

Collector emitter voltage, then  
The collector current is a function only of the base-emitter voltage. then we can write

$$\Delta i_c = \left. \frac{\partial i_c}{\partial V_{BE}} \right|_{Q-pt} \cdot \Delta V_{BE} \quad (or)$$

$$i_c = \left. \frac{\partial i_c}{\partial V_{BE}} \right|_{Q-pt} \cdot V_{be} \quad \text{and collector current}$$

$$i_c = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

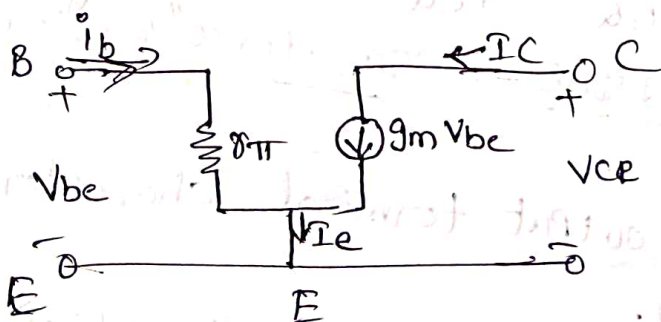
$$\begin{aligned} \text{Then } \frac{\partial i_c}{\partial V_{BE}} &= \frac{\partial}{\partial V_{BE}} \left[ I_S \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \right] \\ &= \frac{1}{V_T} \cdot I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Big|_{Q-pt} \end{aligned}$$

$$\left[ \frac{\partial i_c}{\partial V_{BE}} = \frac{I_{CQ}}{V_T} \right] \quad \frac{I_{CQ}}{V_T} = \text{conductance}$$

Since this conductance relates a current in the collector-emitter voltage in the B-E Ckt the parameter is called trans conductance.

$$\textcircled{1} \text{ class } \quad \boxed{g_m = \frac{I_{CQ}}{V_T}}$$

$\Rightarrow \pi$  class  
Using  $\pi$  &  $g_m$  we are developing a simplified small signal hybrid  $\pi$  equivalent model for the "npn" transistor



Trans conductance model

## Transresistance model :-

developing slightly different form for the output terminals, by relating collector current to small signal base current as.

$$\Delta i_c = \left. \frac{\partial i_c}{\partial i_B} \right|_{a-p-t} \cdot \Delta i_B$$

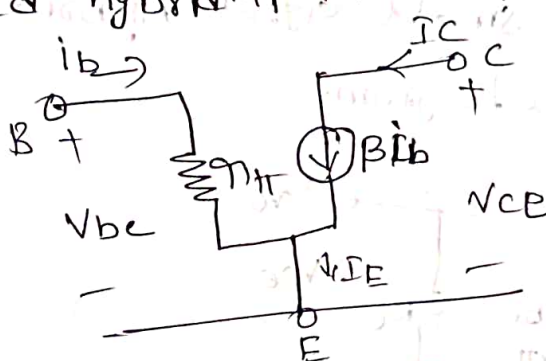
(06)

$$i_c = \left. \frac{\partial i_c}{\partial i_B} \right|_{a-p-t} \cdot i_B$$

$\therefore \frac{\partial i_c}{\partial i_B} = \beta$  Common emitter current gain. We can write above as.

$$i_c = \beta \cdot i_B$$

using  $r_{\pi}$  &  $\beta i_B$  we can develop small signal simplified hybrid  $\pi$  model for "npn" transistor



## Common emitter current gain:-

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{V_T}{I_{BQ}} \rightarrow (1)$$

$$g_m = \frac{I_{CQ}}{V_T} \rightarrow (2)$$

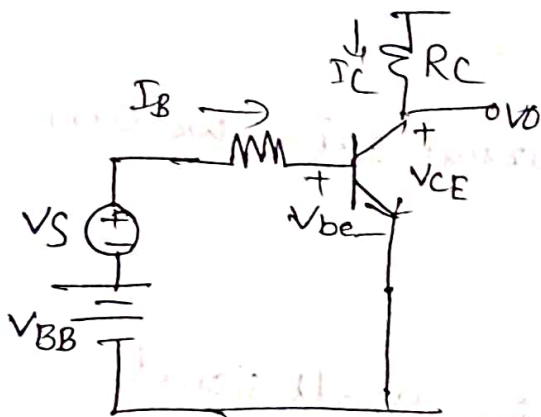
$$\text{current gain} = \frac{I_C}{I_B} = \frac{O/P \text{ current}}{I/P \text{ current}} =$$

If we multiply (1) & (2) we get current gain =  $r_{\pi} g_m = \frac{V_T}{I_{BQ}} \cdot \frac{I_{CQ}}{V_T} = \beta$



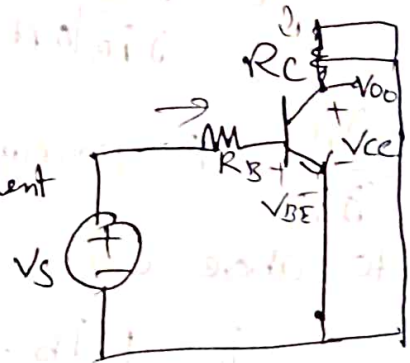
## Small signal voltage gain :-

For calculating voltage gain we need to give input signal to input side and observe the o/p voltage. So, now ~~we~~ we are considering the AC equivalent ckt diagram where we short cktg the DC voltage source and opening the DC current sources for the CE ckt. Diagram.



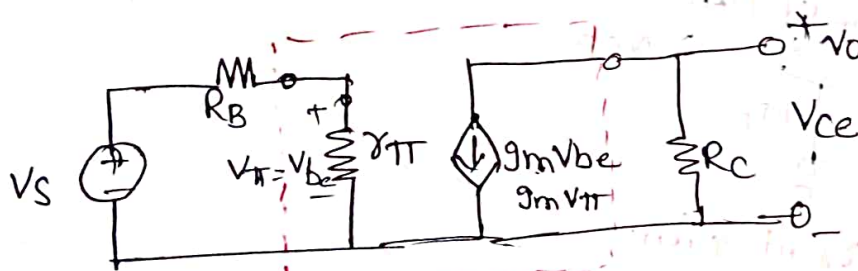
Main ckt

AC equivalent



Simplified small signal hybrid  $\pi$  model

for AC equivalent ckt diagram.



Small signal hybrid  $\pi$  model for n-p-n transistor shown in dotted lines.

Small signal voltage gain :-  $A_{v_s} = \frac{V_o}{V_s} = \frac{\text{o/p signal voltage}}{\text{i/p signal voltage}}$

where  $V_{\pi}$  is small signal Base emitter voltage called control voltage.

$$V_{\pi} = \left( \frac{\tau_{\pi}}{\tau_{\pi} + R_B} \right) \cdot V_S$$

Then the dependent current source  $g_m V_{\pi}$  is producing the negative collector-emitter voltage which flows through  $R_C$ . i.e.

$$V_o = V_{ce} = -(g_m V_{\pi}) \cdot R_C$$

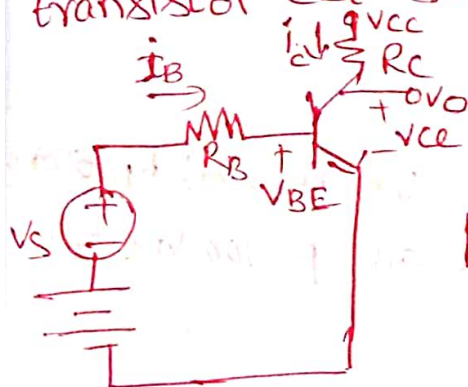
Then small signal voltage gain is

$$A_v = \frac{V_o}{V_S} = \frac{-(g_m V_{\pi}) \cdot R_C}{V_{\pi}} \left( \frac{\tau_{\pi}}{\tau_{\pi} + R_B} \right)$$

$$A_v = (-g_m R_C) \cdot \left( \frac{\tau_{\pi}}{\tau_{\pi} + R_B} \right)$$

example.

calculate small signal voltage gain of the bipolar transistor ckt shown in fig. Assume the transistor



and circuit parameters are  $\beta = 100$ ,  $V_{CC} = 12V$ ,  $V_{BE} = 0.7V$ ,  $R_C = 1k\Omega$

$R_B = 50k\Omega$ , and  $V_{BB} = 1.2V$ .

Sol:  $A_v = ? = \frac{V_o}{V_S} = (-g_m R_C) \left( \frac{\tau_{\pi}}{\tau_{\pi} + R_B} \right)$

$$g_m = \frac{I_{CQ}}{V_T} = ? \quad \frac{\beta I_{BQ}}{V_T} =$$

First we calculate  $I_{BQ}$ . from the ckt diagram

$$I_{BQ} = \frac{V_{BB} - V_{BE(on)}}{R_B} = \frac{1.2 - 0.7}{50} = 10\mu A$$

$$I_{CQ} = \beta I_{BQ}$$

$$= (100)(10 \mu A) = (100)(10 \times 10^{-6}) A$$

$$\text{Then } I_{CQ} = 1 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C$$

$$= 12 - (1)(6) = 6 \text{ V}$$

AC Solution :- From the small signal hybrid- $\pi$  Parameters

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1 \times 10^{-3}}{0.026} = 38.5 \text{ mA/V} \quad \text{then}$$

$$A_v = \frac{V_o}{V_s} = (-g_m R_C) \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right)$$

$$= -(38.5)(6) \left( \frac{2.6 \times 10^3}{2.6 \times 10^3 + 50 \times 10^3} \right)$$

$$= -(38.5)(6) \left( \frac{2.6}{2.6 + 50} \right)$$

$$\boxed{A_v = -11.4}$$

2) The ckt parameters in <sup>above</sup> are  $V_{CC} = 5 \text{ V}$ ,  $V_{BB} = 2 \text{ V}$ ,  $R_B = 650 \text{ k}\Omega$  and  $R_C = 15 \text{ k}\Omega$ , The transistor parameters are  $\beta = 100$ ,  $V_{BE(\text{on})} = 0.7 \text{ V}$  determine

(a) the Q point values  $I_{CQ}$ , and  $V_{CEQ}$

(b) Find the small signal hybrid- $\pi$  parameters  $g_m$ ,  $r_{\pi}$

(c) calculate the small signal voltage gain  $A_v$ .



## Bipolar AC Analysis :

① Analyze the ckt with only the dc sources present. This solution is the dc or a point solution which uses the dc signal models for the elements.

② Replace each element in the circuit ~~with~~ with its small-signal model.

Transformation of elements in dc & small-signal Analysis

element:

element.

DC model

AC model

Resistor

$$I_R = V/R$$

R

R

Capacitor

$$I_C = S \cdot CV$$

Open

C

Inductor

$$I_L = \frac{V}{SL}$$

Short

L

Diode

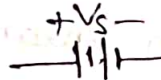
$$I_D = I_S (e^{V/V_T} - 1)$$

cutting voltage  $V_T$ ,  $\eta$  forward resistance

$$r_d = V_T / I_D$$

Independent voltage source

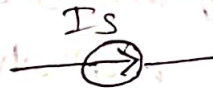
$$V_S = \text{constant}$$



short

Independent current source

$$I_S$$

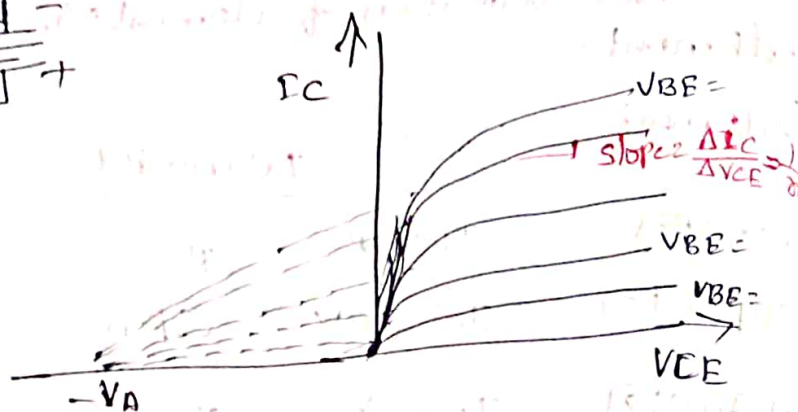
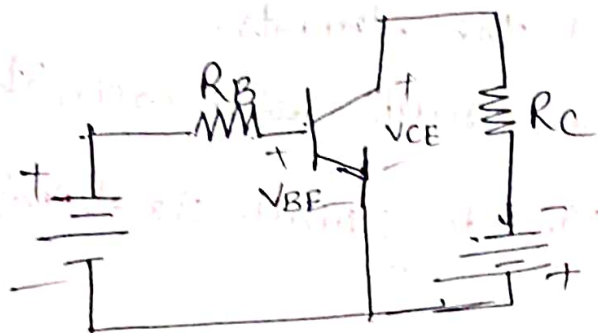
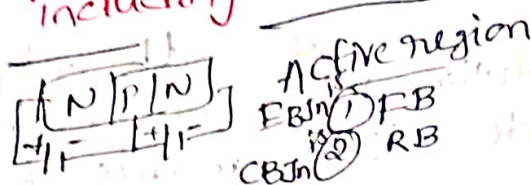


open

③ Analyzing the small signal equivalent ckt, setting the dc source components to zero, to produce the response of the ckt to the time varying input signals only.

$$\left[ \left( \frac{1}{R} + j\omega C \right) \cdot (R + j\omega L) \right] I = V$$

# Hybrid- $\pi$ equivalent ckt including the Early effect:



For a given value of  $V_{BE}$ , in an npn transistor, if  $V_{CE}$  increases, the reverse bias voltage on the collector-base junction increases, which means that the space charge region (or) ~~base~~ width of the B-C region also increases. This reduces the Base width  $w$ . A decrease in base width causes the gradient in minority carrier concentration to increase, which increases the diffusion current through the Base. The collector current then increases as the C-E voltage increases.

The linear dependence of  $I_C$  versus  $V_{CE}$  in the forward active mode can be determined by

$$I_C = I_S (e^{V_{BE}/V_T}) \cdot \left(1 + \frac{V_{CE}}{V_A}\right)$$

$\therefore V_A =$  early voltage

Slope of the curves gives the output resistance  $r_o$ .

$$\frac{1}{r_o} = \frac{\partial I_C}{\partial V_{CE}} \bigg|_{V_{BE} = \text{const}}$$



$$r_o \approx \frac{V_A}{I_C}$$

$V_{CE}$  is small compared to  $V_A$ .

$$r_o = \left. \frac{\partial V_{CE}}{\partial I_C} \right|_{Q-Pt}$$

$$\frac{1}{r_o} = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{Q-Pt} = \frac{\partial}{\partial V_{CE}} \left[ I_S \left( \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right) \right) \right]_{Q-Pt}$$

$$= I_S \left[ \exp\left(\frac{V_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \right]_{Q-Pt}$$

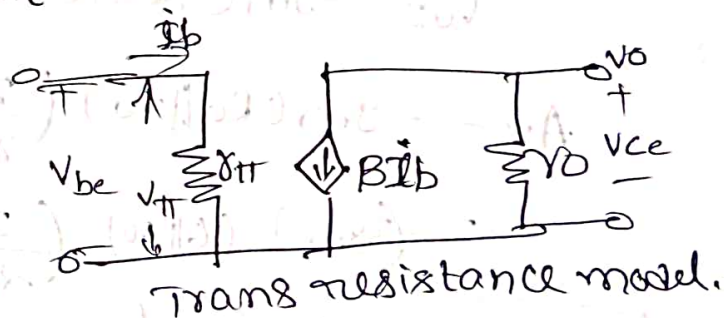
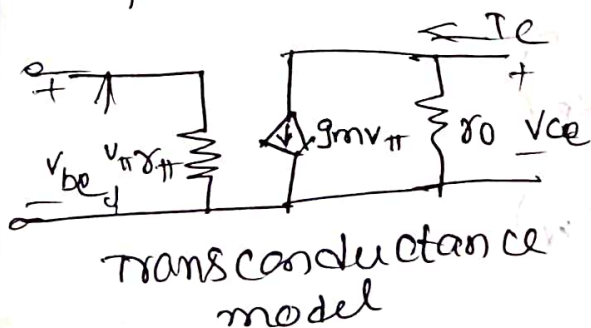
$$\left[ \frac{1}{r_o} = \frac{I_{CQ}}{V_A} \right]$$

$$r_o = \frac{V_A}{I_{CQ}}$$

Small signal  $tr$  output resistance.

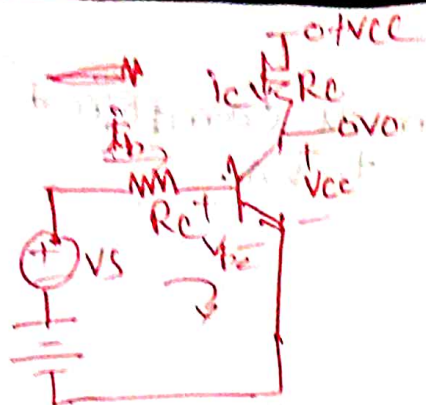
Expanded small signal model of BJT including o/p resistance due to Early effect. for both transconductance & trans resistance model.

This  $r_o$  is Norton resistance which means that  $r_o$  is in parallel with dependent current sources.

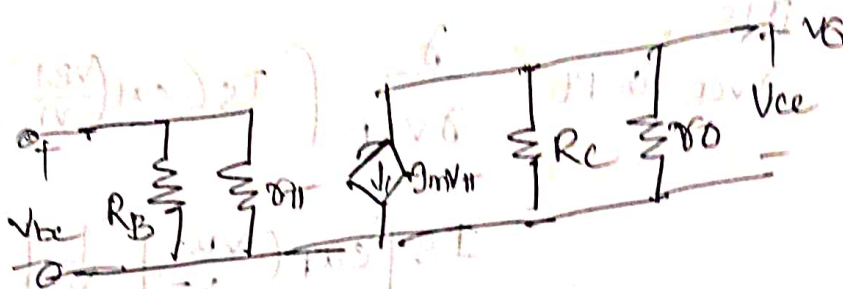


Example (3) Determine the small signal voltage gain, including the effect of the transistor o/p resistance  $r_o$ , for the ckt. with the parameters  $V_{CC} = 12V$ ,  $V_{BE} = 0.7V$ ,  $R_C = 6k\Omega$ ,  $\beta = 100$ ,  $R_B = 50k\Omega$ ,  $V_{BB} = 12V$  and  $V_A = 50V$ .





Sol



$$I_{CQ} = \beta I_{BQ}$$

$$I_{BQ} = \frac{V_{BB} - V_{BE(on)}}{R_B} = \frac{1.2 - 0.7}{50} = 10 \mu A$$

$$I_{CQ} = (10)(100) = 1 mA$$

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} = 2.6 k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1}{0.026} = 38.5 mA/V$$

$$A_v = \frac{V_o}{V_{be}} = -g_m (R_C || R_o)$$

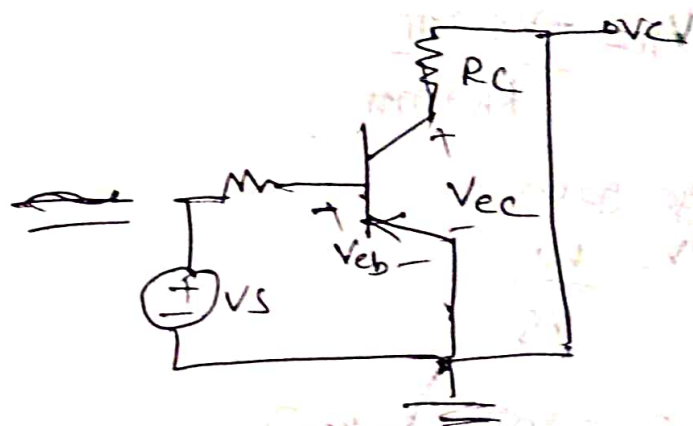
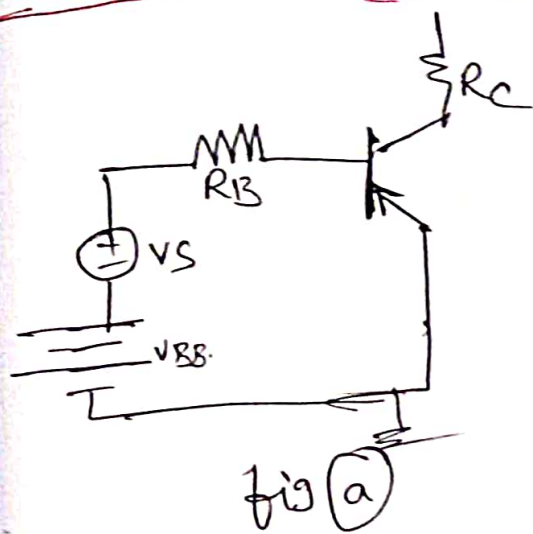
R\_C || R\_o at  
output side

$$A_v = -g_m (R_C || R_o) \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right)$$

$$= -(38.5)(6 || 50) \left( \frac{2.6}{2.6 + 50} \right)$$

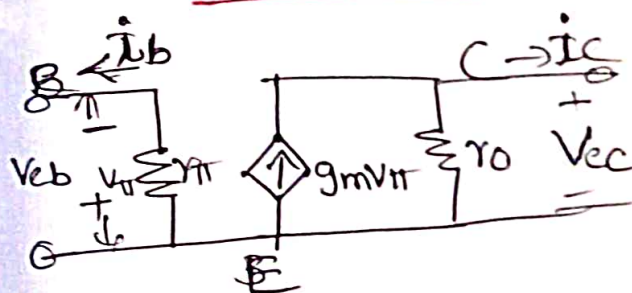
$$\boxed{A_v = -10.2}$$

# Hybrid $\pi$ model for PNP Transistor

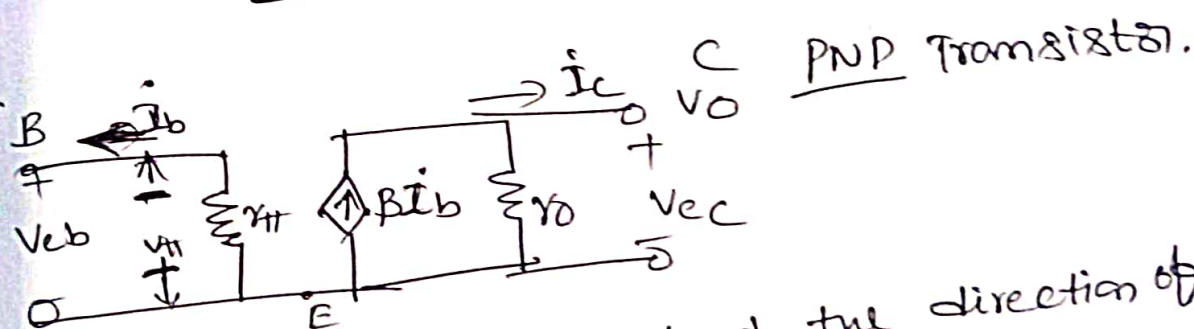


AC equivalent ckt

Small signal hybrid  $\pi$  model <sup>equivalent</sup> for both trans conductance and trans resistance models. including early effect



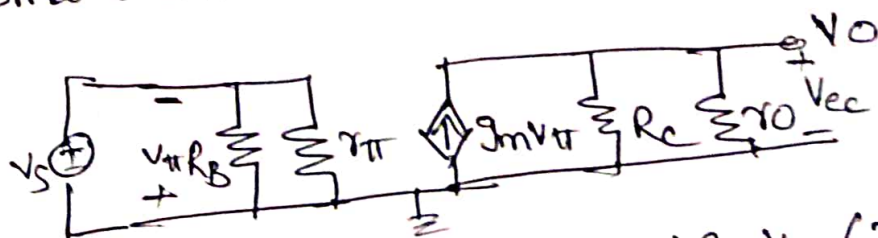
PNP Transistor



PNP Transistor.

$V_{BE}$  polarity is reversed and the direction of dependent current sources are also reversed.

Small signal model for the fig (a) including all resistances



$$V_O = +g_m V_{BE} (r_o || R_C)$$



control voltage  $V_{\pi}$  in terms of input signal voltage  $V_s$ .

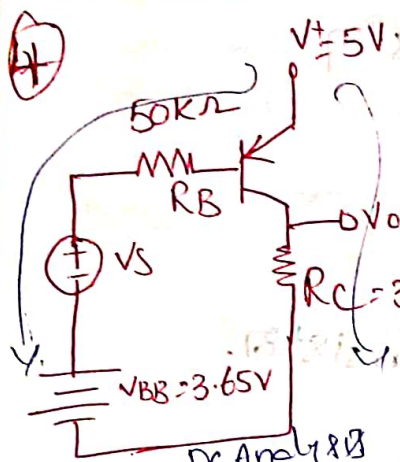
$$V_{\pi} = \frac{-V_s \cdot r_{\pi}}{R_B + r_{\pi}}$$

voltage gain

$$A_v = \frac{V_o}{V_s}$$

$$= \frac{-g_m r_{\pi} (r_o \parallel R_c)}{R_B + r_{\pi}}$$

$$A_v = \frac{-\beta}{R_B + r_{\pi}} (r_o \parallel R_c)$$



Analyze the PNP amplifier ckt!  
consider the circuit shown in fig. Assume transistor parameters  $\beta = 80$ ,  $V_{EB(on)} = 0.7V$  and  $N_A = \infty$ .

sol: A dc KVL equation around the E-B loop

$$V^+ = V_{EB(on)} + I_{BQ} R_B + V_{BB}$$

$$5 = 0.7 + I_{BQ} (50) + 3.65$$

$$I_{BQ} = 13 \mu A$$

$$I_{CQ} = \beta I_{BQ} = 80 (13) \mu A$$

$$= 1.04 mA$$

$$I_{EQ} = I_{BQ} + I_{CQ}$$

$$= 13 \mu A + 1.04 mA$$

$$= 1.05 mA$$

KVL equation around the E-C loop

$$V^+ = V_{ECQ} + I_{CQ} \cdot R_c$$

$$5 = V_{ECQ} + (1.04) \cdot 3$$

$$V_{ECQ} = 1.88V$$

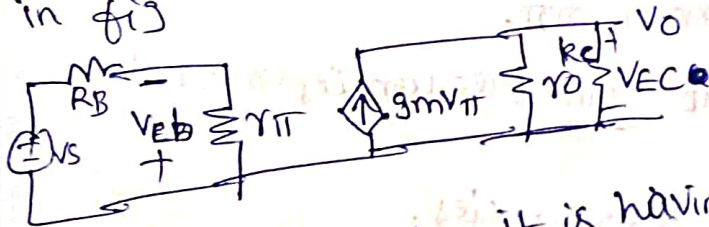
AC Analysis : To find  $A_v$ ,  $r_{\pi}$ ,  $r_o$ .

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.04}{0.026} = 40 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{(90)(0.026)}{1.04} = 2 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{\infty}{1.04} = \infty$$

Small signal equivalent circuit is the same as shown in fig



with  $r_o = \infty$ , i.e. it is having high o/p impedance i.e. it acts as open ckt. then the o/p voltage is

$$v_o = (g_m v_{\pi}) \cdot R_C$$

$$v_{\pi} = - \left( \frac{r_{\pi}}{r_{\pi} + R_B} \right) \cdot v_s = \frac{v_s}{1 + \frac{R_B}{r_{\pi}}}$$

$$A_v \equiv \frac{v_o}{v_s} = - \frac{(g_m v_{\pi}) \cdot R_C}{\left( \frac{v_{\pi}}{1 + \frac{R_B}{r_{\pi}}} \right)} = - \frac{g_m R_C r_{\pi}}{(R_B + r_{\pi})}$$

$$\beta = g_m r_{\pi}$$

$$A_v = \frac{-\beta R_C}{R_B + r_{\pi}} = \frac{-(90)(3)}{2 + 50} = -4.62$$

⑤ For the above ckt in figure. let  $\beta = 90$ ,  $V_A = 120V$ ,  $V_{CC} = 5V$ ,  $V_{EB(on)} = 0.7V$ ,  $R_C = 2.5 \text{ k}\Omega$ ,  $R_B = 50 \text{ k}\Omega$ , and  $V_{BB} = 1.145V$ , (a)

(a) determine the small signal hybrid  $\pi$  Parameters  $r_{\pi}$ ,  $g_m$  and  $r_o$ , (b) find the small signal voltage gain.



## Expanded Hybrid $\pi$ equivalent circuit

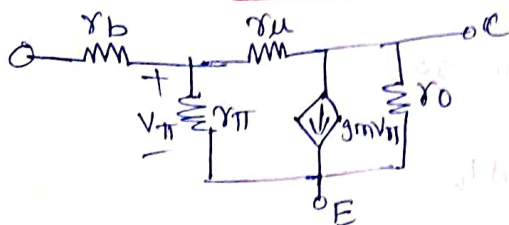


Figure shows an expanded hybrid  $\pi$  equivalent circuit which includes two additional resistances,  $r_b$ ,  $r_u$ .

The parameter  $r_b$  is the "series resistance" of the semiconductor material between the external base terminal  $B_{ext}$  and an idealized internal base region  $B'$ .

The typical value of  $r_b$  is few tenth ohms and is usually much smaller than  $r_{\pi}$ .

hence " $r_b$ " is negligible at low frequencies (or) small frequencies.

$r_u$  is considered at high frequencies.

The parameter " $r_u$ " is the reverse biased diffusion resistance of B-C Junction.

Typical value of  $r_u$  is  $M\Omega$  (megaohms). (open circuit) for small frequencies.