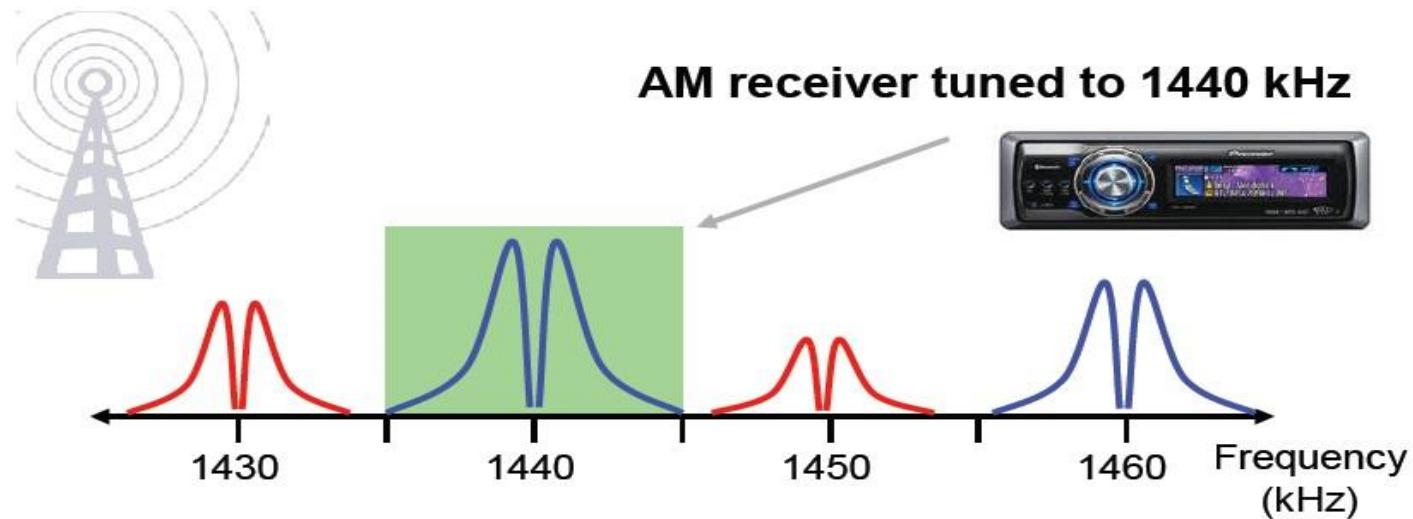
- Ability to amplify weak signals.
- Minimum RF signal level that can be detected at the input to the receiver and still produce a usable demodulated information signal.
- Broadcast receivers/ radio receivers should have reasonably high sensitivity so that it may have good response to the desired signal
- But should not have excessively high sensitivity otherwise it will pick up all undesired noise signals.
- It is function of receiver gain and measures in decibels.

- Sensitivity of a receiver is expressed in microvolts of the received signal.
- Typical sensitivity for commercial broadcast-band AM receiver is  $50 \mu\text{V}$ .
- Sensitivity of the receiver depends on :
  1. Noise power present at the input to the receiver
  2. Receiver noise figure
  3. Bandwidth improvement factor of the receiver

The best way to improve the sensitivity is to reduce the noise level.

# Selectivity

Selectivity of radio receiver is its ability to differentiate desired signal from unwanted signals.



- Selectivity is obtained by using tuned circuits, which are tuned to desired frequency. The quality factor of these LC circuits determines the selectivity. It is given by,

$$Q = XL/R$$

- For better selectivity ‘Q’ should be high.

# Fidelity

- Fidelity is defined as – *a measure of the ability of a communication system to produce an exact replica of the original source information at the output of the receiver.*
- Any variations in the demodulated signal that are not in the original information signal is considered as distortion.
- Radio receiver should have high fidelity or accuracy.
- Example- In an A.M. broadcast the maximum audio frequency is 5 KHz hence receiver with good fidelity must produce entire frequency up to 5KHz.

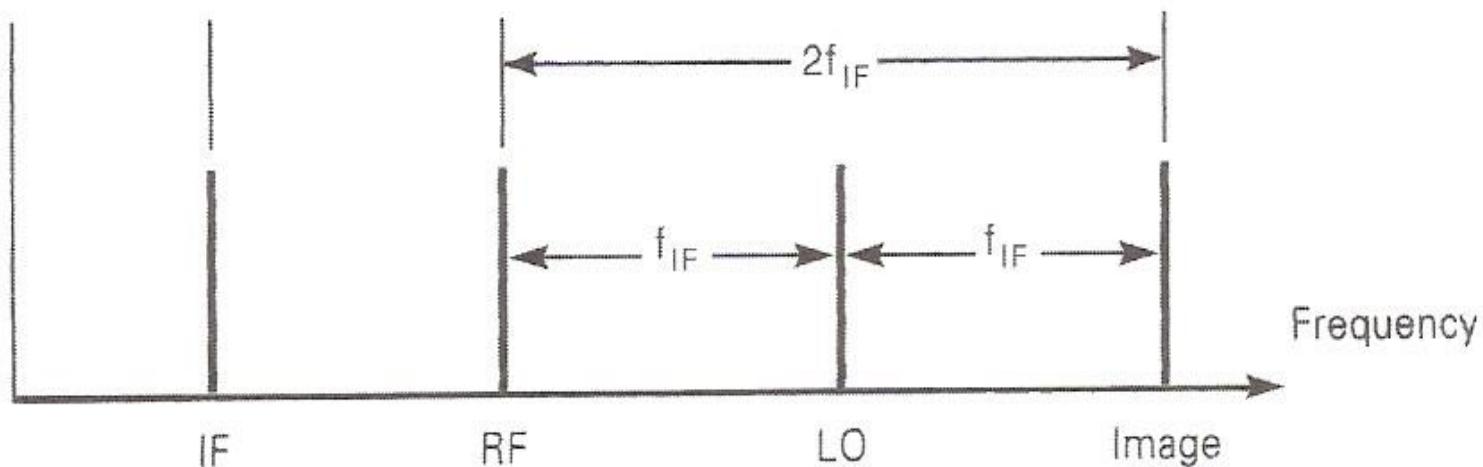
# IMAGE FREQUENCY

- In radio reception using heterodyning in the tuning process, an undesired input frequency that is capable of producing the same intermediate frequency (IF) that the desired input frequency produces.
- Image frequency – *any frequency other than the selected radio frequency carrier that will produce a cross-product frequency that is equal to the intermediate frequency if allowed to enter a receiver and mix with the local oscillator.*
- It is given by signal frequency plus twice the intermediate frequency

$$f_{si} = f_s + 2f_i$$

- It is equivalent to a second radio frequency that will produce an IF that will interfere with the IF from the desired radio frequency.
  - if the selected RF carrier and its image frequency enter a receiver at a same time, they both mix with the local oscillator frequency and produce different frequencies that are equal to the IF.
  - Consequently, two different stations are received and demodulated simultaneously

- The higher the IF, the farther away the image frequency is from the desired radio frequency. Therefore, for better image frequency rejection, a high IF is preferred.
- However, the higher the IF, it is more difficult to build a stable amplifier with high gain. i.e. there is a trade-off when selecting the IF for a radio receiver (image frequency rejection vs IF gain and stability)

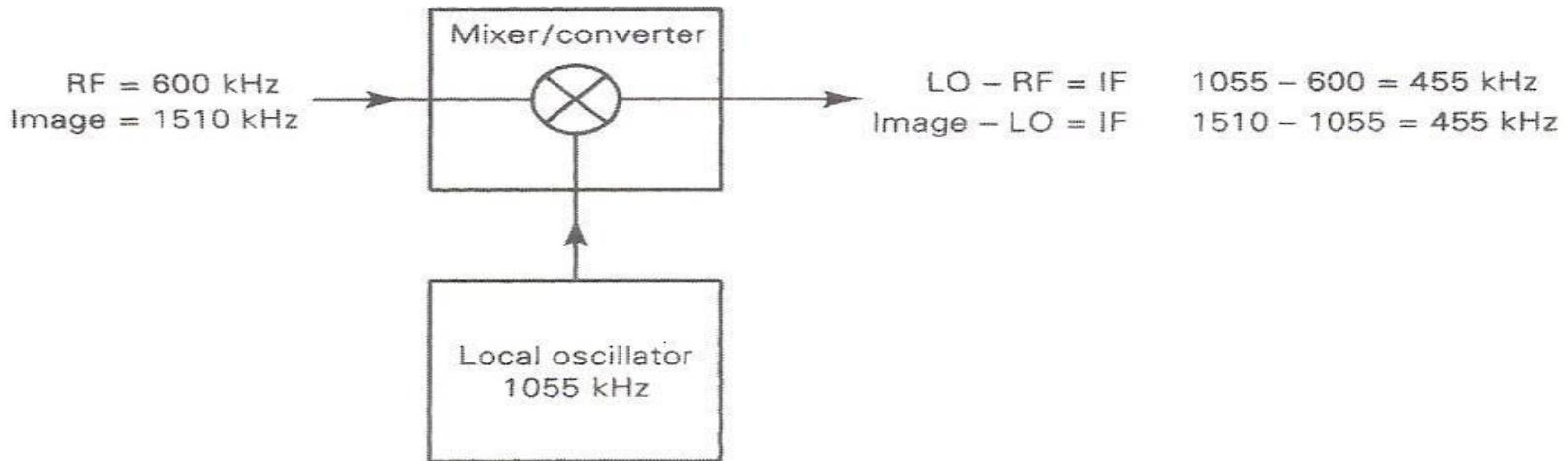


# MATHEMATICAL ANALYSIS

- Basic principle with two frequencies component  $f_1$  and  $f_2$ , we have harmonics  $f_1, f_2, f_1+f_2, f_1-f_2$
- In case of radio receivers, two frequency components are  $f_o$  and  $f_s$
- So harmonic we have  $f_o, f_s, f_o+f_s, f_o-f_s$
- Let Undesired frequency  $f_{si} (=f_o+f_{if})$  able to reach at the mixer
- So now two frequency components will be  $f_o$  (local oscillator) and  $f_{si}$  (undesired freq)
- And harmonics will be  $f_o, f_{si}, f_o+f_{si}, f_o-f_{si}$

- And harmonics will be  $f_o, f_{si}, f_o + f_{si}, f_o - f_{si}$
- Substituting value of  $f_{si}$  we have
  - $f_o, f_o + f_s, 2f_o + f_{if}, f_{if}$
- It was observed that difference component is a mirror image of IF
  - Consequently, 2 different stations are received and demodulated simultaneously

- Once an image frequency has down-converted to IF, it cannot be removed. In order to reject the image frequency, it has to be blocked prior to the mixer stage. the bandwidth of the pre-selector must be sufficiently narrow to prevent image frequency from entering the receiver.



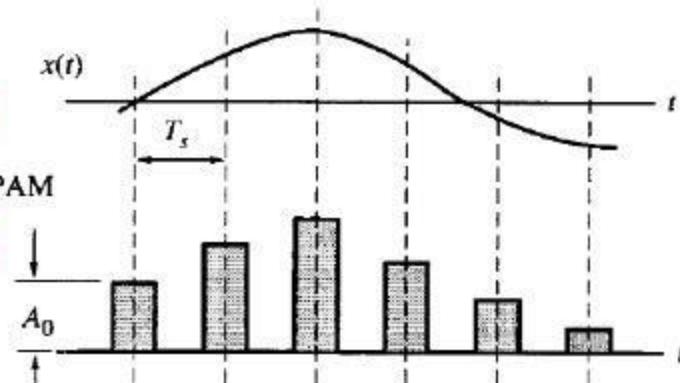
# CHOICE OF IF

- Very high IF will result in poor selectivity and poor adjacent channel rejection
- A high value of IF will result in tracking difficulties
- At low values of IF image frequency rejection is poor. Also the selectivity will be too sharp that cut off the sidebands

## **Pulse Amplitude Modulation (PAM)**

- If a message waveform is adequately described by periodic sample values, it can be transmitted using analogue pulse modulation wherein the sample values modulate the amplitude of pulse train.

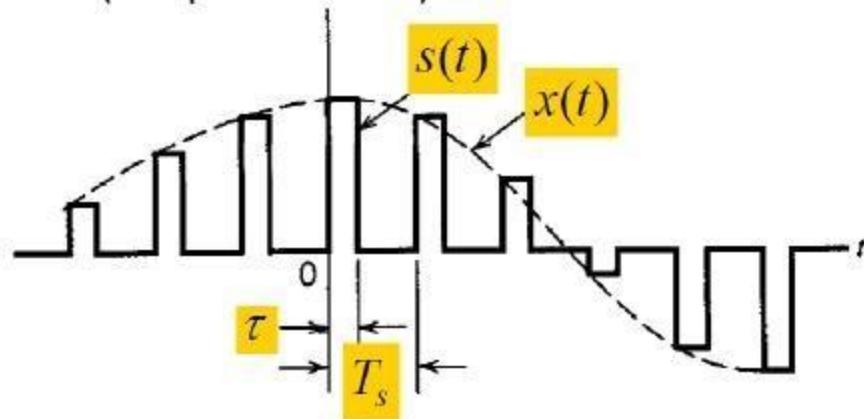
- Therefore, the **an** spaced pulses are  $\text{PAM}$  to the **correspon** a continuous mes



- This technique is termed

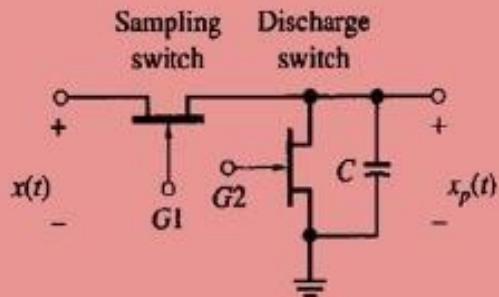
## ***Generation of the PAM signal***

- There are ***two operations*** involved in the generation of the PAM signal:
  2. ***Instantaneous sampling*** of the message signal  $x(t)$  every  $T_s$  seconds, where the sampling rate  $f_s = 1/T_s$  is chosen in accordance with the sampling theorem
  3. ***Lengthening*** the duration of each sample so obtained to some constant value  $\tau$  (sample-and-hold)



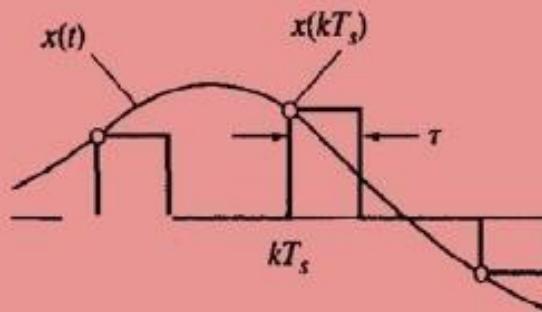
## Flat-top Sampling and PAM

- Practical method for obtaining PAM (or implementing the steps 1. and 2. is the *sample-and-hold* (S/H) technique.
- This method produces flat-top pulses



- *sample-and-hold* circuit

1. G1 closes to sample
2. C Holds value
3. G2 discharges value



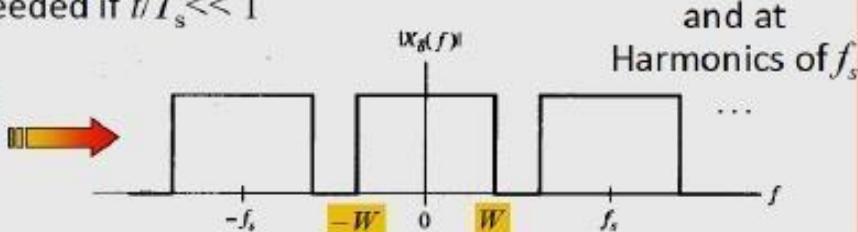
- *waveform obtained*

$$x_p(t) = \sum_k x(kT_s) p(t - kT_s)$$
$$p(t - kT_s) = p(t) * \delta(t - kT_s)$$

## PAM

- Flat-top sampling is equivalent to passing an *ideal sampled wave* through a network having *transfer function*  $P(f) = F[p(t)]$
- Loss of high frequency content is called *aperture effect*
- The larger the pulse duration or aperture  $\tau$ , the larger the effect
- Can be corrected using equalizer  $H_{eq}(f) = K e^{-j\omega t_d} / P(f)$
- No equalization is needed if  $t/T_s \ll 1$

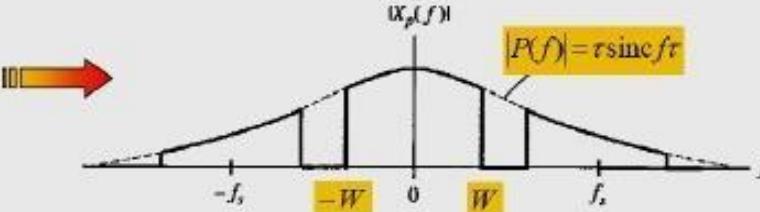
- Spectrum for ideal sampling when  
 $X(f) = \Pi(f/2W)$



- aperture effect* in flat-top sampling

$$p(t) = \Pi\left[\frac{t - \tau/2}{\tau}\right]$$

$$P(f) = \mathcal{F}[p(t)] = \tau \text{sinc} f\tau e^{-j\pi\tau f}$$



$$X_p(f) = P(f)X_\delta(f)$$

Aperture is lowpass filter

## PAM

- There are many similarities between PAM and AM CW modulation
  - Modulation index
  - Spectral impulses
  - DC block
- PAM spectrum extends from DC up through several harmonics of  $f_s$
- Required transmission bandwidth can be estimated based on time-domain considerations
- Assuming small pulse duration compared to time between pulses

$$\tau \ll T_s \leq \frac{1}{2W}$$

- Adequate pulse resolution then requires Transmission Bandwidth

$$B_T \geq \frac{1}{2\tau} \gg W$$

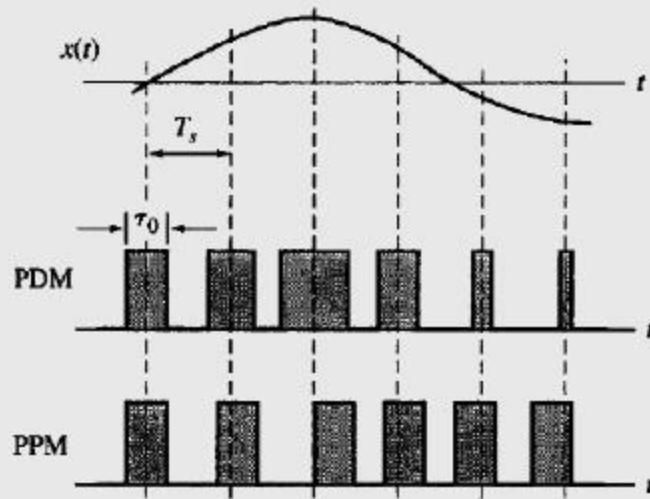
Use PAM when transmitting pulses is advantageous and BW is not an issue

## **Pulse-Time Modulation (PTM)**

- The sample values of a message can also modulate the time parameters of a pulse train:
  1. Pulse width – ***pulse-duration modulation (PDM)***
  2. Pulse position – ***pulse-position modulation (PPM)***
- The pulse width or pulse position varies in direct proportion to the sample values of  $x(t)$

## Pulse-Duration and Pulse-Position Modulation

- In both cases a **time** parameter of the pulse is being modulated
- In both cases **amplitude** remains **constant**
  - robust to nonlinear amplitude distortion (phase is important)
- Methods for producing PDM and PPM are similar



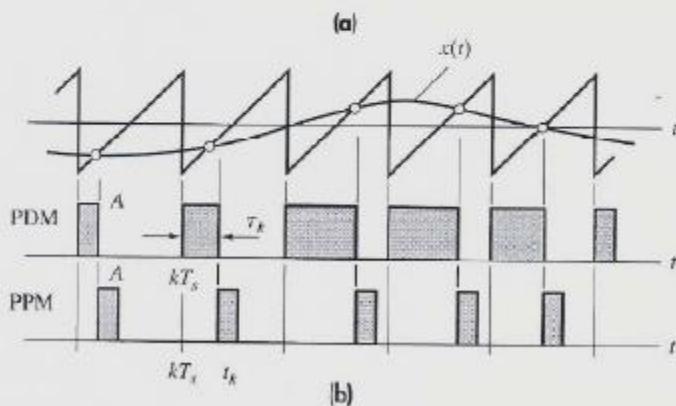
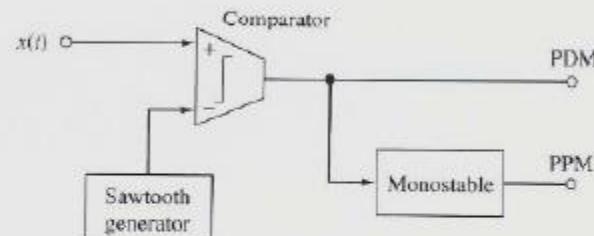
## Generation of PDM and PPM

- When  $x(t)$  exceeds the sawtooth wave (period  $T_s$ ) - comparator output is a positive constant  $A$
- Otherwise comparator output is zero

- This is an example of PDM with *trailing edge* modulation of the pulse duration

- Reverse sawtooth for *leading edge* PDM
- Triangle for both edges

- For PPM signal, the PDM signal triggers a monostable pulse generator (triggers on trailing edge and produces short pulse of fixed duration



## ***Generation of PDM***

- An *approximation* for the PDM can be formulated if we assume rectangular pulses centred at  $t = kT_s$  and assuming  $t_k$  varies slowly from pulse to pulse. Fourier Series expansion is

$$x_p(t) \approx A f_s \tau_0 [1 + \mu x(t)] + \sum_{n=1}^{\infty} \frac{2A}{\pi n} \sin n\phi(t) \cos n\omega_s t$$

- where

$$\phi(t) = \pi f_s \tau_0 [1 + \mu x(t)].$$

- PDM signal contains the message signal plus a dc component and ***phase modulated*** signal at the harmonics of  $f_s$
- Message ( $c_1$ ) recovered with LPF and DC block