

Figure 6.5 Base current versus base-emitter voltage characteristic with superimposed sinusoidal signals. Slope at the Q-point is inversely proportional to r_π , a small-signal parameter.

Using Figure 6.5, we can now determine one quantitative definition of small signal. From the discussion in Chapter 5, in particular, Equation (5.6), the relation between base-emitter voltage and base current can be written as

$$i_B = \frac{I_S}{\beta} \cdot \exp\left(\frac{v_{BE}}{V_T}\right) \quad (6.1)$$

If v_{BE} is composed of a dc term with a sinusoidal component superimposed, i.e., $v_{BE} = V_{BEQ} + v_{be}$, then

$$i_B = \frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BEQ} + v_{be}}{V_T}\right) = \frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BEQ}}{V_T}\right) \cdot \exp\left(\frac{v_{be}}{V_T}\right) \quad (6.2)$$

where V_{BEQ} is normally referred to as the base-emitter turn-on voltage, $V_{BE(on)}$. The term $[I_S/\beta] \cdot \exp(V_{BEQ}/V_T)$ is the quiescent base current, so we can write

$$i_B = I_{BQ} \cdot \exp\left(\frac{v_{be}}{V_T}\right) \quad (6.3)$$

The base current, given in this form, is not linear and cannot be written as an ac current superimposed on a quiescent value. However, if $v_{be} \ll V_T$, then we can expand the exponential term in a Taylor series, keeping only the **linear term**. This approximation is what is meant by **small signal**. We then have

$$i_B \cong I_{BQ} \left(1 + \frac{v_{be}}{V_T}\right) = I_{BQ} + \frac{I_{BQ}}{V_T} \cdot v_{be} = I_{BQ} + i_b$$

where i_b is the time-varying (sinusoidal) base current given by

$$i_b = \left(\frac{I_{BQ}}{V_T} \right) v_{be} \quad (6.4(b))$$

The sinusoidal base current, i_b , is linearly related to the sinusoidal base-emitter voltage, v_{be} . In this case, the term small-signal refers to the condition in which v_{be} is sufficiently small for the linear relationships between i_b and v_{be} given by Equation (6.4(b)) to be valid. As a general rule, if v_{be} is less than 10 mV, then the exponential relation given by Equation (6.3) and its linear expansion in Equation (6.4(a)) agree within approximately 10 percent. Ensuring that $v_{be} < 10$ mV is another useful rule of thumb in the design of linear bipolar transistor amplifiers.

If the v_{be} signal is assumed to be sinusoidal, but if its magnitude becomes too large, then the output signal will no longer be a pure sinusoidal voltage but will become distorted and contain harmonics (see box “Harmonic Distortion”).

Harmonic Distortion

If an input sinusoidal signal becomes too large, the output signal may no longer be a pure sinusoidal signal because of nonlinear effects. A nonsinusoidal output signal may be expanded into a Fourier series and written in the form

where $I_B = \left(\frac{I_{BQ}}{V_I} \right) \cdot V_{be}$

small signal hybrid π model

From the concept of small signal, all time varying signals (ac) continuous signals are superimposed on dc values. Then we can write

$$I_B = I_{BQ} + i_b \quad \begin{matrix} \text{instantaneous} \\ \text{Base current} \end{matrix} \rightarrow (1)$$

\uparrow
dc current

$$I_C = I_{CQ} + i_c \rightarrow (2)$$

$$V_{CE} = V_{CEQ} + v_{ce} \rightarrow (3)$$

and $V_{BE} = V_{BEQ} + v_{be} \rightarrow (4)$

if signal source V_s is zero, then the B-E and C-E emitter loop equations are

$$V_{BB} = I_{BQ} R_B + V_{BEQ} \rightarrow (5)$$

and

$$V_{CC} = I_{CQ} R_C + V_{CEQ} \rightarrow (6)$$

when we consider ac (or) time varying signals V_s is into account, then BE loop equation is

$$V_{BB} + V_s = i_B R_B + V_{BE} \rightarrow (7)$$

$$V_{BB} + V_s = (I_{BQ} + i_b) R_B + (V_{BEQ} + v_{be}) \rightarrow (8)$$

Rearranging terms

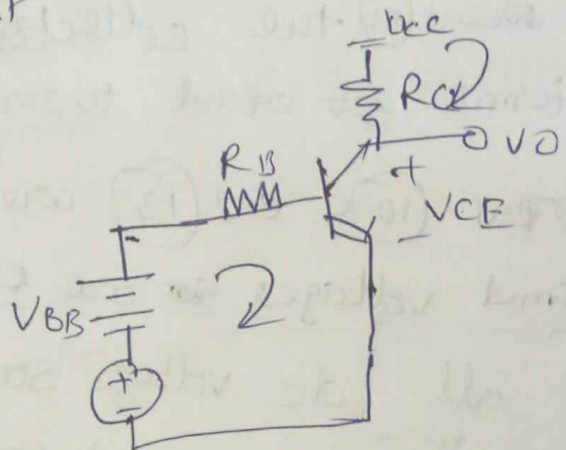
$$V_{BB} - I_{BQ} R_B - V_{BEQ} = i_b R_B + v_{be} - V_s \rightarrow (9)$$

from eq (5), eq (7) and substitute V_{BB} value then

$$0 = i_b R_B + v_{be} - V_s$$

$$V_s = i_b R_B + v_{be} \rightarrow (10)$$

eq (10) which is B-E loop equation with all dc terms equal to zero.



Taking into account the time-varying signals, the collector-emitter loop equation is

$$V_{CC} = i_C R_C + V_{CE} \quad \rightarrow (11)$$

$$= (I_{CQ} + i_c) R_C + (V_{CEQ} + v_{ce})$$

Rearranging terms we find

$$V_{CC} - I_{CQ} R_C - V_{CEQ} = i_c R_C + v_{ce} \quad \rightarrow (12)$$

from eq (11) & eq (12) the left side of eq (12) is becomes

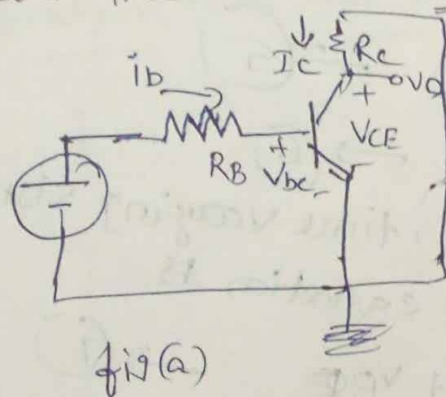
$$0. \quad i_c R_C + v_{ce} = 0 \quad \rightarrow (13)$$

from eq (13) the collector-emitter loop eq with all dc terms get equal to zero.

→ eq (10) & eq (13) are ^{can be obtained} directly by setting all dc voltages ~~so~~ are equal to zero. i_c

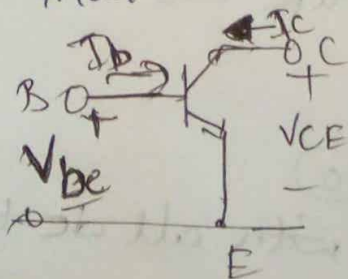
all dc voltage sources become short ckt and
all dc current sources become open ckt.

These will result when we apply superposition to a linear ckt. The below fig "AC equivalent circuit"

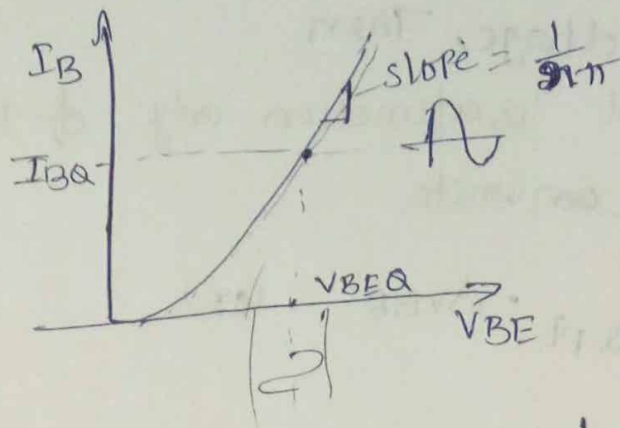


Small signal Hybrid- π model equivalent ckt for BJT

The fig(a) is small signal model equivalent ckt for the Transistor. That such circuit is "hybrid π model"



Transistor is two port network



slope at the Q point as constant, which units of conductance.

The inverse of this conductance is the small signal model resistance defined as r_{π} .

we can relate input base current to the small signal input voltage by

$$V_{be} = i_b r_{\pi}$$

where $1/r_{\pi}$ is slope of the $i_B - V_{BE}$ curve.

$$\left. \frac{\partial i_B}{\partial V_{BE}} \right|_{Q-Pt} = \frac{1}{r_{\pi}} = \left. \frac{\partial}{\partial V_{BE}} \left[\frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \right] \right|_{Q-Pt}$$

where $i_B = \frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BE}}{V_T}\right)$ then

$$\begin{aligned} \frac{1}{r_{\pi}} &= \frac{1}{r_{\pi}} = \frac{1}{V_T} \left[\frac{I_S}{\beta} \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \right] \Big|_{Q-Pt} \\ &= \frac{I_{BQ}}{V_T} \end{aligned}$$

Then

$$\frac{V_{be}}{i_b} = r_{\pi} = \frac{V_T}{I_{BQ}} = \frac{\beta V_T}{I_{CQ}}$$

r_{π} - is called the "diffusion resistance" or "Base-emitter resistance."

we can consider the output terminal characteristics of the bipolar transistor.

\Rightarrow If we initially consider the case in which the output collector current i_c is independent of the

collector emitter voltage, then

The collector current is a function only of the base-emitter voltage, then we can write

$$\Delta i_c = \left. \frac{\partial i_c}{\partial V_{BE}} \right|_{Q-pt} \cdot \Delta V_{BE} \quad (\text{or})$$

$$i_c = \left. \frac{\partial i_c}{\partial V_{BE}} \right|_{Q-pt} \cdot V_{be} \quad \text{and collector current}$$

$$i_c = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\text{Then } \frac{\partial i_c}{\partial V_{BE}} = \frac{\partial}{\partial V_{BE}} \left[I_S \exp\left(\frac{V_{BE}}{V_T}\right) \right]$$

$$= \frac{1}{V_T} \cdot I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Big|_{Q-pt}$$

$$\boxed{\frac{\partial i_c}{\partial V_{BE}} = \frac{I_{CQ}}{V_T}}$$

$\frac{I_{CQ}}{V_T}$ = conductance.

Since this conductance relates a current in the collector-emitter voltage in the B-E Ckt, the parameter is called trans conductance.