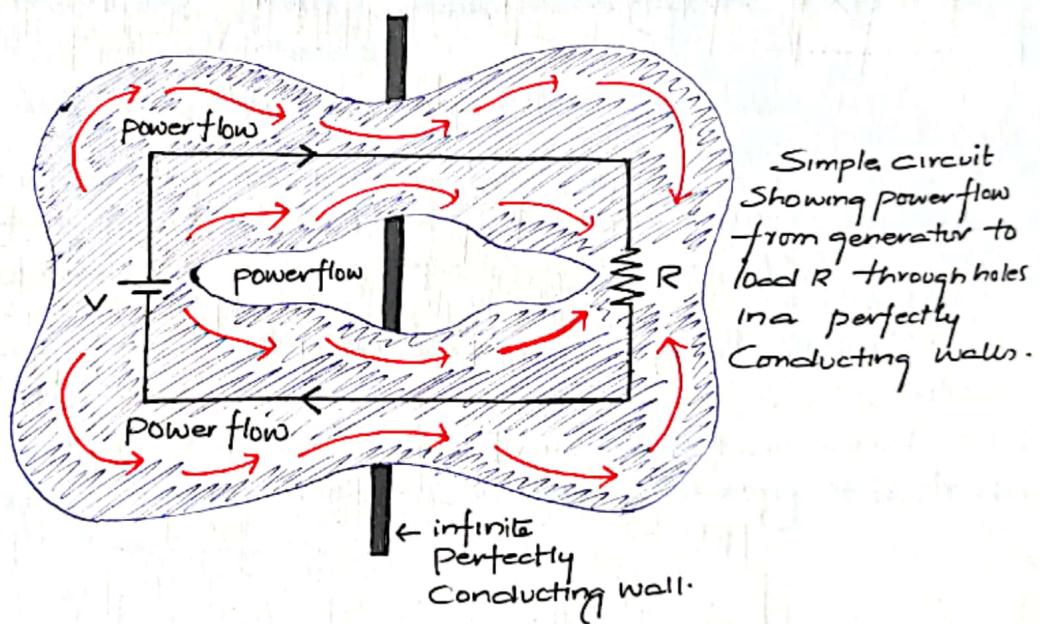


ELECTROMAGNETICS : ITS IMPORTANCE

Electromagnetics is important because it provides a real-world, three-dimensional understanding of electricity and magnetism.

By Circuit theory the battery applies a voltage v sending a current I through the wires to the load. This is a simplistic view. The energy is conveyed from the battery to the load almost entirely by electromagnetic fields external to the wires, the wires acting as guides for the energy as in the figure.

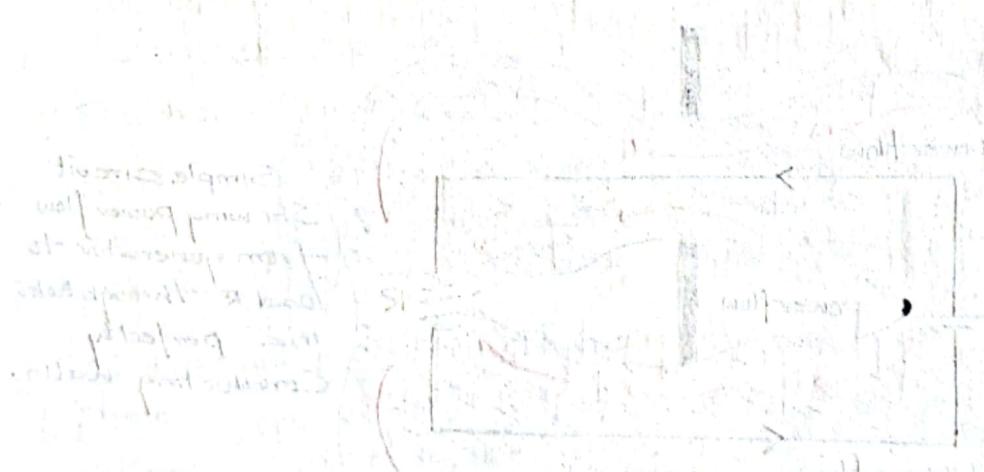


An electric field extends between the wires and a magnetic field surrounds them with alternating currents. Some energy is radiated into space. At high enough frequencies nearly all may be radiated, the circuit acting as an antenna.

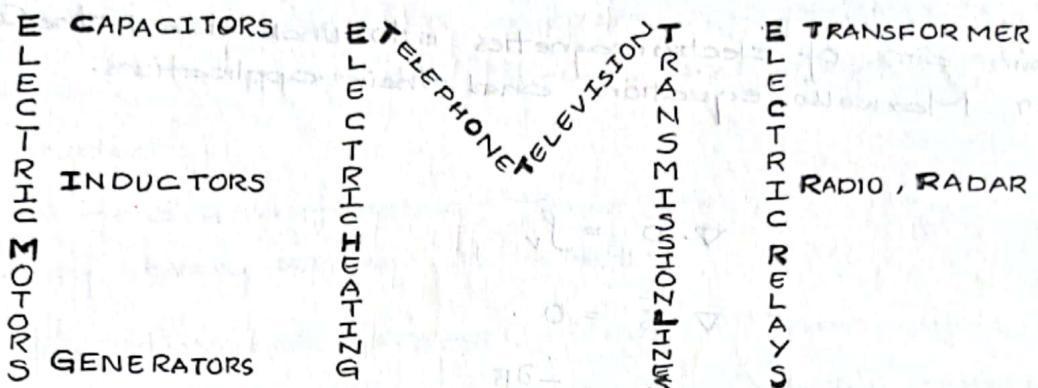
The vast grids of the earth's 50 and 60-Hz power transmission lines not only supply the world's electrical needs but their "hum" is the most powerful continuous electromagnetic signal sent out from our planet.

Many electromagnetic devices are inadvertently coupled to other systems. Video display Unit (VDU) of Computers and television sets may emit sufficient unintentional radiation that it can be picked up, decoded and the screen displays reproduced at distances of a kilometer. Any device that radiates is coupled, in principle, to the entire universe. By reciprocity, the universe is coupled to it.

This is the real world and field theory is essential to understand it. The purpose of Electromagnetism is to understand how electric and magnetic fields interact with matter.



Complete understanding of the behaviour of materials and Components as used in electrical and electronics engineering Such as



Satellite Communication etc requires knowledge of Electro magnetic fields.

Electro magnetic theory deals with electric field intensity and magnetic field intensity.

Electro magnetic theory has more Variables.

In most of EMF problems three Space Variables are involved. Hence the Solution become Complex. Vector algebra is a powerful tool for solving field problems.

Classical electro dynamics was worked out in bits and pieces by FRANKLIN, COULOMB, AMPERE, FARADAY and others, but the man who put it all together and built it into the Compact and Consistent theory it is today, was "JAMES CLERK MAXWELL"

James Clerk Maxwell (1831-1879) giving interdependence of electricity and magnetism. In fact he is Considered as the founder of electro magnetic theory.

Electro magnetics is the Study of the effects of electric charges at rest and in motion.

There are two kinds of charges: positive and negative. Both positive and negative charges are sources of an electric field.

Radar: System for detecting the direction, range or presence of objects by sending out pulses of high frequency electro magnetic waves which they reflect.

The are four fundamental vector field quantities in electromagnetism

- i. Electric field intensity, E
- ii. Electric flux density, D
- iii. Magnetic field intensity, H
- iv. Magnetic flux density, B

The main aim of electromagnetics is to understand the concept of four Maxwell's equations and their applications.

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$$

Where ∇ is a vector differential operator

ρ_v is the volume charge density

J is current density.

Electromagnetics deals with space concepts

and requires thinking in the three dimensions of real world

Hence, we must understand the three dimensional Coordinate Systems (rectangular - Cartesian, Cylindrical and Spherical)

ELECTROMAGNETIC

fields

Introduction:

Electromagnetic is a branch of physics or Electrical Engineering in which the electric and magnetic phenomena are studied.

A field is a function that specifies the particular quantity everywhere in a region.

Electromagnetic devices include Transformers, Transmissions lines, Electric relays, radio/T.V., Electric motors and radars.

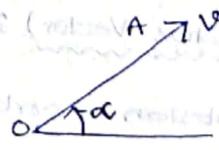
Any physical quantity may be represented either as a scalar (or) as a vector.

Scalar: Scalar is a quantity that has only magnitude.

for Eg: Time, mass, area, Speed, distance, temperature etc length, electric potential, current & energy.

Vector: It is a quantity that has both, magnitude and direction.

for Eg: Velocity, acceleration, force, displacement, force, electric field intensity etc.



The line 'OA' represents a velocity v , the length OA denoting the numerical value of velocity i.e; in m/sec and ' α ' be the orientation of the vector with reference.

The arrow suggests that the velocity of motion is directed away from the origin 'O' along OA.

Scalar field: If the quantity is scalar, the field is said to be scalar field.

Eg: Temperature of hot water in a Container.

Sound intensity in a theater.

Electric potential in a region.

Temp distribution in a building.

Vector field: If the (vector) quantity is vector, the field is said to be vector field.

Eg: Gravitational force on a body in Space.

Velocity of rain drops in the atmosphere.

Earth's magnetic field, but its magnitude is not same every where.

Unit Vector: A vector 'A' has both magnitude and direction.

A unit vector 'a_A' along 'A' is defined on a vector.

whose magnitude is unity and its direction is along A.

$$\text{i.e., } a_A = \frac{A}{|A|} = \frac{A}{A}$$

where $A = |A| a_A = A a_A$ where A is magnitude of 'A' and its (A) direction on

In Cartesian Coordinate System, a vector A may be represented

as

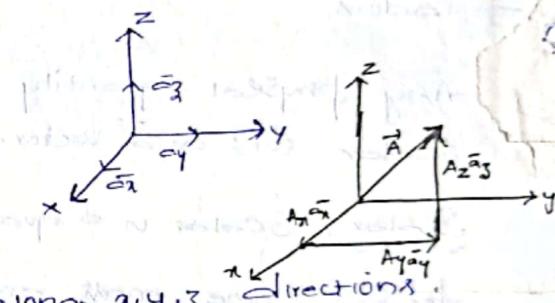
$$A = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

where A_x, A_y, A_z are the components of A along $\hat{a}_x, \hat{a}_y, \hat{a}_z$ directions.

$\hat{a}_x, \hat{a}_y, \hat{a}_z$ are unit vectors along $\hat{a}_x, \hat{a}_y, \hat{a}_z$ directions.

$$\text{where } A = |A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$a_A = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

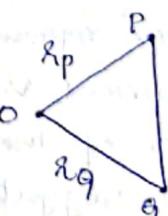


Position Vector (Radius Vector):

A point 'P' in Cartesian coordinates may be represented by (x, y, z) .

A position vector \vec{r}_P of point 'P' is on the directed distance from the origin to the point 'P'.

$$\text{i.e., } \vec{r}_P = \vec{OP} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z.$$

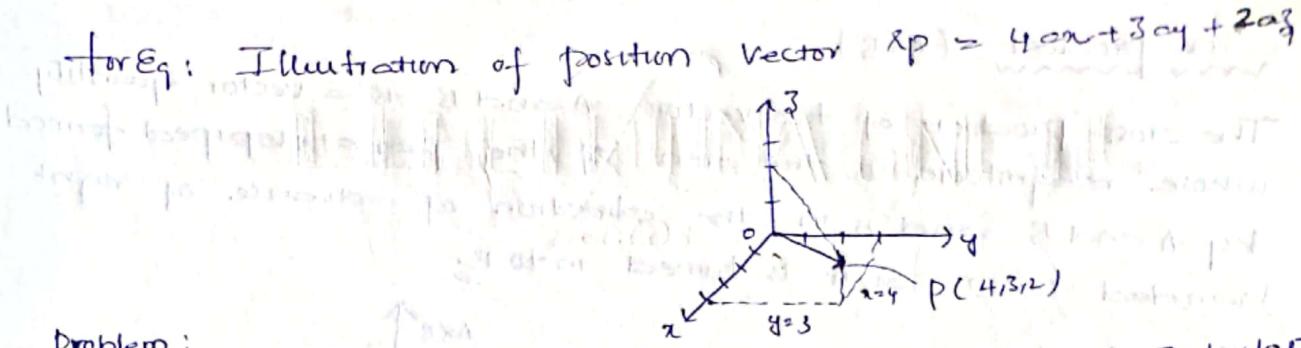


Distance Vector:

The distance vector is the displacement from one point to another point.

i.e., displacement from 'P' to 'Q'

$$\vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P$$



Problem:

Points P and Q are located at (0,2,4) and (-3,1,5). Calculate
a) position vector 'P'. b) distance vector from P to Q c) distance
between P and Q.

$$\text{a) } \vec{OP} = 0\hat{ox} + 2\hat{oy} + 4\hat{oz} = 2\hat{oy} + 4\hat{oz}$$

$$\text{b) } \vec{PQ} = \vec{Q} - \vec{P} = (-3\hat{ox} + \hat{oy} + 5\hat{oz}) - (2\hat{oy} + 4\hat{oz}) = -3\hat{ox} - \hat{oy} + \hat{oz}$$

$$\text{c) distance between P and Q is } \| \vec{PQ} \| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11} = 3.317$$

Vectors:

A vector may be represented graphically by a line with an arrow. The line has an origin and an end point. The orientation of the line and the arrow indicates the direction of a vector.

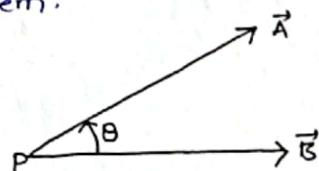
A vector
 Origin Endpoint

Dot Product: (A · B)

The dot product of two vectors A and B is defined as the product of the magnitudes of A and B and the cosine of phase angle between them.

$$A \cdot B = AB \cos \theta_{AB}$$

• It is a scalar product.



If $A = (A_x, A_y, A_z)$ and $B = (B_x, B_y, B_z)$ then

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

Note:

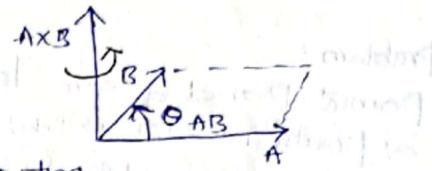
$$\hat{ox} \cdot \hat{ox} = \hat{oy} \cdot \hat{oy} = \hat{oz} \cdot \hat{oz} = 1$$

$$\hat{ox} \cdot \hat{oy} = \hat{oy} \cdot \hat{oz} = \hat{oz} \cdot \hat{ox} = 0.$$

Cross product ($\mathbf{A} \times \mathbf{B}$)

The cross product of two vectors \mathbf{A} and \mathbf{B} is a vector quantity whose magnitude is the area of the parallelopiped formed by \mathbf{A} and \mathbf{B} and is in the direction of advance of right handed screw as \mathbf{A} is turned into \mathbf{B} .

$$\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB} \hat{\mathbf{a}}_n$$



where $\hat{\mathbf{a}}_n$ is a unit vector normal to the

Plane containing \mathbf{A} and \mathbf{B} .

If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$

then $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_n & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

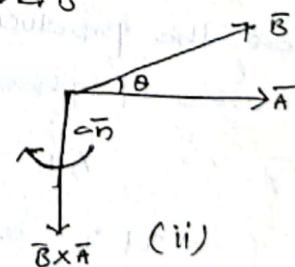
Note: $\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$, $\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$, $\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$.

- Cross product is a vector product

The cross product of two vectors $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ is another vector whose magnitude is the product of magnitude of $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ and the Sine of angle θ between $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ and whose direction is the direction of a right hand screw as it is turned from $\bar{\mathbf{A}}$ towards $\bar{\mathbf{B}}$ through angle θ as shown in figure.

$$\bar{\mathbf{A}} \times \bar{\mathbf{B}} = (\bar{A} \|\bar{B}) \sin \theta_{AB} \hat{\mathbf{a}}_n$$

where $\hat{\mathbf{a}}_n$ is a unit vector (vector of length 1) pointing perpendicular to the plane of $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$



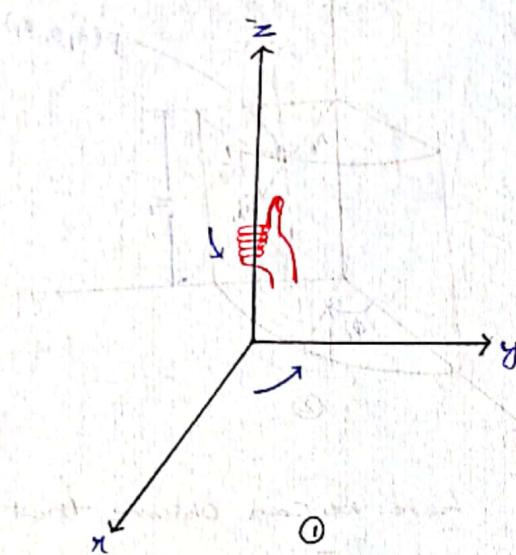
Introduction to coordinate Systems:

i) Rectangular (or) Cartesian Coordinates:

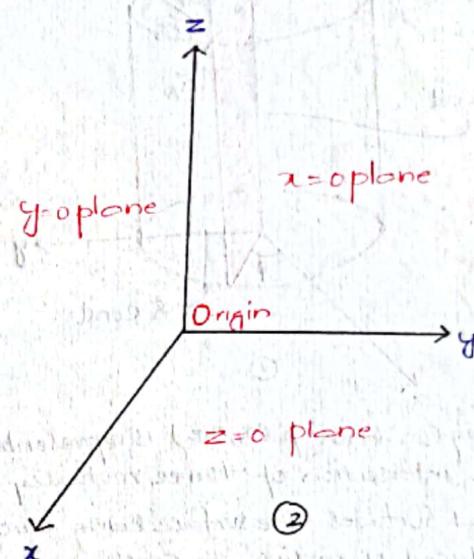
ii) Cylindrical Coordinates

iii) Spherical Coordinates.

The Cartesian Coordinate System (rectangular) (x, y, z)



The thumb pointing the direction of positive z -axis, x -axis & y -axis
In this System we set up three coordinates axes mutually at right angles to each other. It is customary to choose a right handed coordinate system in which the rotation from the axis towards the y -axis (in the direction of fingers of right hand), as thumb pointing in the direction of the z axis.



The planes $x = \text{Constant}$, $y = \text{Constant}$ and $z = \text{Constant}$, the constants being coordinate values of the point.

figure 2 shows the three orthogonal planes.

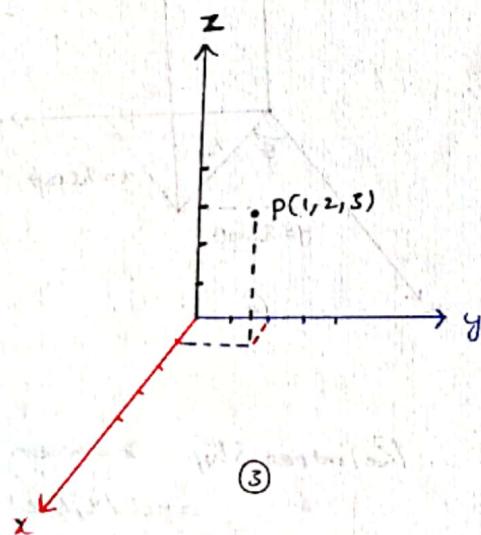


Fig 3 shows the location of point P whose coordinates are $(1, 2, 3)$. Point P is therefore located at the common point of intersection of the planes $x=1$, $y=2$ & $z=3$.

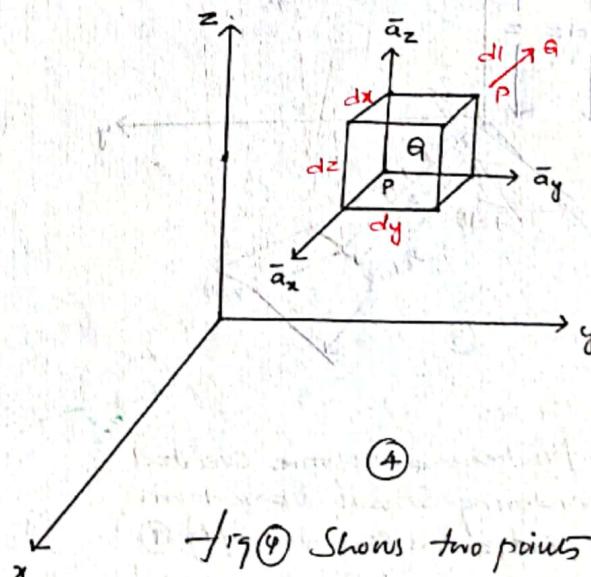
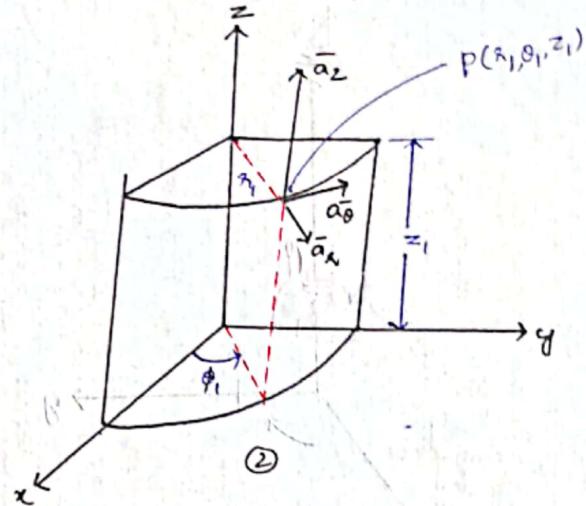
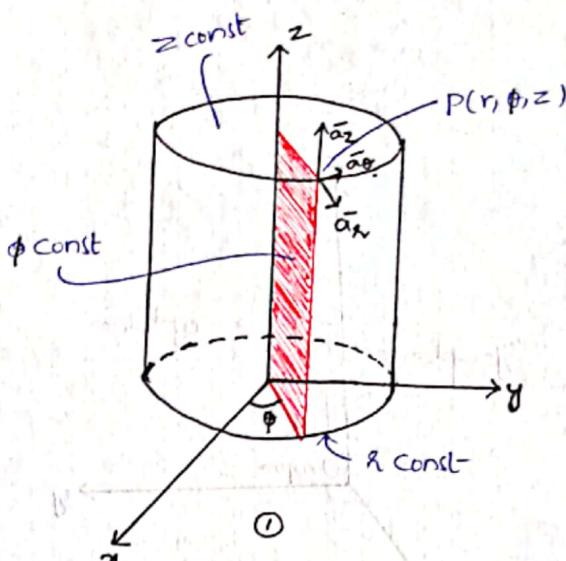


Fig 4 shows two points $P(x, y, z)$ and $Q(x+dx, y+dy, z+dz)$, where Q is obtained by incrementally increasing each coordinate from its unit at P .

The increments are shown as dx , dy and dz in the form of a small box of volume dx, dy, dz .

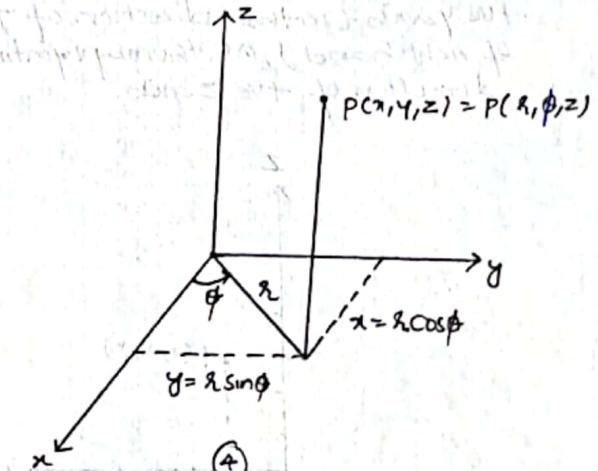
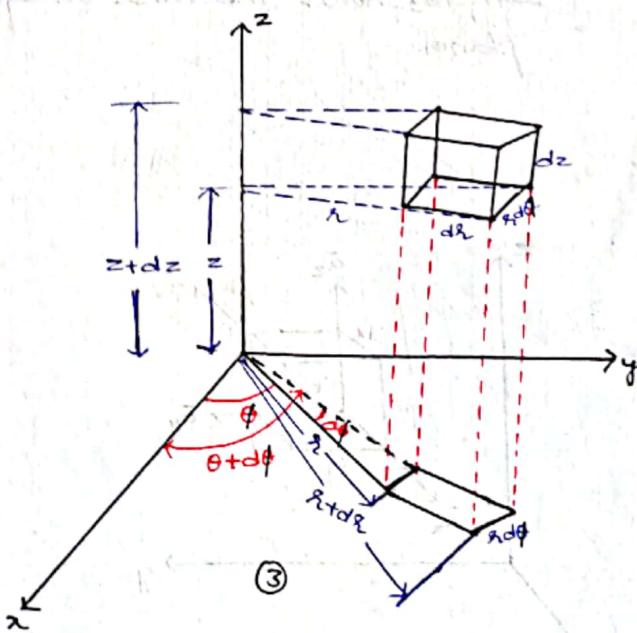
circular Cylindrical Coordinate System (r, ϕ, z)



The fig ① shows $P(r, \phi, z)$ is represented by the intersection of three mutually orthogonal Surfaces. The surface being Circular Cylindrical of radius $r = \text{constant}$ in a Cylinder, $\phi = \text{constant}$ in a plane (made by shifting xy plane by angle ϕ from $y = \text{plane}$) and $z = \text{constant}$ plane as shown.

Here we can observe that

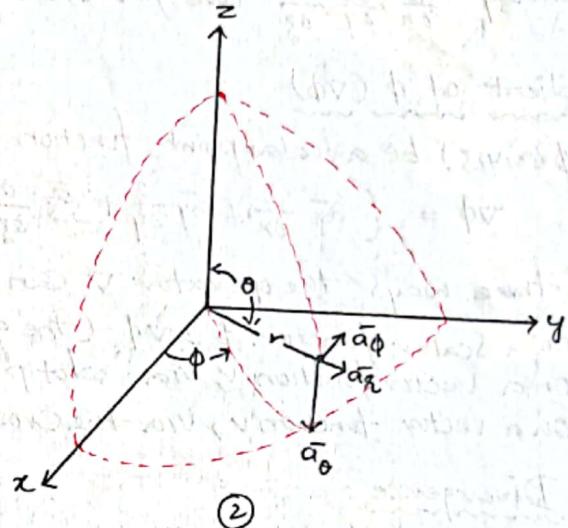
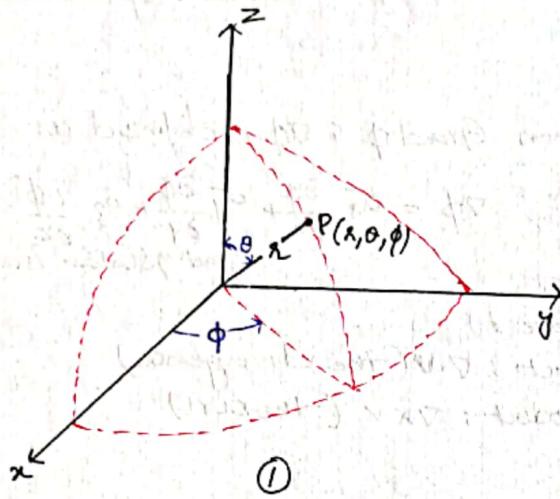
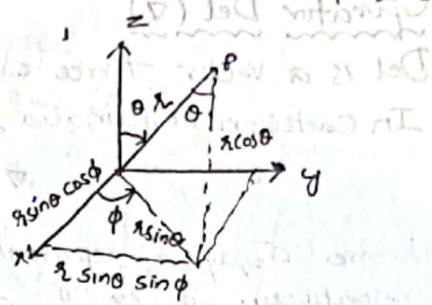
$$\bar{a}_r \times \bar{a}_\phi = \bar{a}_z$$



An infinitesimal volume created by Considering Small variations along r, ϕ, z as shown in fig ③
The differential lengths are dr , $r d\phi$ & dz .

Relationship between (x, y, z) and (r, ϕ, z)

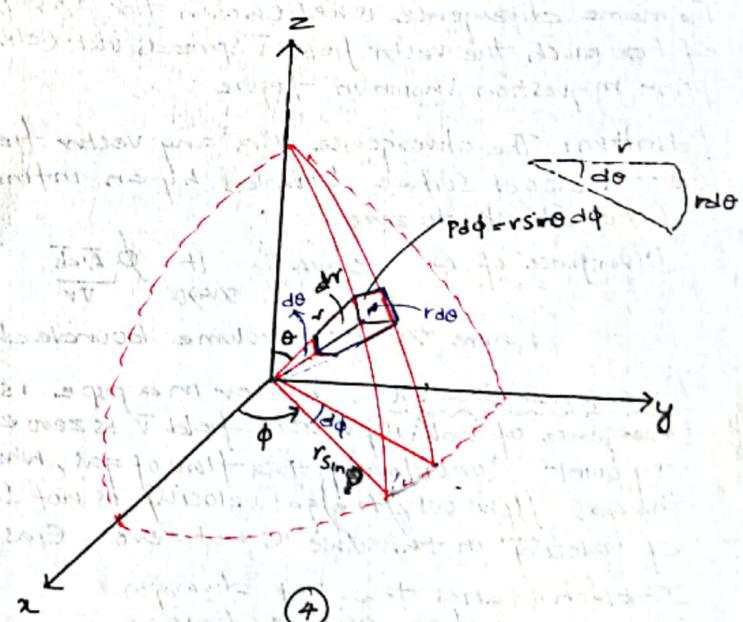
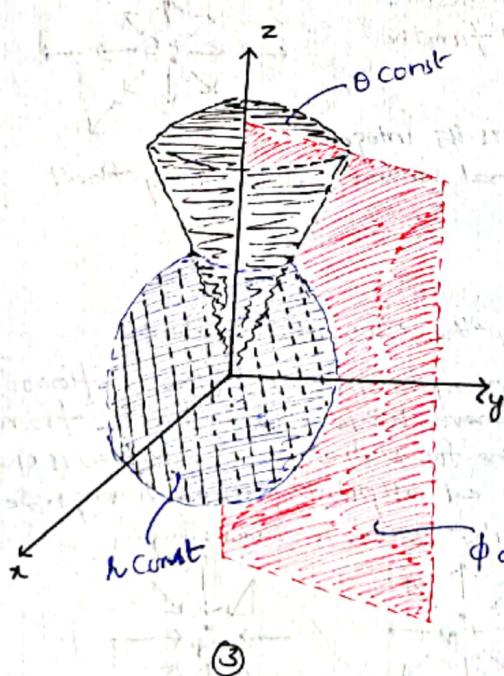
Spherical Coordinate System (r, θ, ϕ)



Spherical Coordinates

$$P(r, \theta, \phi)$$

Unit vectors in Spherical System



Constant r, θ and ϕ Surfaces

r = Constant is a sphere

θ = Constant in a cone

ϕ = Constant in a plane

The differential volume formed by considering small variation along r, θ and ϕ is shown in fig ④

The differential lengths are dr , $r d\theta$ and $r \sin \theta d\phi$

Operator Del (∇)

Del is a vector three dimensional partial differential operator or gradient
In Cartesian Coordinates, it is defined as follows

$$\nabla = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z$$

where \hat{a}_x, \hat{a}_y & \hat{a}_z represent the unit vectors along the x-axis, y-axis and z-axis respectively. $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ are first order partial differentiation operators.

Gradient of ϕ ($\nabla\phi$)

Let $\phi(x, y, z)$ be a scalar point function, then Grad ϕ & $\nabla\phi$ is defined as

$$\nabla\phi = \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z} \right) \phi; \quad \nabla\phi = \hat{a}_x \frac{\partial\phi}{\partial x} + \hat{a}_y \frac{\partial\phi}{\partial y} + \hat{a}_z \frac{\partial\phi}{\partial z}$$

grad of scalar is a vector

The three ways the operator ∇ can act

1. On a Scalar function ϕ : $\nabla\phi$ (the gradient)
2. On a vector function V , via dot product: $\nabla \cdot V$ (the divergence)
3. On a vector function V , via the cross product: $\nabla \times V$ (the curl)

The Divergence:

Let $\vec{V} = V_x \hat{a}_x + V_y \hat{a}_y + V_z \hat{a}_z$ where V_x, V_y & V_z are function of x, y & z by vector-point functions.

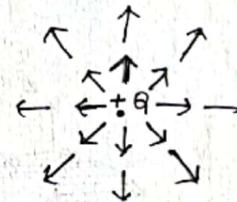
The divergence of \vec{V} is denoted by $\nabla \cdot \vec{V}$ or $\text{div } \vec{V}$ is defined as

$$\nabla \cdot \vec{V} = \left(\frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z \right) \cdot (V_x \hat{a}_x + V_y \hat{a}_y + V_z \hat{a}_z) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Observe that the divergence of a vector function \vec{V} is itself a scalar

Geometrical interpretation:

The name divergence is well-chosen for $\nabla \cdot \vec{V}$ is a measure of how much the vector field \vec{V} spreads out (diverges) from the point in question shown in figure



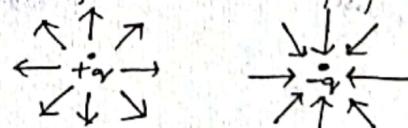
Definition: The divergence of a any vector field A is its integral over the closed surface bounded by an infinitesimal volume divided by that volume shrinks to zero.

$$\text{Divergence of } A \quad \text{div } A = \lim_{\Delta V \rightarrow 0} \frac{\oint_A \vec{A} \cdot d\vec{s}}{\Delta V}$$

where ΔV is the volume bounded by the closed surfaces.

Divergence example: Water in a pipe is in compressible i.e., flow-in = flow-out
Divergence of velocity vector field \vec{V} is zero everywhere. Water can't diverge from any point. Considering the flow of gas, when the top valve of a cylinder is opened the gas flows out, the gas velocity is not same at all points, the divergence of velocity in this case is not zero. Gas velocity vector is diverging.

In electrostatics the E-field diverges (spread out) from positive charge and converges (negative divergence) at a negative charge. Shown →



The Curl

$$\text{curl } \vec{V} \text{ or } \nabla \times \vec{V}, \quad \vec{V} = V_x \hat{a}_x + V_y \hat{a}_y + V_z \hat{a}_z$$

$$\text{curl } \vec{V} = \nabla \times \vec{V} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

We may describe curl as circulation per unit area

mathematical form of definition

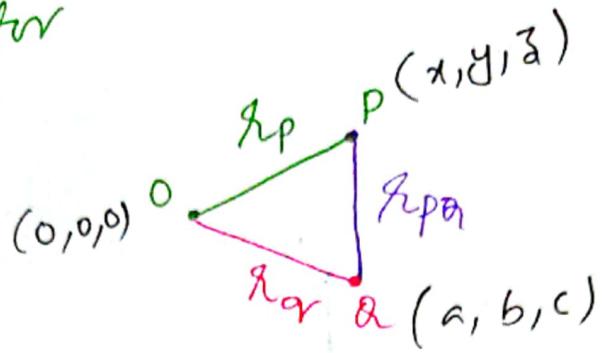
$$\text{curl } \vec{V} = \nabla \times \vec{V} = \iint \frac{\vec{V} - \vec{V}_0}{AS} \cdot d\vec{l}$$

where AS is planar area enclosed by the closed line integral,

Interpretation

- Curl deals with rotation.
- Consider a water whirlpool, which is down wards. If a leaf is thrown into the pool it rotates and moves in downward direction.
- In this case direction of curl of vel vector is downward.
- Consider wind whirlpool, which is upwards. If a piece of paper is released here, it rotates and finally moves upwards.

Position Vector



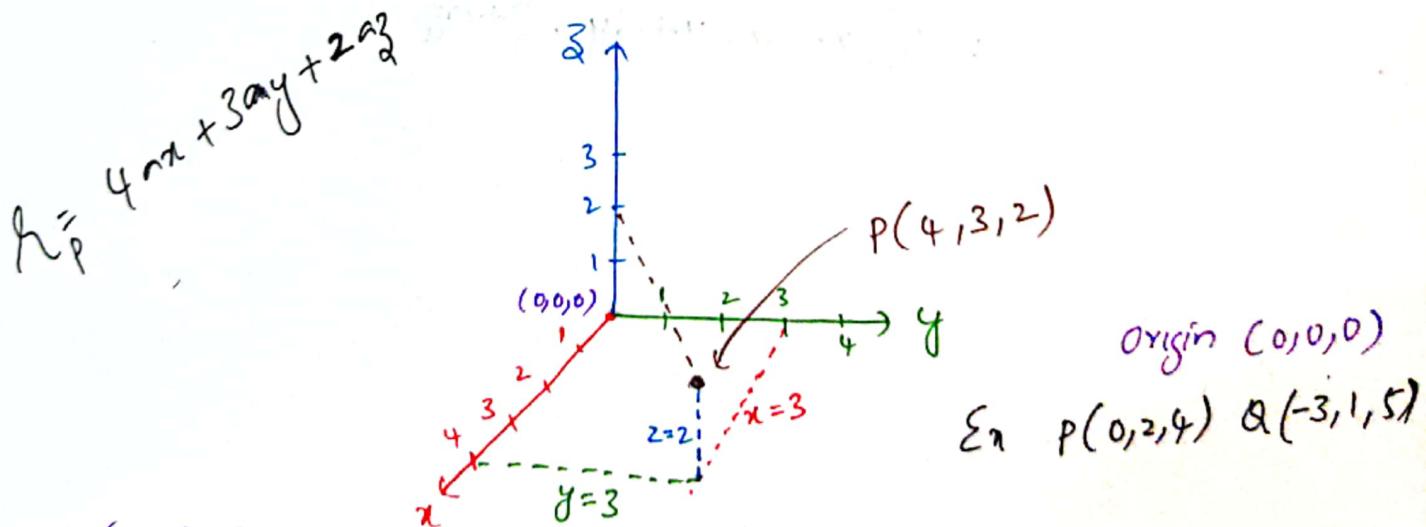
position vector r_p

$$\begin{aligned} r_p = \vec{OP} &= (x-0)\hat{a}_x + (y-0)\hat{a}_y + (z-0)\hat{a}_z \\ &= x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \end{aligned}$$

Distance Vector

$$\begin{aligned} r_{PA} &= \vec{r}_Q - \vec{r}_P \\ &= (a-x)\hat{a}_x + (b-y)\hat{a}_y + (c-z)\hat{a}_z \end{aligned}$$

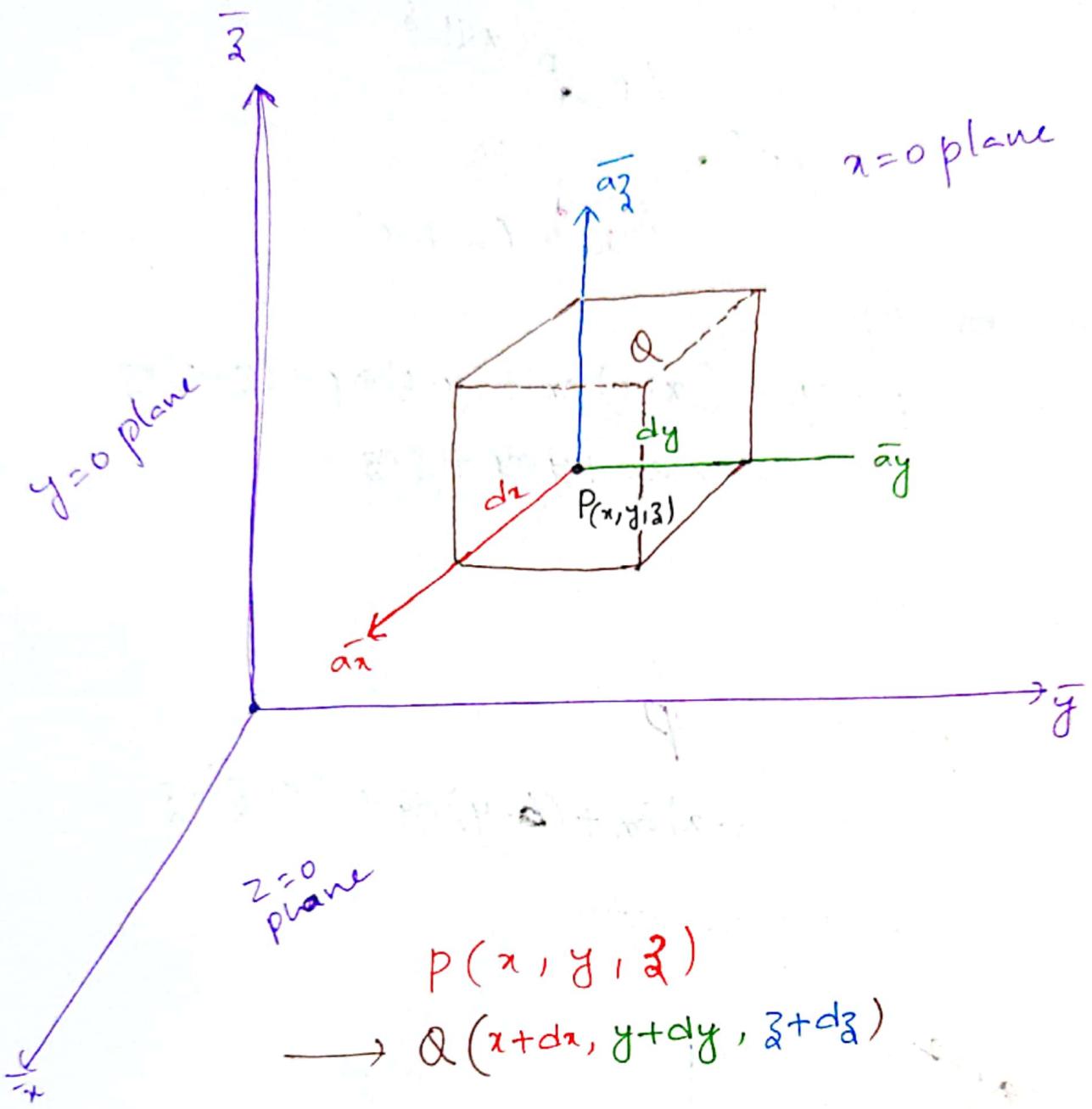
Illustration of position vector



$$\underline{\text{Ex}} \quad (0,0,0) - (0,2,4) \quad r_p = (0-0)\hat{a}_x + (2-0)\hat{a}_y + (4-0)\hat{a}_z = 0\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z = 2\hat{a}_y + 4\hat{a}_z$$

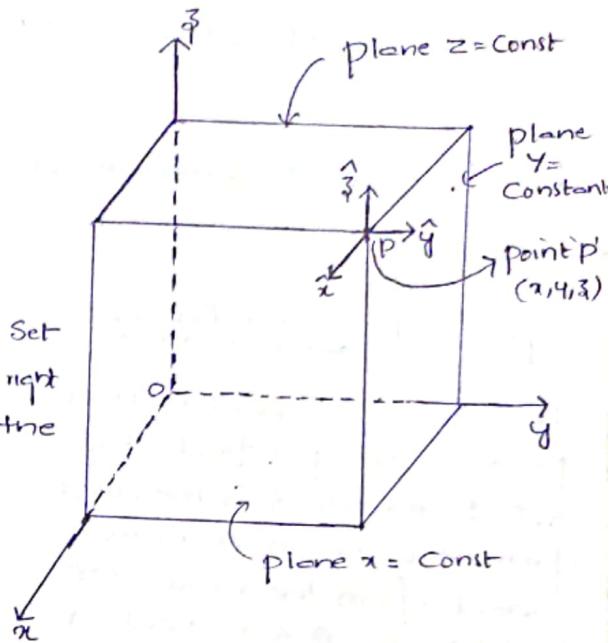
$$r_{PQ} = \vec{r}_Q - \vec{r}_P = (-3-0)\hat{a}_x + (1-2)\hat{a}_y + (5-4)\hat{a}_z = -3\hat{a}_x + \hat{a}_y + \hat{a}_z$$

$$\|r_{PQ}\| = \sqrt{(-3)^2 + (-1)^2 + 1^2} = \sqrt{11} = 3.317$$



$$P(x, y, z) \rightarrow Q(x+dx, y+dy, z+dz)$$

In rectangular coordinates a point 'P' is specified by x, y, z . where these values are all measured from the origin. A vector at the point 'P' is specified in terms of 3-mutually perpendicular components with unit vectors $\hat{x}, \hat{y}, \hat{z}$ form a right handed set i.e; turning \hat{x} into \hat{y} like a right-handed screw we move in the \hat{z} -direction.



Differential displacement $dl = dx \hat{x} + dy \hat{y} + dz \hat{z}$

An infinite length in the rectangular system $dl = \sqrt{dx^2 + dy^2 + dz^2}$

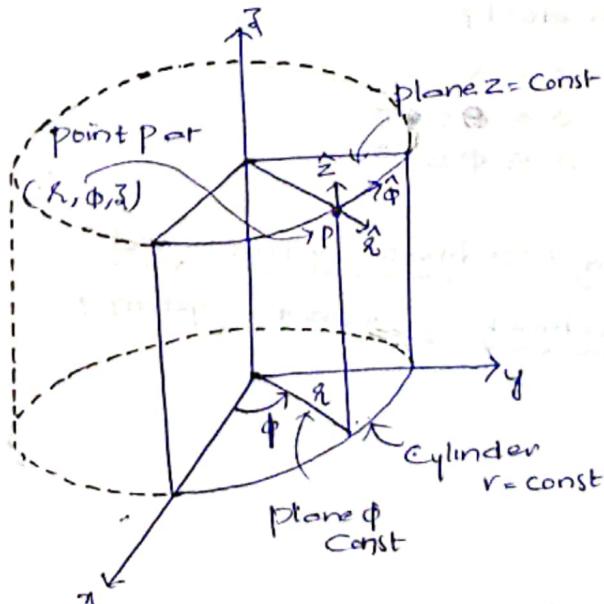
The magnitude of incremental displacement is differential area $ds = \hat{x} dy dz$
or Surface $= \hat{z} dx dy$
 $= \hat{y} dx dz$.

The infinitesimal Volume $dV = dx dy dz$

The ranges of the coordinate system are

$$\begin{array}{ll} 0 \leq x \leq \infty & -\infty < x < \infty \\ 0 \leq y \leq \infty & -\infty < y < \infty \\ 0 \leq z \leq \infty & -\infty < z < \infty \end{array}$$

Cylindrical Coordinates: (r, ϕ, z)



In cylindrical coordinates a point P is specified by r, ϕ, z where ϕ is measured from the x-axis (or) x-z plane. A vector at the point 'P' is specified in terms of 3-mutually perpendicular components with the unit vectors. \hat{r} \perp to cylinder of radius r , $\hat{\phi}$ \perp to the plane through the z-axis at an angle ϕ and \hat{z} \perp to the x-plane at distance z . The unit vectors $\hat{r}, \hat{\phi}, \hat{z}$ form a right handed set.

differential displacement $dl = dr\hat{r} + r d\theta \hat{\theta} + dz\hat{z}$

Infinite length $dl = \sqrt{dr^2 + r^2 d\theta^2 + dz^2}$

Volume $dv = r dr d\theta dz$

Differential area $ds = r d\lambda d\phi \hat{r}$
 $= r d\phi dz \hat{r}$
 $= r dz d\phi \hat{r}$

Ranges of Coordinates

$$0 \leq r \leq \infty$$

$$0 \leq \theta \leq \pi$$

$$-\pi \leq \phi \leq \pi$$

Spherical Coordinates: (r, θ, ϕ)

In Spherical Coordinates a point 'P' is specified by r, θ, ϕ where r is measured from the origin 'O' is measured from the x -axis (on) $X-Z$ plane, θ is called as Zenith (or) Azimuth angle.

A vector at the point P is specified in terms of 3 mutually \perp components

with unit vectors \hat{r} \perp to Sphere of radius r , $\hat{\theta}$ \perp to the Cone of angle θ and $\hat{\phi}$ \perp to the plane through the z -axis

at angle ϕ . The magnitude of incremental displacement is

$$dl = \sqrt{dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2}$$

diff displ $dl = dr\hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$

diff area $ds = r dr d\theta d\phi$
 $= r^2 \sin \theta d\theta d\phi \hat{r}$
 $= r \sin \theta d\phi dr$

diff vol $dv = dr r d\theta r \sin \theta d\phi$
 $= r^2 \sin \theta dr d\theta d\phi$

Ranges of Coordinates

$$0 \leq r \leq \infty$$

$$0 \leq \theta \leq \pi$$

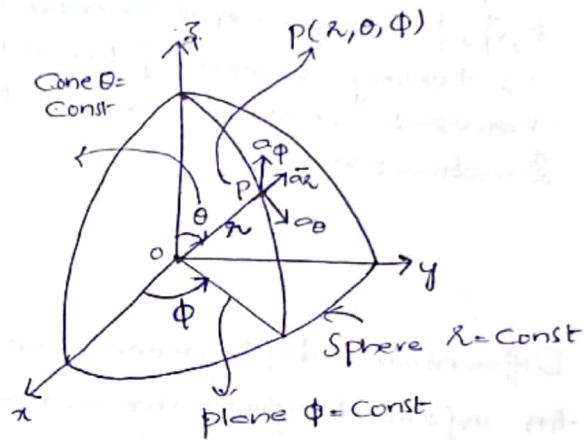
$$0 \leq \phi \leq 2\pi$$

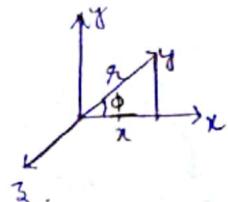
Relation between Cartesian coordinate System and Cylindrical Coordinate System:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$





$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = ?$$

Relation between C.C.S and S.C.S:

$$x = r \sin \theta \cos \phi$$

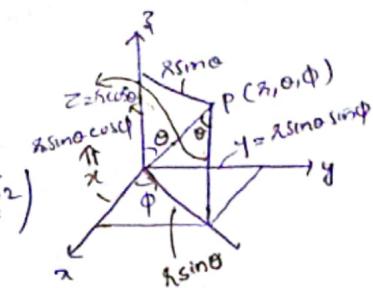
$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$$

$$\phi = \tan^{-1}(y/x)$$



Point charge:

It is the charge whose volume is very small compared with distances under consideration.

The point charges are assumed to exist at isolated points in space.

Line charge:

It is the charge whose surface area is very small when compared with its length.

where σ_L = line charge distribution in Coulomb/m

If the line charge distribution is uniform then the line charge

$$\sigma_L \text{ is } \sigma_L = Q/L \text{ C/m}$$

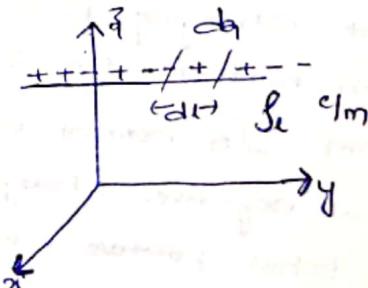
Q = total charge contained along the line

L = length of the line.

If the charge distribution is not uniform as shown in below fig

Consider a small differential element of length (dl) having differential charge dq then

$$\sigma_L = \lim_{\Delta l \rightarrow 0} \frac{\delta Q}{\delta l} = \frac{dq}{dl}$$



$$\text{Total charge Contained } Q = \int \sigma_L dl$$

Surface Charge:

It is a charge having cylindrical form in 3rd dimension (thickness) of which is very small when compared to its area.

Consider a surface area $s \text{ m}^2$ having a charge of q . Then Surface charge density σ_s is

$$\sigma_s = q/s \text{ C/m}^2 \text{ for uniform field}$$

Containing a differential charge of ' δq ' then

$$\sigma_s = \lim_{\delta s \rightarrow 0} \frac{\delta q}{\delta s} = \frac{dq}{ds}$$

$$\text{Total charge contained } Q = \int_S \sigma_s ds = \iint \sigma_s dA$$

Volume charge:

It is a 3 dimensional object

If the field is uniform

Volume charge distribution

$$\sigma_v = q/v$$

If the field is non uniform

$$\sigma_v = \lim_{\delta v \rightarrow 0} \frac{\delta q}{\delta v} = \frac{dq}{dv}$$

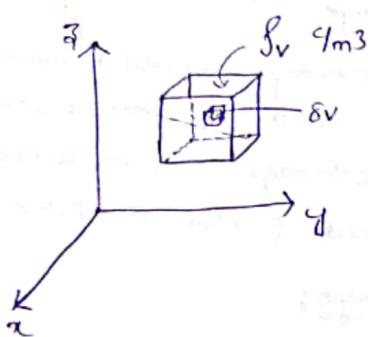
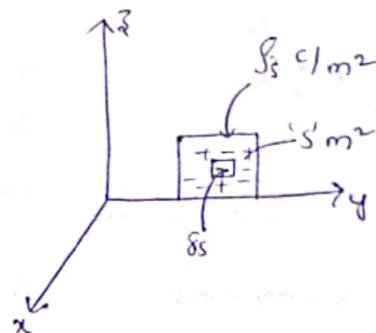
$$\text{Total volume charge } Q = \int_V \sigma_v dv = \iiint \sigma_v dV$$

ELECTROSTATICS

UNIT - I

Static Electric field is due to charge at rest, we know that charging is a process of either removal of electrons from neutral atom (or) addition of electron to neutral atom. In normal atom which is electrically neutral has total negative charge carried by orbital e^- is balanced by total positive charge of protons in the nucleus.

If by ~~any~~ means one or more of valence e^- are removed it is obvious that the atom becomes +ve charged, likewise if e^- is added, atom becomes -ve charged. Thus electrical charge implies either deficiency or excess of e^- in atom of material.



Electric field:

The space or region around an electric charge in which the effect of the charge is felt.

Electro Static field:

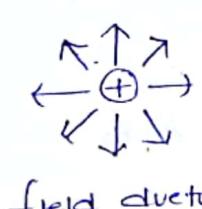
A static electric field may be due to +ve charge or -ve charge unlike magnetic poles which exist always in pairs (N & S).

The presence of or absence of electric field in a region can be ascertained by bringing a test charge into the region.

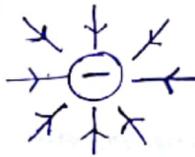
5 If test charge experiences force (attraction or repulsion) it confirms the presence of field.

Like charges repel and unlike charges attract

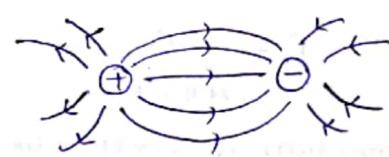
Electro static field can be visualised on comparing lines of forces (or) flux.



field due to
+ve charge

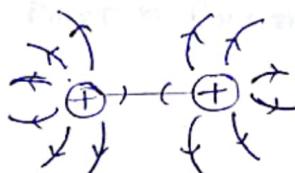


field due to
-ve charge



Two charges of opposite polarity are placed near, pattern of distribution of flux.

for same charges



Laws of Electro Statics:

Two basic laws of electro statics are:

1) Like charges repel and unlike charges attract

2) Coulomb's Inverse Square law:

Coulomb's inverse Square law

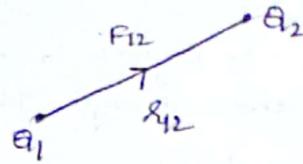
Coulomb's inverse square law states that

"The force of attraction (or repulsion) between two point charges is directly proportional to the magnitudes of the charges and inversely proportional to the square of distance between them."

Q_1, Q_2 are point charges

let r denote the distance between two charges.

F is the force produced.



$$F \propto Q_1 Q_2 \text{ and}$$

$$F \propto 1/r^2$$

$$\therefore \text{therefore } F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{k Q_1 Q_2}{r^2} \text{ where } k \text{ is constant.}$$

Q_1, Q_2 are expressed in Coulombs

R in meters

F in newtons

$$\text{where } k = \frac{1}{4\pi\epsilon_0} \text{ where } \epsilon_0 \text{ is permittivity of medium}$$

$$\text{We have } F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

if medium is vacuum (or) free space (or) air i.e; $\epsilon = \epsilon_0$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ Farads/metre}$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \text{ Newtons in vacuum (or) free space.}$$

$$\frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 8.988 \times 10^9 \approx 9 \times 10^9$$

$$F = 9 \times 10^9 \left(\frac{Q_1 Q_2}{R^2} \right) \text{ N}$$

for any medium we have $\epsilon = \epsilon_0 \epsilon_r$

ϵ = absolute permittivity of medium

ϵ_r = relative permittivity ($\epsilon_r = 1.000 \approx 1$)

ϵ_0 = permittivity of vacuum

for vacuum (or) free space $\epsilon_r = 1$

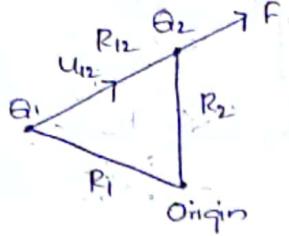
$$\epsilon = \epsilon_0$$

for any other medium $\epsilon_r > 1$

in general

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 \epsilon_r R^2} \text{ Newtons}$$

Vector form of Coulomb's law equation:



Q_1, Q_2 are both +ve charges, force produced is repulsion, thus force acts along the line joining Q_1, Q_2 as shown in figure.

Let Q_1 located at distance R_1 from origin 'O'.
 Q_2 located at distance R_2 from origin 'O'.

R_1 and R_2 may be considered as vectors extending from the origin up to Q_1 and Q_2 respectively.

The force F on Q_2 exerted by Q_1 is along R_{12} the directed line segment from Q_1 to Q_2 .

$$\text{In vector form - Coulomb's law } F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} u_{12} \text{ Newtons}$$

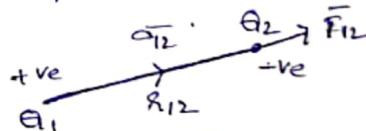
In above expression for force F , u_{12} is a unit vector directed along R_{12} direction of force produced.

$$\begin{aligned} \text{we have } u_{12} &= \frac{\vec{R}_{12}}{|\vec{R}_{12}|} \quad \text{i.e. } \vec{R}_{12} = \vec{R}_2 - \vec{R}_1 \\ &= \frac{\vec{R}_2 - \vec{R}_1}{|\vec{R}_2 - \vec{R}_1|} \end{aligned}$$

Problems:

A two nano +ve charges is located in vacuum at pt $(3, -2, -4)$ and 5μ coulombs -ve charge is located at pt $(1, -4, 2)$. Find the force acting on the -ve charge.

Given that



$$Q_1 = 2 \times 10^{-9} \text{ C at P}(3, -2, -4)$$

$$Q_2 = -5 \times 10^{-6} \text{ C at P}(1, -4, 2)$$

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \hat{u}_{12}$$

$$\begin{aligned} R_{12} &= \vec{R}_2 - \vec{R}_1 = (x_2 - 4x_1 + 2z_1) - (3x_1 - 2x_2 - 4z_1) \\ &= -2x_1 - 2x_2 + 6z_1 \end{aligned}$$

$$a_{12} = \frac{R_{12}}{|R_{12}|} = \frac{-2x_1 - 2x_2 + 6z_1}{\sqrt{44}}$$

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \hat{u}_{12} = \frac{2 \times 5 \times 10^{-9} \times 10^{-6}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 44} \left(\frac{-2x_1 - 2x_2 + 6z_1}{\sqrt{44}} \right)$$

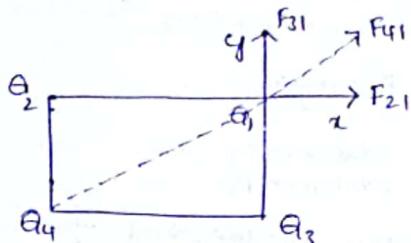
$$= -3.08 \times 10^{-7} (-2x_1 - 2x_2 + 6z_1)$$

$$F_{12} = 6.16 \times 10^{-7} \hat{x}_1 + 6.16 \times 10^{-7} \hat{x}_2 + 18.48 \times 10^{-7} \hat{z}_1 \text{ N}$$

four Concentrated charges are located at the vertices of the plane rectangle. find the magnitude & direction of charge

q_1 located at the origin

Sol:



$$q_2 = 0.1 \mu C \quad 5 \text{ cm}$$

$$q_1 = 0.2 \mu C$$

$$2 \text{ cm}$$

$$q_4 = 0.1 \mu C$$

$$q_3 = -0.2 \mu C$$

$$\begin{aligned} \text{Total force exerted on } q_1 &= \text{Force on } q_1 \text{ due to } q_2 + \text{force on } q_1 \\ &\text{due to } q_3 + \text{force on } q_1 \text{ due to } q_4 \\ &= \bar{F}_{21} + \bar{F}_{31} + \bar{F}_{41} \end{aligned}$$

$$\bar{F}_{21} = \frac{q_1 q_2}{4\pi \epsilon_0 (R_{21})^2} \hat{a}_{21} = \frac{0.2 \times 0.1 \times 10^{-12}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 25 \times 10^{-4}} \hat{a}_{21} \text{ N}$$

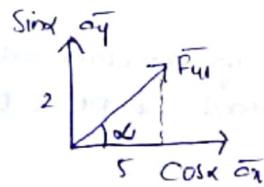
$$= 0.071 \hat{a}_x \text{ N along +ve x-axis N}$$

$$\bar{F}_{31} = \frac{q_1 q_3}{4\pi \epsilon_0 (R_{31})^2} \hat{a}_{31} = \frac{0.2 \times (-0.2) \times 10^{-12}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 4 \times 10^{-4}} \hat{a}_{31} \text{ N}$$

$$= -0.899 \hat{a}_y \text{ N}$$

$$\bar{F}_{41} = \frac{q_1 q_4}{4\pi \epsilon_0 (R_{41})^2} \hat{a}_{41} = \frac{-0.1 \times 0.2 \times 10^{-12}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times (\sqrt{29} \times 10^{-2})^2} \hat{a}_{41}$$

from fig



$$\tan \alpha = 2/5$$

$$\sin \alpha = 2/\sqrt{29}$$

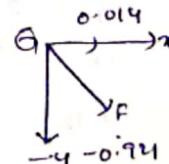
$$\cos \alpha = 5/\sqrt{29}$$

$$\hat{a}_{41} = \cos \alpha \hat{a}_x + \sin \alpha \hat{a}_y$$

$$= 5/\sqrt{29} \hat{a}_x + 2/\sqrt{29} \hat{a}_y$$

$$\begin{aligned} \bar{F}_{41} &= -0.06 [5/\sqrt{29} \hat{a}_x + 2/\sqrt{29} \hat{a}_y] \\ &= -0.057 \hat{a}_x - 0.022 \hat{a}_y \text{ N} \end{aligned}$$

$$F = \bar{F}_{21} + \bar{F}_{31} + \bar{F}_{41} = -0.014 \hat{a}_x - 0.921 \hat{a}_y \text{ N}$$



find the force on a point charge of $50 \mu C$ at pt. $(0,0,5)$ m due to a charge of $500\pi \mu C$ i.e; uniformly distributed over the circular disc $R \leq 5$ m $z=0$

Sol:

Charge distribution is uniform

$$S_s = q/s$$

$$= Q/\pi R^2$$

$$= \frac{500\pi \times 10^{-6}}{\pi (5)^2} = 20 \times 10^{-6} C/m^2$$

