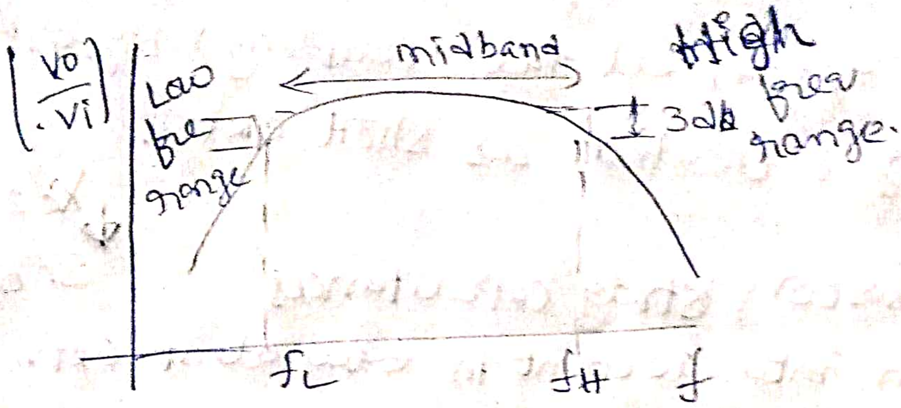
Unit - II

Frequency response

→ All Amplifier gain factors are functions of signal frequency.

→ These include voltage, current, transconductance and transresistance.



The curve drawn between frequency & gain factor is called frequency response curve.

⇒ The frequency ranges are divided as:

- low frequency $f < f_L$ $20Hz < f < 20kHz$
- High frequency $f > f_H$ $f_H > 20kHz$
- Medium frequency. f_H

Frequency ranges

low frequency range

$\uparrow X_C = \frac{1}{2\pi f C}$ high capacitance

→ In this region, coupling and bypass capacitors must be included in the equivalent ckt and in the amplification factor equations.

→ The stray and transistor capacitances are treated as open ckts.

→ In low frequencies range, $f < f_L$, the gain ↓ as freq ↓ because of coupling & bypass capacitor effects.

High frequency Range

→ In this range all the stray & parasitic capacitors are present.

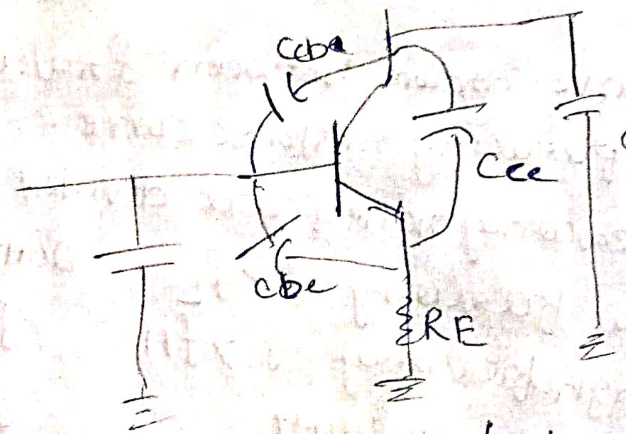
→ due to these capacitances the gain decreases as frequency increase.

→ In this range we use High frequency equivalent ckt.

→ In this region all the coupling & Bypass capacitors are going to treated as short ckts.

$\downarrow X_C = \frac{1}{2\pi f C} \uparrow$
Short ckts.

→ The transistor & stray capacitances are taken into account in equivalent ckt.



→ due to PN junction (between collector, base and emitter).

→ C_μ & C_π

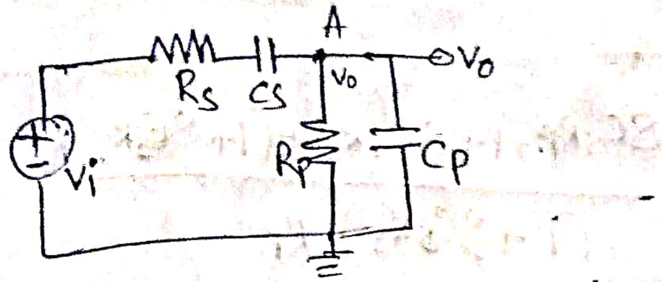
Midband Range :-

coupling & bypass capacitors are treated as short ckt

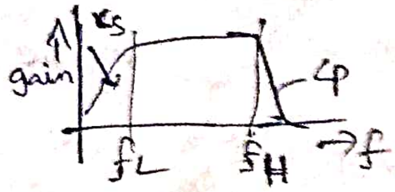
→ stray and transistor capacitancy are treated as open ckts.

→ so In this range no capacitances in the equivalent ckt.

short ckt and open ckt Time constant



- Cs is the coupling capacitor
- Cp is the load capacitor and is in parallel with the output and ground.



- Applying KCL at output node
→ due to Cs at low frequencies the gain is going to be reduced
→ due to Cp at High frequencies the gain is going to be reduced. at mid fre the stray, parasitic capacitances are appear so, gain maintains constant.
→ Applying KCL at node A ie) output node

$$\frac{V_o - V_i}{R_s + 1/sC_s} + \frac{V_o}{R_P} + \frac{V_o}{1/sC_P} = 0$$

$$\therefore \frac{(V_o - V_i)sC_s}{1 + sC_sR_s} + \frac{V_o}{R_P} + V_o \cdot sC_P = 0$$

separate V_o & V_i terms, to get the transfer function $\frac{V_o}{V_i}$. so, the equation becomes

$$V_o\left[\frac{sC_s}{1+sC_sR_s} + \frac{1}{R_P} + sC_P\right] = \frac{(V_i)sC_s}{1+sC_sR_s}$$

$$\frac{V_o}{V_i} = \frac{sC_s}{1+sC_sR_s\left[\frac{sC_s}{1+sC_sR_s} + \frac{1}{R_P} + sC_P\right]}$$

take LCM

$$\frac{V_o}{V_i} = \frac{S\cancel{C_s}}{1+\cancel{SC_s}R_s\left[\frac{SC_sR_p+1+SC_sR_s+(1+\cancel{SC_s}R_s)R_p\,SC_p}{(1+\cancel{SC_s}R_s)\,R_p}\right]}$$

$$X = \frac{SC_s\cdot\cancel{R_p}}{1+SC_sR_s\left[\cancel{\{1+SC_sR_s\}R_p}\cdot\frac{C_p}{C_s}+R_p+R_s+\right.}$$

$$= \frac{\cancel{SC_s}\,R_p}{\cancel{SC_s}\left[\frac{1}{SC_s}+R_s+R_p+(1+SC_sR_s)R_p\frac{C_p}{C_s}\right]}$$

$$= \frac{R_p}{\left[R_s+R_p+\frac{1}{SC_s}+(R_p+SC_sR_pR_s)\cdot\frac{C_p}{C_s}\right]}$$

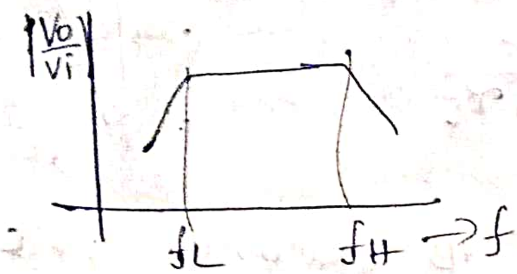
$$= \left[\frac{R_p}{R_s+R_p}\right]\left[\frac{1}{1+\underbrace{\frac{1}{SC_s(R_p+R_s)}}_{\text{Series combination } ST_s}+\left(\frac{R_p}{R_s+R_p}\right)\frac{C_p}{C_s}+\underbrace{\left(\frac{R_pR_s}{R_s+R_p}\right)\cdot SC_p}_{\text{Parallel combination } ST_p}}\right]$$

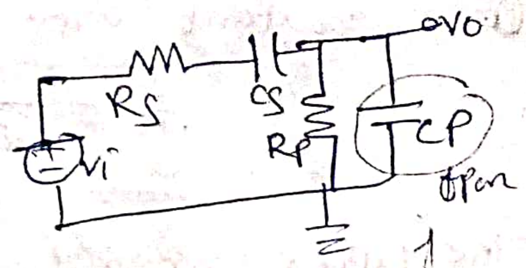
$$\frac{V_o(s)}{V_i(s)} = \left[\frac{R_p}{R_s+R_p}\right]\left[\frac{1}{1+\left(\frac{R_p}{R_s+R_p}\right)\frac{C_p}{C_s}+\frac{1}{ST_s}+ST_p}\right]$$

where

$$\tau_s = (R_s + R_p) C_s$$

$$\tau_p = \left(\frac{R_s R_p}{R_s + R_p}\right) C_p$$ Time constants

→ 



C_s - affects the low frequency response

C_p - affects the High frequency response.

if $C_p \ll C_s$ — at low frequencies C_p as an open ckt

To find the equivalent resistance seen by a capacitor, set all independent sources equal to zero.

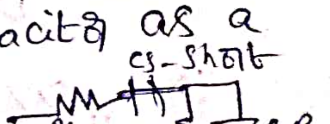
∴ The effective resistance seen by C_s is the series combination of R_s & R_p.

The Time constant of ckt is

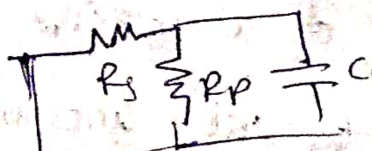
$$\tau_s = (R_s + R_p) C_s.$$

Since C_p was made an open ckt, τ_s is called an "open ckt" time constant.

⇒ At high frequencies, C_s coupling capacitor act as a short ckt.



∴ The effective resistance seen by C_p is the parallel combination of R_s and R_p and the associated time constant is



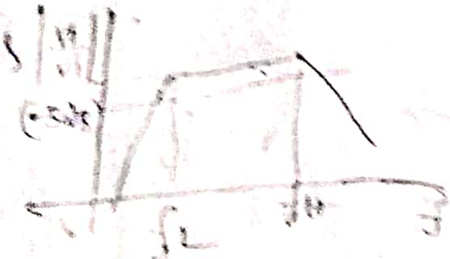
$$\tau_p = \left(\frac{R_s R_p}{R_s + R_p}\right) \cdot C_p$$

$(R_s \| R_p) C_p$

∴ τ_p is called the "short ckt time constant.

let consider
Bode plot of the voltage transfer function magnitude

* The lower corner (or) 3dB frequency, which is at the low end of the frequency scale, is a function of the open ckt time constant and is defined as



$$f_L = \frac{1}{2\pi\tau_s}$$

→ The upper corner (or) 3dB frequency which is at the high end of the frequency scale, is a function of the short ckt time constant and is defined as

$$f_H = \frac{1}{2\pi\tau_P}$$

→ The amplifier gain is constant over a wide frequency range, called the "mid band".

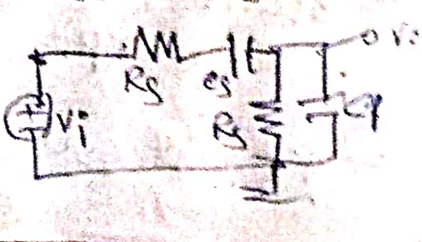
→ In "mid band" frequency range all capacitance effects are negligible (stray and parasitic capacitances)

→ The mid band range (or) bandwidth is defind by the corner frequencies f_L and f_H as

$$BW = f_H - f_L$$

is $f_H >> f_L$ then $\boxed{f_{BW} \cong f_H}$

Determine the corner frequencies and bandwidth of a passive ckt containing two capacitors, consider the ckt shown in figuere, with parameters $R_s = 1k\Omega$, $R_P = 10k\Omega$, $C_s = 1\mu F$ and $C_P = 3pF$.



Sol:

$$\tau_s = (R_s + R_p)C_s = (10^3 + 10\times10^3)(10^{-6}) = 1.1\times10^{-2}\ s$$

$$\tau_p = \left(\frac{R_s R_p}{R_s + R_p}\right).C_p = \frac{10^3\times10\times10^3}{10^3+10\times10^3}(3\times10^{-12}) = 2.73\times10^{-9}\ s$$

$$f_L = \frac{1}{2\pi\tau_s} = \frac{1}{2\pi\times1.1\times10^{-2}} = 14.5\ Hz$$

$$f_H = \frac{1}{2\pi\tau_p} = \frac{1}{2\pi(2.73\times10^{-9})} = 58.3\ MHz$$

$$f_{BW} = f_H - f_L = 58.3\times10^{-6} - 14.5\ Hz$$
$$= 58.3\ MHz \qquad f_H >> f_L \quad \boxed{BW \simeq f_H}$$

Time Response :-

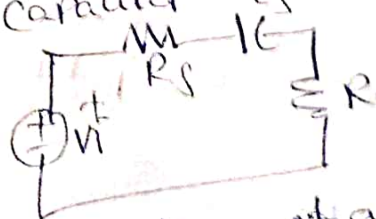
→ upto now we observed steady state sinusoidal frequency response, we for sinusoidal inputs.

→ In some cases, we may need to amplify other than sinusoidal signal also, ie Pulse, square, triangular signals. ie digital signals.

→ In these cases, we need to consider the time response of output signals.

series coupling capacitor

consider ckt



The voltage transfer function of above ckt

$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_s + R_P + \frac{1}{sC_s}}$$

$$= \frac{sC_sR_P}{1 + s(R_s + R_P)C_s}$$

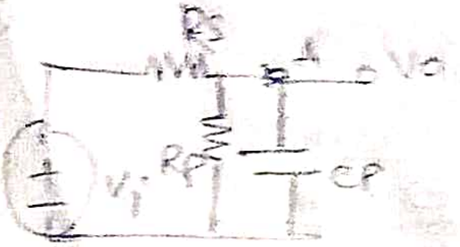
$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S+R_P}\right) \frac{S(R_S+R_P)C_S}{1+S(R_S+R_P)C_S}$$

$$\left(\frac{V_o}{V_i}\right) = K_2 \frac{S\tau_2}{1+S\tau_2}$$

$$\therefore \tau_2 = (R_S+R_P)SC_S$$

$$\therefore K_2 = \frac{R_P}{R_S+R_P}$$

Parallel load capacitor ckt



KCL at output node A

$$\frac{V_o - V_i}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{1/SC_P} = 0$$

$$\frac{V_o}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{1/SC_P} = \frac{V_i}{R_S}$$

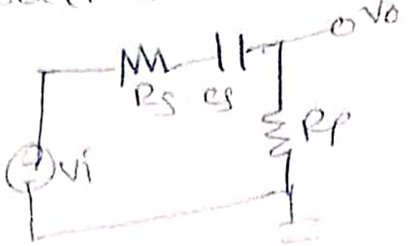
$$\frac{V_o}{V_i} = \left(\frac{R_P}{R_S+R_P}\right) \frac{1}{1+S\left(\frac{R_SR_P}{R_S+R_P}\right)C_P}$$

$$= \left(\frac{R_P}{R_S+R_P}\right)\left(\frac{1}{S\tau_P+1}\right)$$

$$\tau_P = (R_S \parallel R_P)C_P.$$

(9)

In the input voltage is a step function $V_i(s) = 1/s$. be can series coupling capacitor ckt. Then output voltage $V_o(s) =$



$$V_o(s) = K_2 \left[\frac{s\tau_2}{1+s\tau_2}\right] =$$

$$= \left(\frac{Rp}{Rs+Rp}\right)\left[\frac{s(Rs+Rp)\,Cs}{1+s(Rs+Rp)\,Cs}\right]$$

$$\frac{V_o(s)}{V_i(s)} = K_2\left[\frac{s\tau_2}{1+s\tau_2}\right] \quad \text{if} \quad V_i(s) = 1/s$$

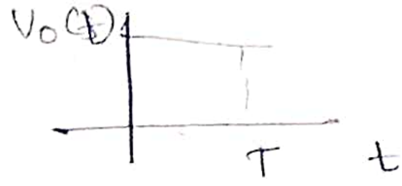
$$= K_2\left[\frac{1}{s+\tau_2}\right] \qquad V_o(s) = K_2\left[\frac{s\tau_2}{1+s\tau_2}\right] \times \frac{1}{s}$$

$$= K_2\left[\frac{\tau_2}{1+s\tau_2}\right]$$

$$= K_2\left[\frac{1}{s+1/\tau_2}\right]$$

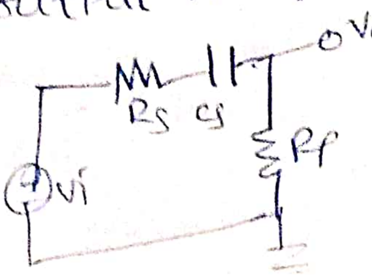
Taking inverse laplace transfer function, then

$$V_o(s) = K_2\, e^{-t/\tau_2}$$



If we apply pulse input voltage, to above ckt the voltage applied to load ckt is slowly decreases.

Inthe input voltage is a step function $V_i(S) = 1/S$. V_o can be Sereis Coupling capacitor ckt. Then output voltage $V_o(S) =$



$$V_o(S) = K_2 \left[\frac{S\tau_2}{1+S\tau_2}\right] =$$

$$= \left(\frac{Rp}{Rs+Rp}\right)\left[\frac{S(Rs+Rp)\,Cs}{1+S(Rs+Rp)\,Cs}\right]$$

$$\frac{V_o(S)}{V_i(S)} = K_2\left[\frac{S\tau_2}{1+S\tau_2}\right] \quad \text{if } V_i(S) = 1/S$$

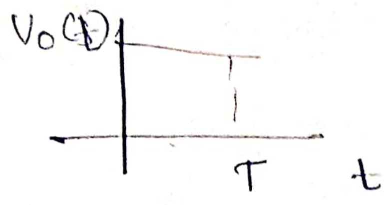
$$= K_2\left[\frac{1}{S+1/\tau_2}\right] \qquad V_o(S) = K_2\left[\frac{S\tau_2}{1+S\tau_2}\right] \times \frac{1}{S}$$

$$= K_2\left[\frac{\tau_2}{1+S\tau_2}\right]$$

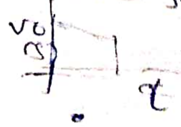
$$= K_2\left[\frac{1}{S+1/\tau_2}\right]$$

Taking inverse Laplace transfer function, then

$$V_o(S) = K_2\, e^{-t/\tau_2}$$



If we apply pulse input voltage, to above ckt the voltage applied to load ckt is slowly decreases.



we can rearrange the terms and write the function as

$$T(s) = K_1\left(\frac{1}{1+s\tau_1}\right) \qquad \text{(7.2(b))}$$

where τ_1 is a **time constant.** Other transfer functions may be written as

$$T(s) = K_2\left(\frac{s\tau_2}{1+s\tau_2}\right) \qquad \text{(7.2(c))}$$

where τ_2 is also a time constant. In most cases, we will write the transfer functions in terms of the time constants.

To introduce the frequency response analysis of transistor circuits, we will examine the circuits shown in Figures 7.2 and 7.3. The voltage transfer function for the circuit in Figure 7.2 can be expressed in a voltage divider format, as follows:

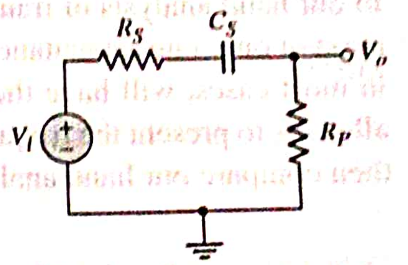


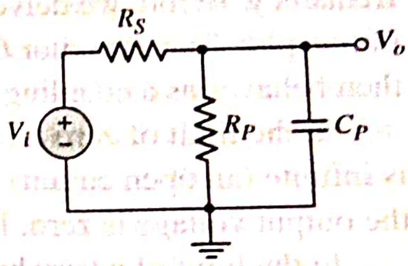
Figure 7.2 Series coupling capacitor circuit

$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P + \dfrac{1}{sC_S}} \qquad \text{(7.3)}$$

The elements R_S and C_S are in series between the input and output signals, and the element R_P is in parallel with the output signal. Equation (7.3) can be written in the form

$$\frac{V_o(s)}{V_i(s)} = \frac{sR_PC_S}{1+s(R_S+R_P)C_S} \qquad \text{(7.4)}$$

which can be rearranged and written as



$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S+R_P}\right)\left[\frac{s(R_S+R_P)C_S}{1+s(R_S+R_P)C_S}\right] = K_2\left(\frac{s\tau}{1+s\tau}\right) \qquad \text{(7.5)}$$

Figure 7.3 Parallel load capacitor circuit

In this equation, the time constant is

$$\tau = (R_S + R_P)C_S$$

Writing a Kirchhoff current law (KCL) equation at the output node, we can determine the voltage trans fer function for the circuit shown in Figure 7.3, as follows:

$$\frac{V_o - V_i}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{(1/sC_P)} = 0 \qquad \text{(7.}$$

In this case, the element R_S is in series between the input and output signals, and the elements R_P and C_P a in parallel with the output signal. Rearranging the terms in Equation (7.6) produces

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S+R_P}\right)\left[\frac{1}{1+s\left(\dfrac{R_SR_P}{R_S+R_P}\right)C_P}\right] \qquad \text{(7.7(}$$

or

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s(R_S \| R_P)C_P}\right] \tag{7.7(b)}$$

In Equation (7.7(b)), the time constant is

$$\tau = (R_S \| R_P)C_P$$

7.2.2 First-Order Functions

of the pulse.

Consider, now, the circuit shown in Figure 7.18, which is a repeat of Figure 7.3. In this case, the capacitor C_P may represent the input capacitance of an amplifier. The transfer function was given in Equation (7.7(b)) as

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right) \cdot \left[\frac{1}{1 + s\,(R_P \| R_S)\,C_P}\right] \tag{7.26}$$

or

$$\frac{V_o(s)}{V_i(s)} = K_1 \left(\frac{1}{1 + s\tau_1}\right) \tag{7.27}$$

where the time constant is $\tau_1 = (R_P \| R_S)C_P$.

Again, if the input signal is a step function, then $V_i(s) = 1/s$. The output voltage can then be written as

$$V_o(s) = \frac{K_1}{s}\left(\frac{1}{1 + s\tau_1}\right) = \frac{K_1}{s}\left(\frac{1/\tau_1}{s + 1/\tau_1}\right) \tag{7.28}$$

Taking the inverse Laplace transform, we find the output voltage time response as

$$v_O(t) = K_i(1 - e^{-t/\tau_1}) \tag{7.29}$$

If we are trying to amplify an input voltage pulse, we need to ensure that the time constant τ_1 is short compared to the pulse width T, so that the signal $v_O(t)$ reaches a steady-state value. The output voltage is shown in Figure 7.19 for a square wave input signal. A short time constant implies a very small capacitor C_P as an input capacitance to an amplifier.

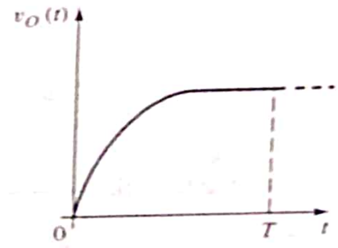


Figure 7.19 Output response of circuit in Figure 7.18 for a square-wave input signal and for a short time constant

In this case, if the cutoff frequency of the transfer function is $f_{3\text{-dB}} = 1/2\pi\tau_1 = 10$ MHz, then the time constant is $\tau_1 = 15.9$ ns.

Figure 7.20 summarizes the steady-state output responses for square wave input signals of the two circuits we've just been considering. Figure 7.20(a) shows the steady-state output response of the circuit in Figure 7.16 (coupling capacitor) for a long time constant, and Figure 7.20(b) shows the steady-state output response of the circuit in Figure 7.18 (load capacitor) for a short time constant.

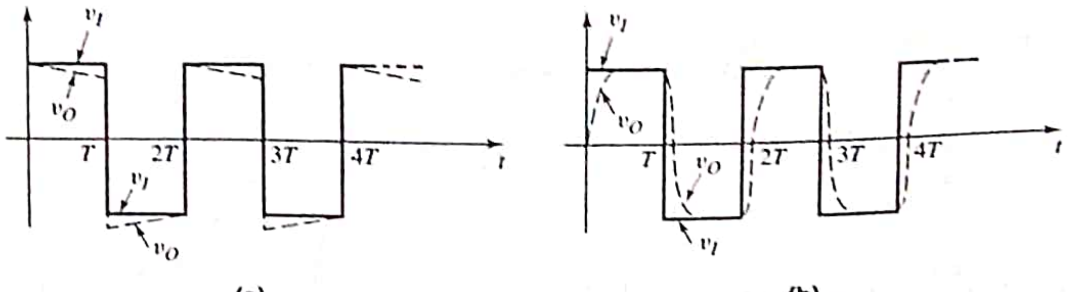


Figure 7.20 Steady-state output response for a square-wave input response for (a) circuit in Figure 7.16 (coupling capacitor) and a large time constant, and (b) circuit in Figure 7.18 (load capacitor) and a short time constant

7.3 FREQUENCY RESPONSE: TRANSISTOR AMPLIFIERS WITH CIRCUIT CAPACITORS

Objective: • Analyze the frequency response of transistor circuits with capacitors.

In this section, we will analyze the basic single-stage amplifier that includes circuit capacitors. Three types of capacitors will be considered: coupling capacitor, load capacitor, and bypass capacitor. In our hand analysis, we will consider each type of capacitor individually and determine its frequency response. In the last part of this section, we will consider the effect of multiple capacitors using a PSpice analysis.

The frequency response of multistage circuits will be considered in Chapter 12 when the stability of amplifiers is considered.

7.3.1 Coupling Capacitor Effects

Input Coupling Capacitor: Common-Emitter Circuit

Figure 7.21(a) shows a bipolar common-emitter circuit with a coupling capacitor. Figure 7.21(b) shows the corresponding small-signal equivalent circuit, with the transistor small-signal output resistance r_o assumed to

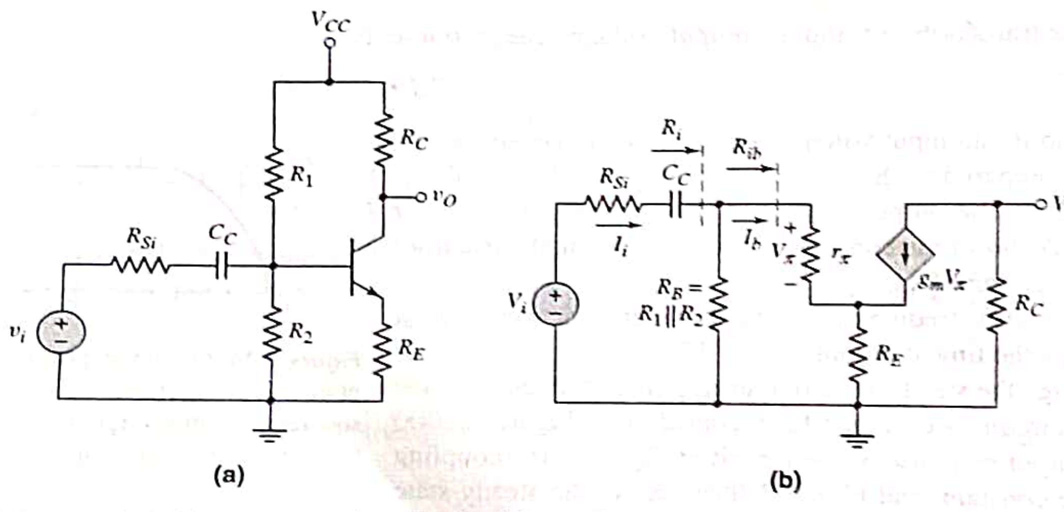


Figure 7.21 (a) Common-emitter circuit with coupling capacitor and (b) small-signal equivalent circuit

be infinite. This assumption is valid since $r_o \gg R_C$ and $r_o \gg R_E$ in most cases. Initially, we will use a current–voltage analysis to determine the frequency response of the circuit. Then, we will use the equivalent time constant technique.

From the analysis in the previous section, we note that this circuit is a high-pass network. At high frequencies, the capacitor C_C acts as a short circuit, and the input signal is coupled through the transistor to the output. At low frequencies, the impedance of C_C becomes large and the output approaches zero.

Current–Voltage Analysis: The input current can be written as

$$I_i = \frac{V_i}{R_{Si} + \dfrac{1}{sC_C} + R_i} \tag{7.30}$$

where the input resistance R_i is given by

$$R_i = R_B \| [r_\pi + (1+\beta)R_E] = R_B \| R_{ib} \tag{7.31}$$

In writing Equation (7.31), we used the resistance reflection rule given in Chapter 6. To determine the input resistance to the base of the transistor, we multiplied the emitter resistance by the factor $(1+\beta)$.

Using a current divider, we determine the base current to be

$$I_b = \left(\frac{R_B}{R_B + R_{ib}}\right) I_i \tag{7.32}$$

and then

$$V_\pi = I_b r_\pi \tag{7.33}$$

The output voltage is given by

$$V_o = -g_m V_\pi R_C \tag{7.34}$$

Combining Equations (7.30) through (7.34) produces

$$V_o = -g_m R_C (I_b r_\pi) = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right) I_i$$

$$= -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{V_i}{R_{Si} + \dfrac{1}{sC_C} + R_i}\right) \tag{7.35}$$

Therefore, the small-signal voltage gain is

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{sC_C}{1 + s(R_{Si} + R_i)C_C}\right) \tag{7.36}$$

which can be written in the form

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m r_\pi R_C}{(R_{Si} + R_i)} \left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{s\tau_S}{1 + s\tau_S}\right) \tag{7.37}$$

where the time constant is

$$\tau_S = (R_{Si} + R_i)C_C \tag{7.38}$$

The form of the voltage transfer function as given in Equation (7.37) is the same as that of Equation (7.5), for the coupling capacitor circuit in Figure 7.2. The Bode plot is therefore similar to that shown in Figure 7.5. The corner frequency is

$$f_L = \frac{1}{2\pi \tau_S} = \frac{1}{2\pi(R_{Si} + R_i)C_C} \tag{7.39}$$

and the maximum magnitude, in decibels, is

$$|A_v(\max)|_{dB} = 20\log_{10}\left(\frac{g_m r_\pi R_C}{R_{Si} + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right) \tag{7.40}$$

EXAMPLE 7.3

Objective: Calculate the corner frequency and maximum gain of a bipolar common-emitter circuit with a coupling capacitor.

For the circuit shown in Figure 7.21, the parameters are: $R_1 = 51.2\ \text{k}\Omega$, $R_2 = 9.6\ \text{k}\Omega$, $R_C = 2\ \text{k}\Omega$, $R_E = 0.4\ \text{k}\Omega$, $R_{Si} = 0.1\ \text{k}\Omega$, $C_C = 1\ \mu\text{F}$, and $V_{CC} = 10$ V. The transistor parameters are: $V_{BE}(\text{on}) = 0.7$ V, $\beta = 100$, and $V_A = \infty$.

Solution: From a dc analysis, the quiescent collector current is $I_{CQ} = 1.81$ mA. The transconductance is therefore

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.81}{0.026} = 69.6\ \text{mA/V}$$

and the diffusion resistance is

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.81} = 1.44\ \text{k}\Omega$$

The input resistance is

$$R_i = R_1 \| R_2 \| [r_\pi + (1+\beta)R_E]$$
$$= 51.2 \| 9.6 \| [1.44 + (101)(0.4)] = 6.77\ \text{k}\Omega$$

and the time constant is therefore

$$\tau_S = (R_{Si} + R_i)C_C = (0.1\times 10^3 + 6.77\times 10^3)(1\times 10^{-6}) = 6.87\times 10^{-3}\ \text{s}$$

or

$$\tau_S = 6.87\ \text{ms}$$

The corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(6.87\times 10^{-3})} = 23.2\ \text{Hz}$$

Finally, the maximum voltage gain magnitude is

$$|A_v|_{\max} = \frac{g_m r_\pi R_C}{(R_{Si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)$$

where

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.44 + (101)(0.4) = 41.8\ \text{k}\Omega$$

Therefore,

$$|A_v|_{\max} = \frac{(69.6)(1.44)(2)}{(0.1 + 6.775)}\left(\frac{8.084}{8.084 + 41.84}\right) = 4.72$$