*6.2.5 Other Small-Signal Parameters and Equivalent Circuits

Other small-signal parameters can be developed to model the bipolar transistor or other transistors described in the following chapters.

One common equivalent circuit model for bipolar transistor uses the **h-parameters,** which relate the small-signal terminal currents and voltages of a two-port network. These parameters are normally given in bipolar transistor data sheets, and are convenient to determine experimentally at low frequency.

Figure 6.20(a) shows the small-signal terminal current and voltage phasors for a common-emitter transistor. If we assume the transistor is biased at a Q-point in the forward-active region, the linear relationships between the small-signal terminal currents and voltages can be written as

$$V_{be} = h_{ie} I_b + h_{re} V_{ce} \tag{6.38(a)}$$

$$I_c = h_{fe} I_b + h_{oe} V_{ce} \tag{6.38(b)}$$

These are the defining equations of the common–emitter h-parameters, where the subscripts are: i for input, r for reverse, f for forward, o for output, and e for common emitter.

These equations can be used to generate the small-signal h-parameter equivalent circuit, as shown in Figure 6.20(b). Equation (6.38(a)) represents a Kirchhoff voltage law equation at the input, and the resistance h_{ie} is in series with a dependent voltage source equal to $h_{re} V_{ce}$. Equation (6.38(b)) represents a Kirchhoff current law equation at the output, and the conductance h_{oe} is in parallel with a dependent current source equal to $h_{fe} I_b$.

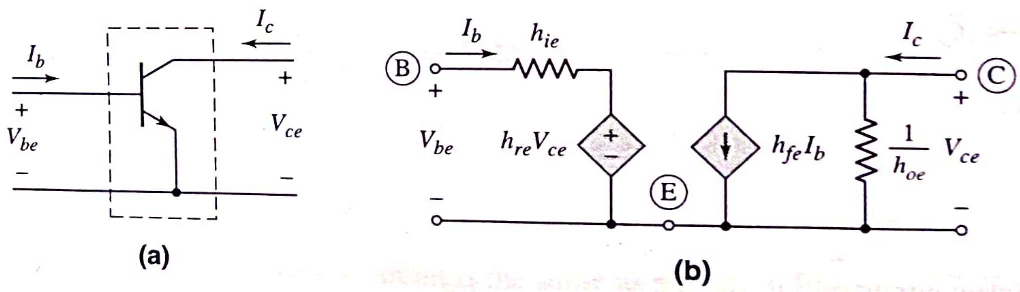

Figure 6.20 (a) Common-emitter npn transistor and (b) the h-parameter model of the common-emitter bipolar transistor

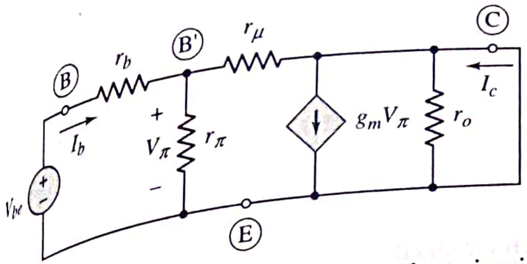

Figure 6.21 Expanded hybrid-π equivalent circuit with the output short-circuited

Since both the hybrid-π and h-parameters can be used to model the characteristics of the same transistor, these parameters are not independent. We can relate the hybrid-π and h-parameters using the equivalent circuit shown in Figure 6.19. The **small-signal input resistance** h_{ie}, from Equation (6.38(a)), can be written as

$$h_{ie} = \left.\frac{V_{be}}{I_b}\right|_{V_{ce}=0} \tag{6.39}$$

where the small-signal C–E voltage is held at zero. With the C–E voltage equal to zero, the circuit in Figure 6.19 is transformed to the one shown in Figure 6.21. From this figure, we see that

$$h_{ie} = r_b + r_\pi \| r_\mu \tag{6.40}$$

In the limit of a very small r_b and a very large r_μ, $h_{ie} \cong r_\pi$.

The parameter h_{fe} is the **small-signal current gain.** From Equation (6.38(b)), this parameter can be written as

$$h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{ce}=0} \tag{6.41}$$

Since the collector–emitter voltage is again zero, we can use Figure 6.21, for which the short-circuit collector current is

$$I_c = g_m V_\pi \tag{6.42}$$

If we again consider the limit of a very small r_b and a very large r_μ, then

$$V_\pi = I_b r_\pi$$

and

$$h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{ce}=0} = g_m r_\pi = \beta \tag{6.43}$$

Consequently, at low frequency, the small-signal current gain h_{fe} is essentially equal to β in most situations.

The parameter h_{re} is called the **voltage feedback ratio,** which, from Equation (6.38(a)), can be written as

$$h_{re} = \left.\frac{V_{be}}{V_{ce}}\right|_{I_b=0} \tag{6.44}$$

Since the input signal base current is zero, the circuit in Figure 6.19 transformed to that shown in Figure 6.22, from which we can see that

$$V_{be} = V_\pi = \left(\frac{r_\pi}{r_\pi + r_\mu}\right) \cdot V_{ce} \tag{6.45(a)}$$

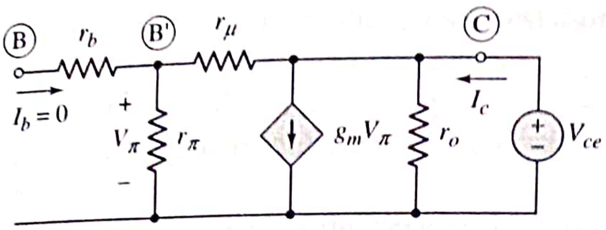

Figure 6.22 Expanded hybrid-π equivalent circuit with the input open-circuited

and

$$h_{re} = \left.\frac{V_{be}}{V_{ce}}\right|_{I_b=0} = \frac{r_\pi}{r_\pi + r_\mu} \tag{6.45(b)}$$

Since $r_\pi \ll r_\mu$, this can be approximated as

$$h_{re} \cong \frac{r_\pi}{r_\mu} \tag{6.46}$$

Since r_π is normally in the kilohm range and r_μ is in the megohm range, the value of h_{re} is very small and can usually be neglected.

The fourth h-parameter is the **small-signal output admittance** h_{oe}. From Equation (6.38(b)), we can write

$$h_{oe} = \left.\frac{I_c}{V_{ce}}\right|_{I_b=0} \tag{6.47}$$

Since the input signal base current is again set equal to zero, the circuit in Figure 6.22 is applicable, and a Kirchhoff current law equation at the output node produces

$$I_c = g_m V_\pi + \frac{V_{ce}}{r_o} + \frac{V_{ce}}{r_\pi + r_\mu} \tag{6.48}$$

where V_π is given by Equation (6.45(a)). For $r_\pi \ll r_\mu$, Equation (6.48) becomes

$$h_{oe} = \left.\frac{I_c}{V_{ce}}\right|_{I_b=0} = \frac{1+\beta}{r_\mu} + \frac{1}{r_o} \tag{6.49}$$

In the ideal case, r_μ is infinite, which means that $h_{oe} = 1/r_o$.

The h-parameters for a pnp transistor are defined in the same way as those for an npn device. Also, the small-signal equivalent circuit for a pnp transistor using h-parameters is identical to that of an npn device, except that the current directions and voltage polarities are reversed.

6.4.1 Basic Common-Emitter Amplifier Circuit

Figure 6.28 shows the basic common-emitter circuit with voltage-divider biasing. We see that the emitter is at ground potential—hence the name common emitter. The signal from the signal source is coupled into the base of the transistor through the coupling capacitor C_C, which provides dc isolation between the amplifier and the signal source. The dc transistor biasing is established by R_1 and R_2, and is not disturbed when the signal source is capacitively coupled to the amplifier.

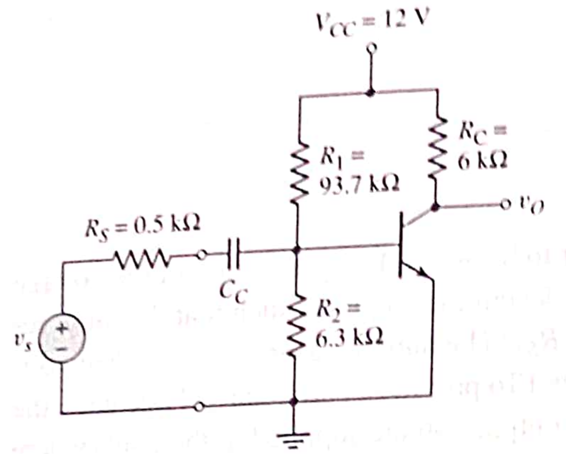

Figure 6.28 A common-emitter circuit with a voltage-divider biasing circuit and a coupling capacitor

If the signal source is a sinusoidal voltage at frequency f, then the magnitude of the capacitor impedance is $|Z_c| = [1/(2\pi f C_C)]$. For example, assume that $C_C = 10\ \mu$F and $f = 2$ kHz. The magnitude of the capacitor impedance is then

$$|Z_c| = \frac{1}{2\pi f C_C} = \frac{1}{2\pi(2 \times 10^3)(10 \times 10^{-6})} \cong 8\ \Omega \tag{6.52}$$

The magnitude of this impedance is much less than the Thevenin resistance at the capacitor terminals, which in this case is $R_1 \| R_2 \| r_\pi$. We can therefore assume that the capacitor is essentially a short circuit to signals with frequencies greater than 2 kHz. We are also neglecting any capacitance effects within the transistor. Using these results, our analyses in this chapter assume that the signal frequency is sufficiently high that any coupling capacitance acts as a perfect short circuit, and is also sufficiently low that the transistor capacitances can be neglected. Such frequencies are in the midfrequency range, or simply at the midband of the amplifier.

The small-signal equivalent circuit in which the coupling capacitor is assumed to be a short circuit is shown in Figure 6.29. The small-signal variables, such as the input signal voltage and input base current, are given in phasor form. The control voltage V_π is also given as a phasor.

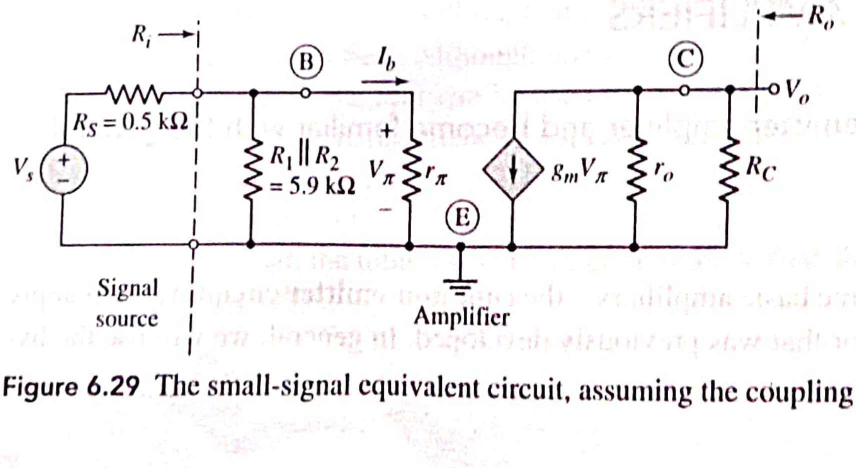

Figure 6.29 The small-signal equivalent circuit, assuming the coupling capacitor is a short circuit

EXAMPLE 6.5

Objective: Determine the small-signal voltage gain, input resistance, and output resistance of the circuit shown in Figure 6.28.

Assume the transistor parameters are: $\beta = 100$, $V_{BE}(\text{on}) = 0.7$ V, and $V_A = 100$ V.

DC Solution: We first perform a dc analysis to find the Q-point values. We find that $I_{CQ} = 0.95$ mA and $V_{CEQ} = 6.31$ V, which shows that the transistor is biased in the forward-active mode.

AC Solution: The small-signal hybrid-π parameters for the equivalent circuit are

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(0.95)} = 2.74\ \text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.95}{0.026} = 36.5\ \text{mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.95} = 105\ \text{k}\Omega$$

Assuming that C_C acts as a short circuit, Figure 6.29 shows the small-signal equivalent circuit. The small-signal output voltage is

$$V_o = -(g_m V_\pi)(r_o \| R_C)$$

The dependent current $g_m V_\pi$ flows through the parallel combination of r_o and R_C, but in a direction that produces a negative output voltage. We can relate the control voltage V_π to the input voltage V_s by a voltage divider. We have

$$V_\pi = \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) \cdot V_s$$

We can then write the small-signal voltage gain as

$$A_v = \frac{V_o}{V_s} = -g_m \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) (r_o \| R_C)$$

or

$$A_v = -(36.5) \left(\frac{5.9 \| 2.74}{5.9 \| 2.74 + 0.5} \right) (105 \| 6) = -163$$

We can also calculate R_i, which is the resistance to the amplifier. From Figure 6.29, we see that

$$R_i = R_1 \| R_2 \| r_\pi = 5.9 \| 2.74 = 1.87\ \text{k}\Omega$$

The output resistance R_o is found by setting the independent source V_s equal to zero. In this case, there is no excitation to the input portion of the circuit so $V_\pi = 0$, which implies that $g_m V_\pi = 0$ (an open circuit). The output resistance looking back into the output terminals is then

$$R_o = r_o \| R_C = 105 \| 6 = 5.68\ \text{k}\Omega$$

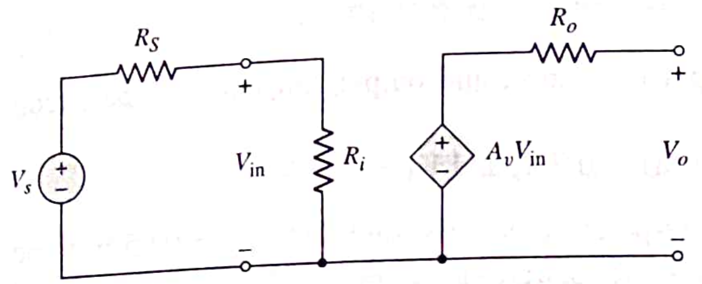

Figure 6.30 Two-port equivalent circuit for the amplifier in Example 6.5

Comment: In this circuit, the effective series resistance between the voltage source V_s and the base of the transistor is much less than that given in Example 6.1. For this reason, the magnitude of the voltage gain for the circuit given in Figure 6.28 is much larger than that found in Example 6.1.

Discussion: The two-port equivalent circuit along with the input signal source for the common-emitter amplifier analyzed in this example is shown in Figure 6.30. We can determine the effect of the source resistance R_S in conjunction with the amplifier input resistance R_i. Using a voltage-divider equation, we find the input voltage to the amplifier is

$$V_{in} = \left(\frac{R_i}{R_i + R_S}\right)V_s = \left(\frac{1.87}{1.87 + 0.5}\right)V_s = 0.789\ V_s$$

Because the input resistance to the amplifier is not very much greater than the signal source resistance, the actual input voltage to the amplifier is reduced to approximately 80 percent of the signal voltage. This is called a **loading effect.** The voltage V_{in} is a function of the amplifier connected to the source. In other amplifier designs, we will try to minimize the loading effect, or make $R_i \gg R_S$, which means that $V_{in} \cong V_s$.

EXERCISE PROBLEM

Ex 6.5: The circuit parameters in Figure 6.28 are changed to $V_{CC} = 5$ V, $R_1 = 35.2\ \text{k}\Omega$, $R_2 = 5.83\ \text{k}\Omega$, $R_C = 10\ \text{k}\Omega$, and $R_S = 0$. Assume the transistor parameters are the same as listed in Example 6.5. Determine the quiescent collector current and collector-emitter voltage, and find the small-signal voltage gain. (Ans. $I_{CQ} = 0.21$ mA, $V_{CEQ} = 2.9$ V, $A_v = -79.1$)

6.4.2 Circuit with Emitter Resistor

For the circuit in Figure 6.28, the bias resistors R_1 and R_2 in conjunction with V_{CC} produce a base current of 9.5 μA and a collector current of 0.95 mA, when the B–E turn-on voltage is assumed to be 0.7 V. If the transistor in the circuit is replaced by a new one with slightly different parameters so that the B–E turn-on voltage is 0.6 V instead of 0.7 V, then the resulting base current is 26 μA, which is sufficient to drive the transistor into saturation. Therefore, the circuit shown in Figure 6.28 is not practical. An improved dc biasing design includes an emitter resistor.

In the last chapter, we found that the Q-point was stabilized against variations in β if an emitter resistor were included in the circuit, as shown in Figure 6.31. We will find a similar property for the ac signals, in that the voltage gain of a circuit with R_E will be less dependent on the transistor current gain β. Even though the emitter of this circuit is not at ground potential, this circuit is still referred to as a common-emitter circuit.

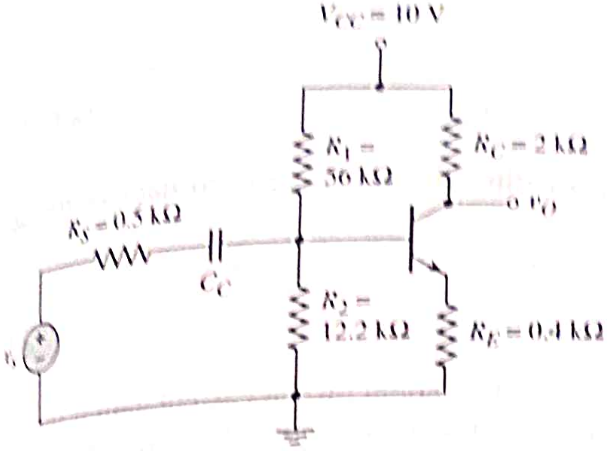

Figure 6.31 An npn common-emitter circuit with an emitter resistor, a voltage-divider biasing circuit, and a coupling capacitor

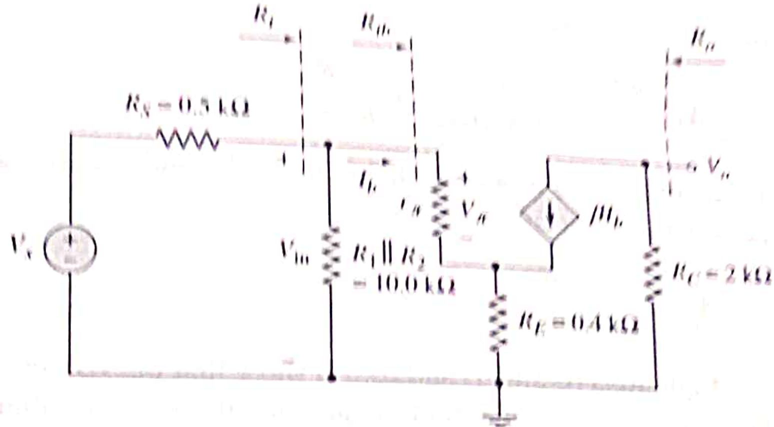

Figure 6.32 The small-signal equivalent circuit of the circuit shown in Figure 6.31

Assuming that C_C acts as a short circuit, Figure 6.32 shows the small-signal hybrid-π equivalent circuit. As we have mentioned previously, to develop the small-signal equivalent circuit, start with the three terminals of the transistor. Sketch the hybrid-π equivalent circuit between the three terminals and then sketch in the remaining circuit elements around these three terminals. In this case, we are using the equivalent circuit with the current gain parameter β, and we are assuming that the Early voltage is infinite so the transistor output resistance r_o can be neglected (an open circuit). The ac output voltage is

$$V_o = -(\beta I_b) R_C \tag{6.53}$$

To find the small-signal voltage gain, it is worthwhile finding the input resistance first. The resistance R_{ib} is the input resistance looking into the base of the transistor. We can write the following loop equation

$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E \tag{6.54}$$

The input resistance R_{ib} is then defined as, and found to be,

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta) R_E \tag{6.55}$$

In the common-emitter configuration that includes an emitter resistance, the small-signal input resistance looking into the base of the transistor is r_π plus the emitter resistance multiplied by the factor $(1 + \beta)$. This effect is called the **resistance reflection rule.** We will use this result throughout the text without further derivation.

The input resistance to the amplifier is now

$$R_i = R_1 \| R_2 \| R_{ib} \tag{6.56}$$

We can again relate V_{in} to V_s through a voltage-divider equation as

$$V_{in} = \left(\frac{R_i}{R_i + R_S}\right) \cdot V_s \tag{6.57}$$

Combining Equations (6.53), (6.55), and (6.57), we find the small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{-(\beta I_b) R_C}{V_s} = -\beta R_C \left(\frac{V_{in}}{R_{ib}}\right) \cdot \left(\frac{1}{V_s}\right) \tag{6.58(a)}$$

or

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}\left(\frac{R_i}{R_i + R_S}\right) \tag{6.58(b)}$$

From this equation, we see that if $R_i \gg R_S$ and if $(1+\beta)R_E \gg r_\pi$, then the small-signal voltage gain is approximately

$$A_v \cong \frac{-\beta R_C}{(1+\beta)R_E} \cong \frac{-R_C}{R_E} \tag{6.59}$$

Equations (6.58(b)) and (6.59) show that the voltage gain is less dependent on the current gain β than in the previous example, which means that there is a smaller change in voltage gain when the transistor current gain changes. The circuit designer now has more control in the design of the voltage gain, but this advantage is at the expense of a smaller gain.

In Chapter 5, we discussed the variation in the Q-point with variations or tolerances in resistor values. Since the voltage gain is a function of resistor values, it is also a function of the tolerances in those values. This must be considered in a circuit design.

EXAMPLE 6.6

Objective: Determine the small-signal voltage gain and input resistance of a common-emitter circuit with an emitter resistor.

For the circuit in Figure 6.31, the transistor parameters are: $\beta = 100$, $V_{BE}(\text{on}) = 0.7$ V, and $V_A = \infty$.

DC Solution: From a dc analysis of the circuit, we can determine that $I_{CQ} = 2.16$ mA and $V_{CEQ} = 4.81$ V, which shows that the transistor is biased in the forward-active mode.

AC Solution: The small-signal hybrid-π parameters are determined to be

$$r_\pi = \frac{V_T\beta}{I_{CQ}} = \frac{(0.026)(100)}{(2.16)} = 1.20\,\text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.16}{0.026} = 83.1\,\text{mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

The input resistance to the base can be determined as

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.20 + (101)(0.4) = 41.6\,\text{k}\Omega$$

and the input resistance to the amplifier is now found to be

$$R_i = R_1 \| R_2 \| R_{ib} = 10 \| 41.6 = 8.06\,\text{k}\Omega$$

Using the exact expression for the voltage gain, we find

$$A_v = \frac{-(100)(2)}{1.20 + (101)(0.4)}\left(\frac{8.06}{8.06 + 0.5}\right) = -4.53$$

we use the approximation given by Equation (6.59), we obtain

$$A_v = \frac{-R_C}{R_E} = \frac{-2}{0.4} = -5.0$$

Comment: The magnitude of the small-signal voltage gain is substantially reduced when an emitter resistor is included. Also, Equation (6.59) gives a good first approximation for the gain, which means that it can be used in the initial design of a common-emitter circuit with an emitter resistor.

Discussion: The amplifier gain is nearly independent of changes in the current gain parameter β. This fact is shown in the following calculations:

β	A_v
50	−4.41
100	−4.53
150	−4.57

In addition to gaining an advantage in stability by including an emitter resistance, we also gain an advantage in the loading effect. We see that, for $\beta = 100$, the input voltage to the amplifier is

$$V_{in} = \left(\frac{R_i}{R_i + R_S}\right) \cdot V_s = (0.942)V_s$$

We see that V_{in} is much closer in value to V_s than in the previous example. There is less loading effect because the input resistance to the base of the transistor is higher when an emitter resistor is included.

The same equivalent circuit as shown in Figure 6.30 applies to this example also. The difference in the two examples is the values of input resistance and gain parameter.

EXERCISE PROBLEM

Ex 6.6: For the circuit in Figure 6.33, let $R_E = 0.6\,\text{k}\Omega$, $R_C = 5.6\,\text{k}\Omega$, $\beta = 120$, $V_{BE}(\text{on}) = 0.7$ V, $R_1 = 250\,\text{k}\Omega$, and $R_2 = 75\,\text{k}\Omega$. (a) For $V_A = \infty$, determine the small-signal voltage gain A_v. (b) Determine the input resistance looking into the base of the transistor. (Ans. (a) $A_v = -8.27$, (b) $R_{ib} = 80.1\,\text{k}\Omega$)

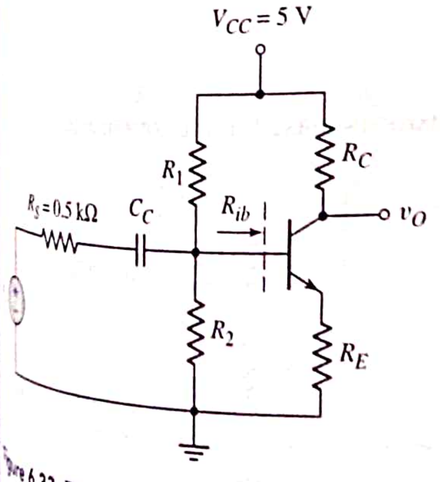

Figure 6.33 Figure for Exercise Ex6.6