Collector emitter voltage, Then
The collector current is a function only of the base-emitter
Voltage. then we can write

$$\Delta i_c = \frac{\delta i_c}{\partial V_{BE}}\bigg|_{Q\text{-}pt} \cdot \Delta V_{BE} \quad (\text{or})$$

$$i_c = \frac{\partial i_c}{\partial V_{BE}}\bigg|_{Q\text{-}pt} \cdot V_{be}$$ and collector current

$$i_c = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

Then $$\frac{\partial i_c}{\partial V_{BE}} = \frac{\partial}{\partial V_{BE}}\left[I_S \cdot \exp\left(\frac{V_{BE}}{V_T}\right)\right]$$

$$= \frac{1}{V_T} \cdot \underbrace{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}_{I_{CQ}}\bigg|_{Q\,pt}$$

$$\boxed{\frac{\partial i_c}{\partial V_{BE}} = \frac{I_{CQ}}{V_T}}$$ — $\frac{I_{CQ}}{V_T}$ - conductance.

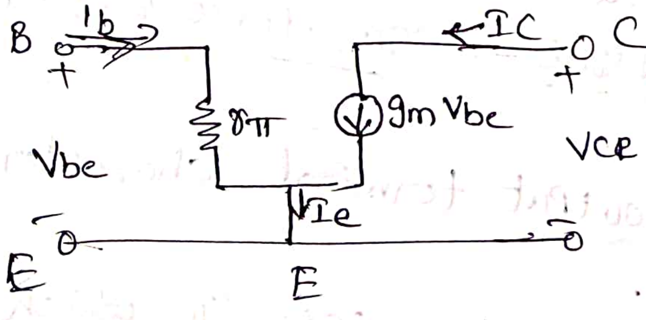
Since this conductance relates a current in the collect-emitter voltage in the B-E Ckt, the Parameter is called trans conductance.

① class

$$\boxed{g_m = \frac{I_{CQ}}{V_T}}$$

II class

Using r_π & g_m we are developing a simplified small signal hybrid-π equivalent model for the "npn" transistor



Trans conductance model

Transresistance model :-

developing slightly different form for the output terminals, by relating collector current to small signal base current as

$$\Delta i_C = \left.\frac{\partial i_C}{\partial i_B}\right|_{Q\text{-}pt} \cdot \Delta i_B$$

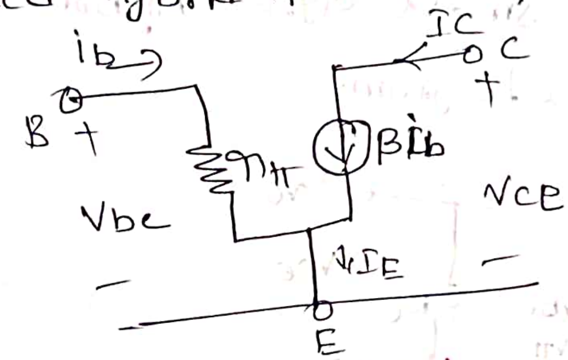
(or)

$$i_c = \left.\frac{\partial i_c}{\partial i_B}\right|_{Q\text{-}pt} \cdot i_b$$

$\because \frac{\partial i_c}{\partial i_B} = \beta$ common emitter current gain. we can write above eq.

$$i_c = \beta . i_b .$$

using r_π & βi_b we can developing small signal simplified hybrid-π model for "npn" transistor



Common emitter current gain:-

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{V_T}{I_{BQ}} \rightarrow (1)$$

$$g_m = \frac{I_{CQ}}{V_T} \rightarrow (2)$$

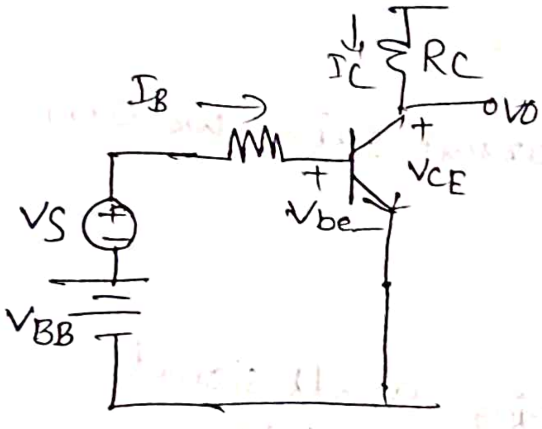
$$\text{current gain} = \frac{I_C}{I_B} = \frac{\text{o/p current}}{\text{i/p current}} =$$

If we multiply (1) & (2) we get current gain $= r_\pi \, g_m = \frac{V_T \beta}{I_{CQ}} \cdot \frac{I_{CQ}}{V_T}$

$$\boxed{r_\pi . g_m = \beta}$$

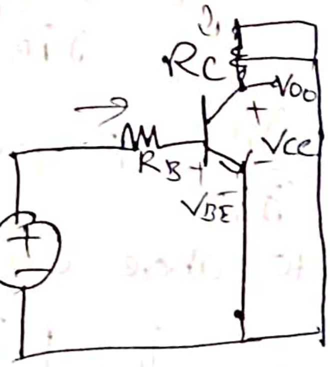
Small signal voltage gain :-

For calculating voltage gain we need to give input signal to input side and observe the o/p voltage. So now we are considering the AC equivalent ckt diagram where we short ckting the DC voltage source and opencktng the DC current sources for the CE ckt diagram.

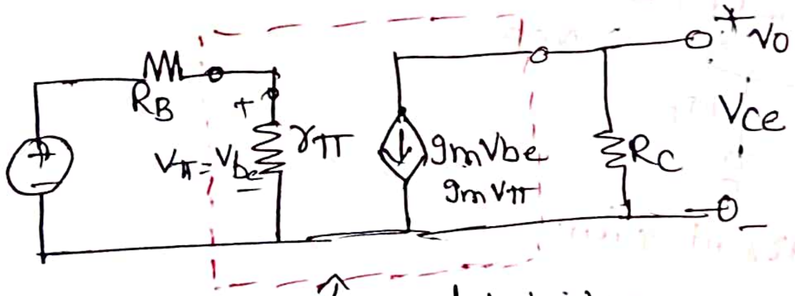


Main ckt

Ac equivalent



Simplified small signal hybrid π model for Ac equivalent ckt diagram.



small signal hybrid-π model for npn transistor shown in dotted lines.

Small signal voltage gain: $A_v = \frac{V_o}{V_s} = \frac{\text{o/p signal voltage}}{\text{i/p signal voltage}}$

where V_π is small signal Base emitter voltage called control voltage.

TV

$$\boxed{V_\pi = \left(\frac{r_\pi}{r_\pi + R_B}\right) \cdot V_S}$$

Then the dependent current source $g_m V_\pi$ is producing the negative collector-emitter voltage which flows through R_C. ie

$$\boxed{V_0 = V_{ce} = -(g_m V_\pi) \cdot R_C}$$

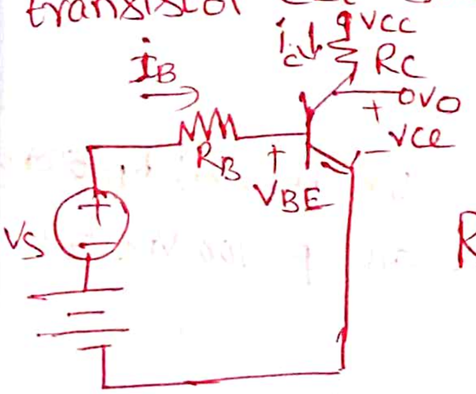
Then small signal voltage gain is

$$A_v = \frac{V_0}{V_S} = \frac{-(g_m \cancel{V_\pi}) \cdot R_C \left(\frac{r_\pi}{r_\pi + R_B}\right)}{\cancel{V_\pi}}$$

$$\boxed{A_v = (-g_m R_C) \cdot \left(\frac{r_\pi}{r_\pi + R_B}\right)}$$

Example.

calculate small signal voltage gain of the bipolar transistor ckt shown in fig. Assume the transistor and circuit parameters are



$\beta = 100$, $V_{CC} = 12V$, $V_{BE} = 0.7V$, $R_C = 6k\Omega$, $R_B = 50k\Omega$, and $V_{BB} = 1.2V$.

Sol: $A_v = ? = \frac{V_o}{V_S} = (-g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$

$$g_m = \frac{I_{CQ}}{V_T} = ? \quad \frac{\beta I_{BQ}}{V_T} =$$

First we calculate I_{BQ}. from the ckt diagram

$$I_{BQ} = \frac{V_{BB} - V_{BE(on)}}{R_B} = \frac{1.2 - 0.7}{50} = 10\mu A$$

$I_{CQ} = \beta I_{BQ}$

$= (100)(10\mu A) = (100)(10\times10^{-6})A$

Then $= 1mA$

$V_{CEQ} = V_{CC} - I_{CQ}R_C$

$= 12 - (1)(6) = 6V$

<u>AC Solution</u> :- From the small signal hybrid-π Parame

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} = 2.6k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1\times10^{-3}}{0.026} = 38.5 mA/V$$ then

$$A_v = \frac{V_o}{V_S} = (-g_m R_C) \cdot \left(\frac{r_\pi}{r_\pi + R_B}\right)$$

$$= -(38.5)(6)\left(\frac{2.6\times10^3}{2.6\times10^3 + 50\times10^3}\right)$$

$$= -(38.5)\cdot(6)\left(\frac{2.6}{2.6+50}\right)$$

$$\boxed{A_v = -11.4}$$

(2) The ckt parameters in fig above are $V_{CC}=5V$, $V_{BB}=2V$, $R_B=650k$ and $R_C = 15k\Omega$, The transistor parameters are $\beta=100$, $V_{BE(on)} = 0.7V$ Determine

(a) the Q point values I_{CQ}, and V_{CEQ}

(b) Find the small signal hybrid-π parameters g_m, r_π

(c) calculate the small signal voltage gain A_v.

Bipolar Ac Analysis :

① Analyze the ckt with only the dc-sources present. This solution is the dc or Q point solution which uses the dc signal models for the elements.

② Replace each element in the circuit ~~ckt~~ with its small-signal model.

Transformation of elements in dc & small-signal Analysis

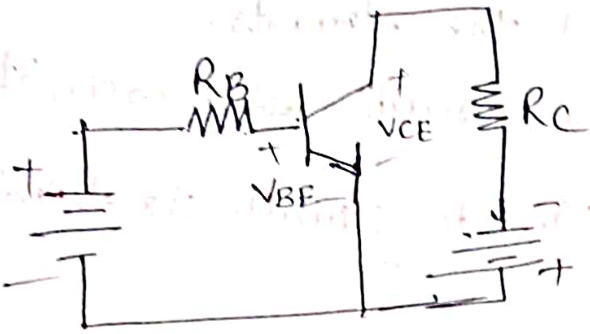
~~element~~:

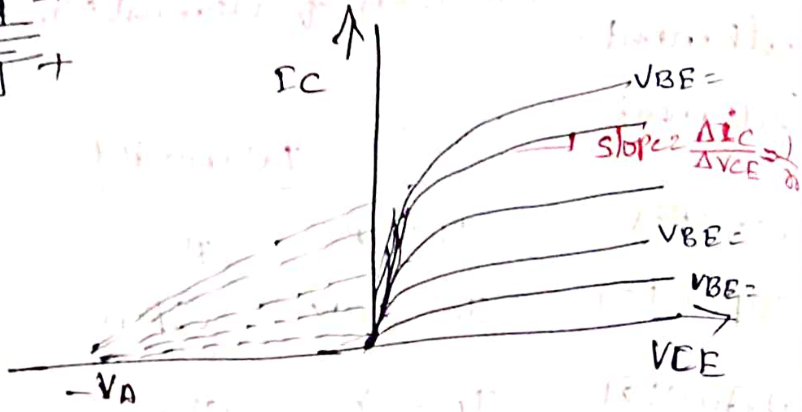
element		DC model	Ac model
Resistor	$I_R = V/R$	R	R
Capacitor	$I_C = s \cdot CV$	Open	C
Inductor	$I_L = \frac{V}{sL}$	Short	L
Diode	$I_D = I_S(e^{V_D/V_T} - 1)$	cutin voltage $+V_\gamma, - r_f$ forward resistance	$r_d = V_T/I_D$
Independent voltage source	V_S = constant	$+V_S-$	short
Independent current source	I_S	I_S	open

③ Analyzing the small signal equivalent ckt, setting the dc source components to zero, to produce the response of the ckt to the time varying input signals only.

Hybrid-π equivalent ckt including the Early effect:

NPN
Active region
EBJn (1) FB
CBJn (2) RB





For a given value of V_{BE}, in an npn transistor, if V_{CE} increases, the reverse bias voltage on the collector-Base Junction increases, which means that the space charge region (or) width of the B-C depletion region also increases. This reduces the Base width W. A decrease in base width causes the gradient in monority carrier concentration to increase, which increases the diffusion current through the Base. The collector current then increases as the C-E voltage increases.

The linear dependence of i_C versus VCE in the forward active mode can be determined by

$$\boxed{i_C = I_S \left(e^{V_{BE}/V_T}\right) \cdot \left(1 + \frac{V_{CE}}{V_A}\right)}$$

∴ V_A = early voltage

Slope of the curves gives the output resistance r_o.

$$\frac{1}{r_o} = \left.\frac{\partial i_C}{\partial V_{CE}}\right|_{V_{BE}\text{-const}}$$

$$\boxed{r_o \simeq \frac{V_A}{I_C}}$$

V_{CE} is small compared to V_A.

$$r_o = \frac{\partial v_{CE}}{\partial i_C}\bigg|_{Q\text{-}pt}$$

$$\frac{1}{r_o} = \frac{\partial i_C}{\partial v_{CE}}\bigg|_{Q\text{-}pt} = \frac{\partial}{\partial v_{CE}}\left[I_S \left(\exp\left(\frac{V_{BE}}{V_T}\right)\right)\left(1+\frac{V_{CE}}{V_A}\right)\right]_{Q\text{-}pt}$$

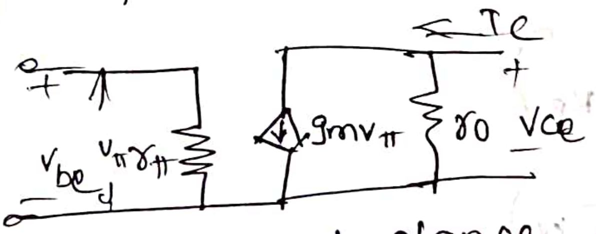
$$= I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right)\cdot\frac{1}{V_A}\right]_{Q\text{-}pt}$$

$$\boxed{\frac{1}{r_o} = \frac{I_{CQ}}{V_A}}$$

$$\boxed{r_o = \frac{V_A}{I_{CQ}}}$$ small signal tr output resistance.

Expanded small signal model of BJT including o/p resistance due to early effect for both transconductance & trans resistance models.

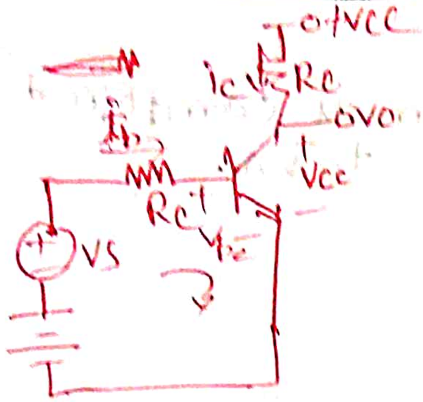
This r_o is norton resistance which means that r_o is in parallel with dependent current sources.



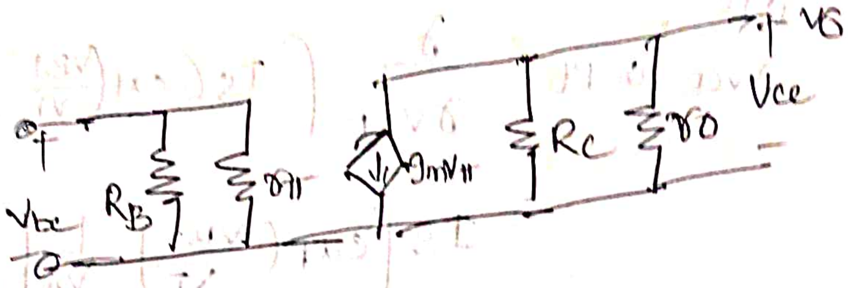
Transconductance model

Trans resistance model.

Example (3) Determine the small signal voltage gain, including the effect of the transistor o/p resistance r_o, for the ckt. with the parameters $V_{CC} = 12V$, $V_{BE} = 0.7V$, $R_C = 6k\Omega$, $\beta = 100$, $R_B = 50k\Omega$, $V_{BB} = 1.2V$ and $V_A = 50V$.



Sol



$I_{CQ} = \beta I_{BQ}$

$$i_{BQ} = \frac{V_{BB} - V_{BE(on)}}{R_B} = \frac{1.2 - 0.7}{50} = 10\mu A$$

$I_{CQ} = (10)(100) = 1mA$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} = 2.6k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1}{0.026} = 38.5 \ mA/V$$

Rc||Ro at o/p side

~~Av=~~

$V_o = -g_m(R_c \| r_o)$

$$A_v = -g_m(R_c \| r_o)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

$$= -(38.5)(6 \| 50)\left(\frac{2.6}{2.6 + 50}\right)$$

$$\boxed{A_v = -10.2}$$

Hybrid π model for PNP Transistor

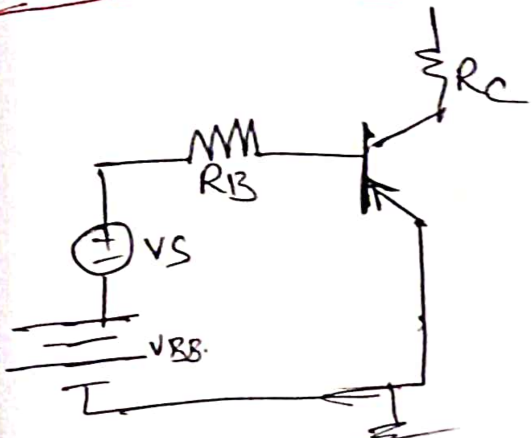
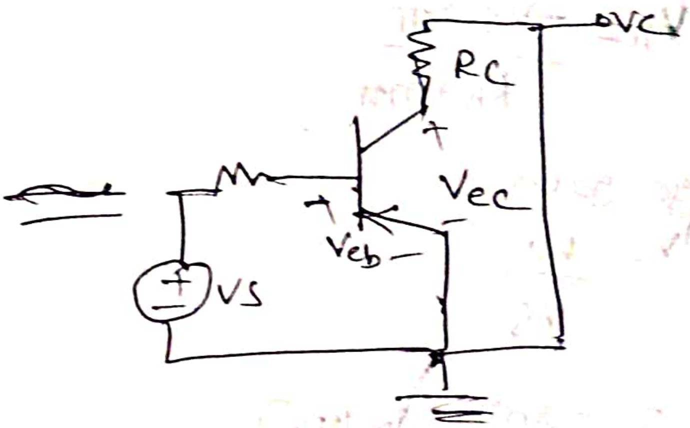


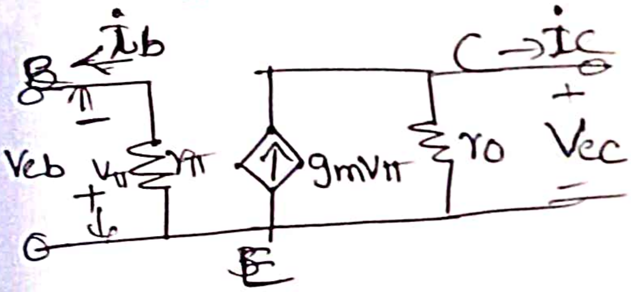
fig (a)



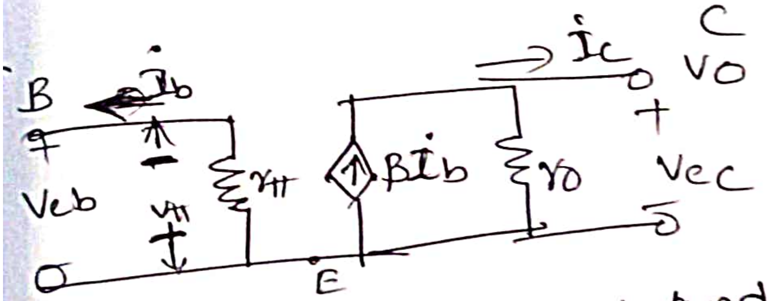
Ac equivalent ckt

Small signal hybrid π equivalent model for both trans conductance and trans resistance models. including early effect

PNP transistor

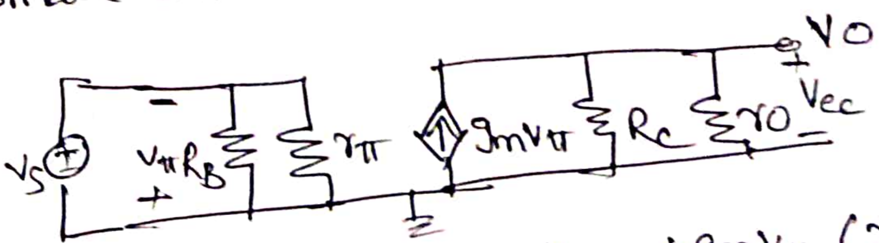


PNP Transistor.



V_π - Polarity is reversed and the direction of dependent current sources are also reversed.

Small signal model for the fig (a) including all resistances



$$V_o = +g_m V_\pi (r_o \| R_C)$$

Control voltage V_π interms of input signal voltage V_S.

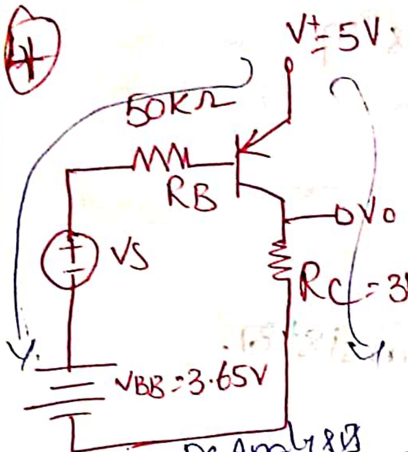
$$V_\pi = \frac{-V_S \cdot r_\pi}{R_B + r_\pi}.$$

Voltage gain

$$A_V = \frac{V_O}{V_S}$$

$$= \frac{-g_m r_\pi}{R_B + r_\pi}(r_o \| R_C)$$

$$\boxed{A_V = \frac{-\beta}{R_B + r_\pi}(r_o \| R_C)}$$

4. 

Analyze the PNP amplifier ckt: consider the circuit shown in fig. Assume transistor parameters of $\beta = 80$, $V_{EB(on)} = 0.7V$ and $V_A = \infty$.

Sol: DC Analysis
A dc KVL equation around the E-B loop

$$V^+ = V_{EB(on)} + I_{BQ} R_B + V_{BB}$$

$$5 = 0.7 + I_{BQ}(50) + 3.65$$

$$I_{BQ} = 13\ \mu A.$$

$$I_{CQ} = \beta I_{BQ} = 80(13)\ \mu A$$
$$= 1.04\ mA.$$

$$I_{EQ} = I_{BQ} + I_{CQ}$$
$$= 13\mu A + 1.04 mA$$
$$= 1.05\ mA$$

KVL equation Around the E-C loop,

$$V^+ = V_{ECQ} + I_{CQ} \cdot R_C$$

$5 = V_{ECQ} + (1.04) \cdot 3$

$V_{ECQ} = 1.88V$

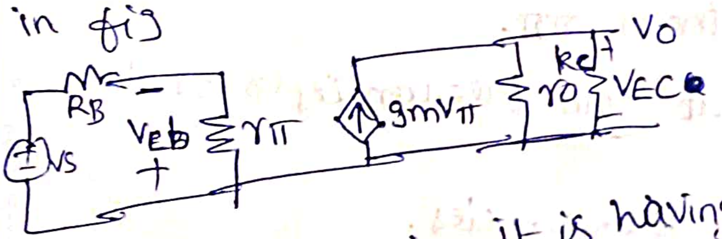
AC Analysis : To find A_v, r_π, r_o.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.04}{0.026} = 40 mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{1.04} = 2k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{\infty}{1.04} = \infty$$

small signal equivalent circuit is the same as shown in fig



with $r_o = \infty$, ie it is having high o/p impedance ie it acts as open ckt. then the o/p voltage is

$$V_o = (g_m V_\pi) \cdot R_C$$

$$V_\pi = -\left(\frac{r_\pi}{r_\pi + R_B}\right) \cdot V_s$$

$$A_V = \frac{V_o}{V_S} = -\frac{(g_m V_\pi) \cdot R_C}{(\cancel{V_\pi}(r_\pi)/r_\pi + R_B)} = -\frac{g_m R_C r_\pi}{(r_\pi + R_B)}$$

$$\beta = g_m r_\pi$$

$$A_V = \frac{-\beta R_C}{R_B + r_\pi} = \frac{-(80)(3)}{2+50} = -4.62$$

⑤ For the above ckt in figure. let $\beta = 90$, $V_A = 120V$, $V_{CC} = 5V$, $V_{EB(on)} = 0.7V$, $R_C = 2.5k\Omega$, $R_B = 50k\Omega$, and $V_{BB} = 1.145V$, ⓐ Determine the small signal hybrid π parameters r_π, g_m and r_o, ⓑ find the small signal voltage gain.)

Expanded Hybrid-π equivalent circuit :-

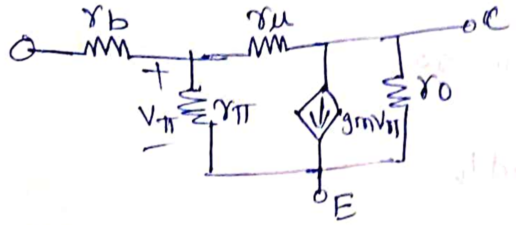


Figure shows an expanded hybrid π equivalent circu[it] which includes two additional resistances, r_b, r_π.

The parameter r_b is the "series resistance" of the semicon[du]ctor material between the external base terminal B an[d] an idealized internal base region B'.

The typical value of r_b is few tens of ohms and is usually much smaller than r_π.

hence "r_b" is negligible at low frequencies (or) small frequencies.

r_b is considered at high frequencies.

The parameter "r_μ" is the reverse biased diffusion resistance of B-C Junction.

Typical value of r_μ is MΩ (megaohms). (open ckt) for small frequencies.