

**EXAMPLE 6.5**

**Objective:** Determine the small-signal voltage gain, input resistance, and output resistance of the circuit shown in Figure 6.28.

Assume the transistor parameters are:  $\beta = 100$ ,  $V_{BE(on)} = 0.7$  V, and  $V_A = 100$  V.

**DC Solution:** We first perform a dc analysis to find the  $Q$ -point values. We find that  $I_{CQ} = 0.95$  mA and  $V_{CEQ} = 6.31$  V, which shows that the transistor is biased in the forward-active mode.

**AC Solution:** The small-signal hybrid- $\pi$  parameters for the equivalent circuit are

$$r_{\pi} = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(0.95)} = 2.74 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.95}{0.026} = 36.5 \text{ mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.95} = 105 \text{ k}\Omega$$

Assuming that  $C_C$  acts as a short circuit, Figure 6.29 shows the small-signal equivalent circuit. The small-signal output voltage is

$$V_o = -(g_m V_{\pi})(r_o \parallel R_C)$$

The dependent current  $g_m V_{\pi}$  flows through the parallel combination of  $r_o$  and  $R_C$ , but in a direction that produces a negative output voltage. We can relate the control voltage  $V_{\pi}$  to the input voltage  $V_s$  by a voltage divider. We have

$$V_{\pi} = \left( \frac{R_1 \parallel R_2 \parallel r_{\pi}}{R_1 \parallel R_2 \parallel r_{\pi} + R_S} \right) \cdot V_s$$

We can then write the small-signal voltage gain as

$$A_v = \frac{V_o}{V_s} = -g_m \left( \frac{R_1 \parallel R_2 \parallel r_{\pi}}{R_1 \parallel R_2 \parallel r_{\pi} + R_S} \right) (r_o \parallel R_C)$$

or

$$A_v = -(36.5) \left( \frac{5.9 \parallel 2.74}{5.9 \parallel 2.74 + 0.5} \right) (105 \parallel 6) = -163$$

We can also calculate  $R_i$ , which is the resistance to the amplifier. From Figure 6.29, we see that

$$R_i = R_1 \parallel R_2 \parallel r_{\pi} = 5.9 \parallel 2.74 = 1.87 \text{ k}\Omega$$

The output resistance  $R_o$  is found by setting the independent source  $V_s$  equal to zero. In this case, there is no excitation to the input portion of the circuit so  $V_{\pi} = 0$ , which implies that  $g_m V_{\pi} = 0$  (an open circuit). The output resistance looking back into the output terminals is then

$$R_o = r_o \parallel R_C = 105 \parallel 6 = 5.68 \text{ k}\Omega$$

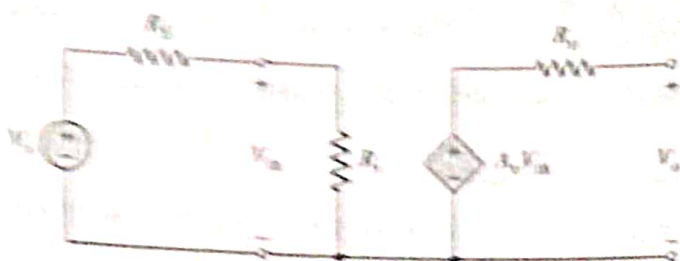


Figure 6.30 Two-port equivalent circuit for the amplifier in Example 6.5

**Comment:** In this circuit, the effective series resistance between the voltage source  $V_s$  and the base of the transistor is much less than that given in Example 6.1. For this reason, the magnitude of the voltage gain for the circuit given in Figure 6.28 is much larger than that found in Example 6.1.

**Discussion:** The two-port equivalent circuit along with the input signal source for the common-emitter amplifier analyzed in this example is shown in Figure 6.30. We can determine the effect of the source resistance  $R_s$  in conjunction with the amplifier input resistance  $R_i$ . Using a voltage-divider equation, we find the input voltage to the amplifier is

$$V_{in} = \left( \frac{R_i}{R_i + R_s} \right) V_s = \left( \frac{1.87}{1.87 + 0.5} \right) V_s = 0.789 V_s$$

Because the input resistance to the amplifier is not very much greater than the signal source resistance, the actual input voltage to the amplifier is reduced to approximately 80 percent of the signal voltage. This is called a **loading effect**. The voltage  $V_{in}$  is a function of the amplifier connected to the source. In other amplifier designs, we will try to minimize the loading effect, or make  $R_i \gg R_s$ , which means that  $V_{in} \cong V_s$ .

## EXERCISE PROBLEM

**Ex 6.5:** The circuit parameters in Figure 6.28 are changed to  $V_{CC} = 5$  V,  $R_1 = 35.2$  k $\Omega$ ,  $R_2 = 5.83$  k $\Omega$ ,  $R_C = 10$  k $\Omega$ , and  $R_E = 0$ . Assume the transistor parameters are the same as listed in Example 6.5. Determine the quiescent collector current and collector-emitter voltage, and find the small-signal voltage gain. (Ans.  $I_{CQ} = 0.21$  mA,  $V_{CEQ} = 2.9$  V,  $A_v = -79.1$ )

### 6.4.2 Circuit with Emitter Resistor

For the circuit in Figure 6.28, the bias resistors  $R_1$  and  $R_2$  in conjunction with  $V_{CC}$  produce a base current of  $9.5$   $\mu$ A and a collector current of  $0.95$  mA, when the B–E turn-on voltage is assumed to be  $0.7$  V. If the transistor in the circuit is replaced by a new one with slightly different parameters so that the B–E turn-on voltage is  $0.6$  V instead of  $0.7$  V, then the resulting base current is  $26$   $\mu$ A, which is sufficient to drive the transistor into saturation. Therefore, the circuit shown in Figure 6.28 is not practical. An improved dc biasing design includes an emitter resistor.

In the last chapter, we found that the  $Q$ -point was stabilized against variations in  $\beta$  if an emitter resistor were included in the circuit, as shown in Figure 6.31. We will find a similar property for the ac signals, in that the voltage gain of a circuit with  $R_E$  will be less dependent on the transistor current gain  $\beta$ . Even though the emitter of this circuit is not at ground potential, this circuit is still referred to as a common-emitter circuit.



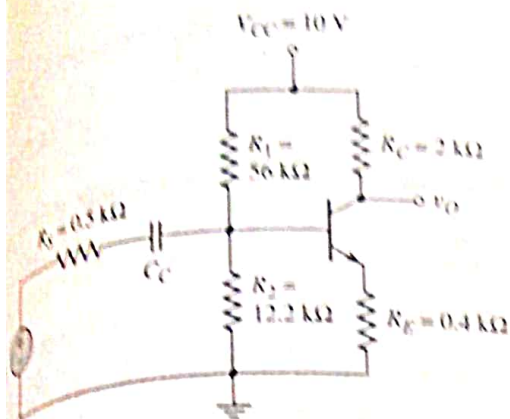


Figure 6.31 An npn common-emitter circuit with an emitter resistor, a voltage-divider biasing network, and a coupling capacitor

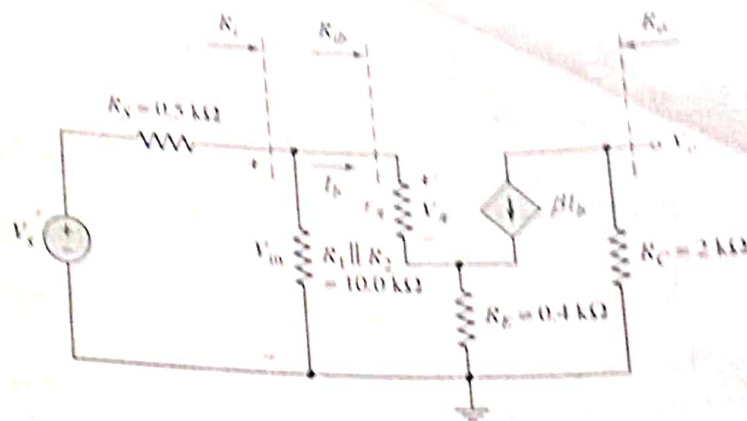


Figure 6.32 The small-signal equivalent circuit of the circuit shown in Figure 6.31

Assuming that  $C_C$  acts as a short circuit, Figure 6.32 shows the small-signal hybrid- $\pi$  equivalent circuit. As we have mentioned previously, to develop the small-signal equivalent circuit, start with the three terminals of the transistor. Sketch the hybrid- $\pi$  equivalent circuit between the three terminals and then sketch in the remaining circuit elements around these three terminals. In this case, we are using the equivalent circuit with the current gain parameter  $\beta$ , and we are assuming that the Early voltage is infinite so the transistor output resistance  $r_o$  can be neglected (an open circuit). The ac output voltage is

$$V_o = -(\beta I_b) R_C \quad (6.53)$$

To find the small-signal voltage gain, it is worthwhile finding the input resistance first. The resistance  $R_{ib}$  is the input resistance looking into the base of the transistor. We can write the following loop equation

$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E \quad (6.54)$$

The input resistance  $R_{ib}$  is then defined as, and found to be,

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta) R_E \quad (6.55)$$

the common-emitter configuration that includes an emitter resistance, the small-signal input resistance looking into the base of the transistor is  $r_\pi$  plus the emitter resistance multiplied by the factor  $(1 + \beta)$ . This effect is called the **resistance reflection rule**. We will use this result throughout the text without further derivation.

The input resistance to the amplifier is now

$$R_i = R_1 \parallel R_2 \parallel R_{ib} \quad (6.56)$$

We can again relate  $V_{in}$  to  $V_s$  through a voltage-divider equation as

$$V_{in} = \left( \frac{R_i}{R_i + R_S} \right) \cdot V_s \quad (6.57)$$

Combining Equations (6.53), (6.55), and (6.57), we find the small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{-(\beta I_b) R_C}{V_s} = -\beta R_C \left( \frac{V_{in}}{R_{ib}} \right) \cdot \left( \frac{1}{V_s} \right) \quad (6.58(a))$$

or

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} \left( \frac{R_i}{R_i + R_S} \right) \quad (6.58(b))$$

From this equation, we see that if  $R_i \gg R_S$  and if  $(1 + \beta)R_E \gg r_\pi$ , then the small-signal voltage gain is approximately

$$A_v \cong \frac{-\beta R_C}{(1 + \beta)R_E} \cong \frac{-R_C}{R_E} \quad (6.59)$$

Equations (6.58(b)) and (6.59) show that the voltage gain is less dependent on the current gain  $\beta$  than in the previous example, which means that there is a smaller change in voltage gain when the transistor current gain changes. The circuit designer now has more control in the design of the voltage gain, but this advantage is at the expense of a smaller gain.

In Chapter 5, we discussed the variation in the  $Q$ -point with variations or tolerances in resistor values. Since the voltage gain is a function of resistor values, it is also a function of the tolerances in those values. This must be considered in a circuit design.

### EXAMPLE 6.6

**Objective:** Determine the small-signal voltage gain and input resistance of a common-emitter circuit with an emitter resistor.

For the circuit in Figure 6.31, the transistor parameters are:  $\beta = 100$ ,  $V_{BE(on)} = 0.7$  V, and  $V_A = \infty$ .

**DC Solution:** From a dc analysis of the circuit, we can determine that  $I_{CQ} = 2.16$  mA and  $V_{CEQ} = 4.81$  V, which shows that the transistor is biased in the forward-active mode.

**AC Solution:** The small-signal hybrid- $\pi$  parameters are determined to be

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(2.16)} = 1.20 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.16}{0.026} = 83.1 \text{ mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

The input resistance to the base can be determined as

$$R_{ib} = r_\pi + (1 + \beta)R_E = 1.20 + (101)(0.4) = 41.6 \text{ k}\Omega$$

and the input resistance to the amplifier is now found to be

$$R_i = R_1 \parallel R_2 \parallel R_{ib} = 10 \parallel 41.6 = 8.06 \text{ k}\Omega$$

Using the exact expression for the voltage gain, we find

$$A_v = \frac{-(100)(2)}{1.20 + (101)(0.4)} \left( \frac{8.06}{8.06 + 0.5} \right) = -4.53$$



If we use the approximation given by Equation (6.59), we obtain

$$A_v = \frac{-R_C}{R_E} = \frac{-2}{0.4} = -5.0$$

**Comment:** The magnitude of the small-signal voltage gain is substantially reduced when an emitter resistor is included. Also, Equation (6.59) gives a good first approximation for the gain, which means that it can be used in the initial design of a common-emitter circuit with an emitter resistor.

**Discussion:** The amplifier gain is nearly independent of changes in the current gain parameter  $\beta$ . This fact is shown in the following calculations:

| $\beta$ | $A_v$ |
|---------|-------|
| 50      | -4.41 |
| 100     | -4.53 |
| 150     | -4.57 |

In addition to gaining an advantage in stability by including an emitter resistance, we also gain an advantage in the loading effect. We see that, for  $\beta = 100$ , the input voltage to the amplifier is

$$V_{in} = \left( \frac{R_i}{R_i + R_s} \right) \cdot V_s = (0.942) V_s$$

We see that  $V_{in}$  is much closer in value to  $V_s$  than in the previous example. There is less loading effect because the input resistance to the base of the transistor is higher when an emitter resistor is included.

The same equivalent circuit as shown in Figure 6.30 applies to this example also. The difference in the two examples is the values of input resistance and gain parameter.

### EXERCISE PROBLEM

**Ex 6.6:** For the circuit in Figure 6.33, let  $R_E = 0.6 \text{ k}\Omega$ ,  $R_C = 5.6 \text{ k}\Omega$ ,  $\beta = 120$ ,  $V_{BE(on)} = 0.7 \text{ V}$ ,  $R_1 = 250 \text{ k}\Omega$ , and  $R_2 = 75 \text{ k}\Omega$ . (a) For  $V_A = \infty$ , determine the small-signal voltage gain  $A_v$ . (b) Determine the input resistance looking into the base of the transistor. (Ans. (a)  $A_v = -8.27$ , (b)  $R_{ib} = 80.1 \text{ k}\Omega$ )

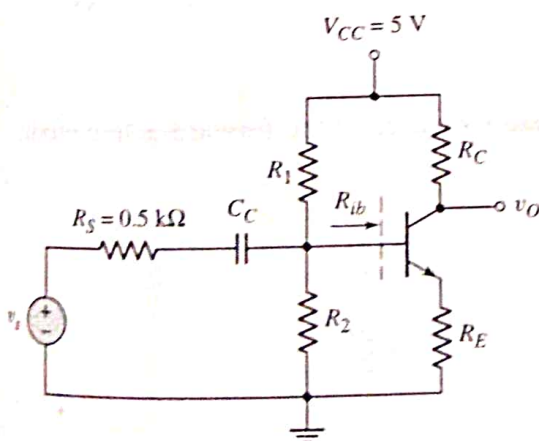


Figure 6.33 Figure for Exercise Ex6.6

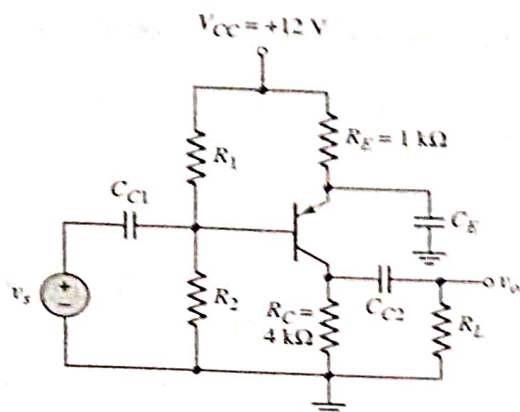


Figure 6.51 Figure for Exercise Ex6.12

## Test Your Understanding

**TYU 6.8** For the circuit in Figure 6.33, use the parameters given in Exercise Ex6.6. If the total instantaneous current must always be greater than 0.1 mA and the total instantaneous C–E voltage must be in the range  $0.5 \leq v_{CE} \leq 5$  V, determine the maximum symmetrical swing in the output voltage. (Ans. 3.82 V peak-to-peak)

**TYU 6.9** For the circuit in Figure 6.40, assume the transistor parameters are:  $\beta = 100$ ,  $V_{BE(on)} = 0.7$  V, and  $V_A = \infty$ . Determine a new value of  $R_E$  that will achieve a maximum symmetrical swing in the output voltage, for  $i_C > 0$  and  $0.7 \leq v_{CE} \leq 19.5$  V. What is the maximum symmetrical swing that can be achieved? (Ans.  $R_E = 16.4$  k $\Omega$ , 10.6 V peak-to-peak)

## 6.6 COMMON-COLLECTOR (EMITTER-FOLLOWER) AMPLIFIER

**Objective:** • Analyze the emitter-follower amplifier and become familiar with the general characteristics of this circuit.

The second type of transistor amplifier to be considered is the **common-collector circuit**. An example of this circuit configuration is shown in Figure 6.52. As seen in the figure, the output signal is taken off of the emitter with respect to ground and the collector is connected directly to  $V_{CC}$ . Since  $V_{CC}$  is at signal ground in the ac equivalent circuit, we have the name common-collector. The more common name for this circuit is **emitter follower**. The reason for this name will become apparent as we proceed through the analysis.

### 6.6.1 Small-Signal Voltage Gain

The dc analysis of the circuit is exactly the same as we have already seen, so we will concentrate on the small-signal analysis. The hybrid- $\pi$  model of the bipolar transistor can also be used in the small-signal analysis of this circuit. Assuming the coupling capacitor  $C_C$  acts as a short circuit, Figure 6.53 shows the small-signal

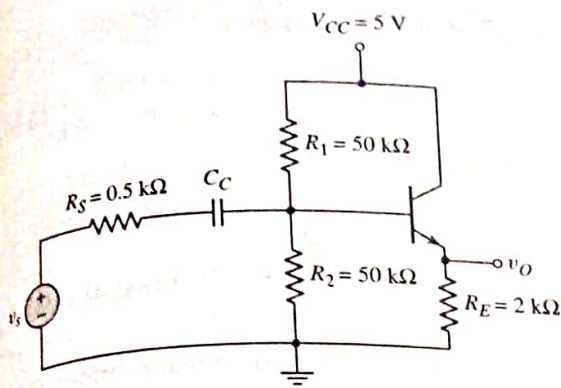


Figure 6.52 Emitter-follower circuit. Output signal is at the emitter terminal with respect to ground.

equivalent circuit of the circuit shown in Figure 6.52. The collector terminal is at signal ground and the transistor output resistance  $r_o$  is in parallel with the dependent current source.

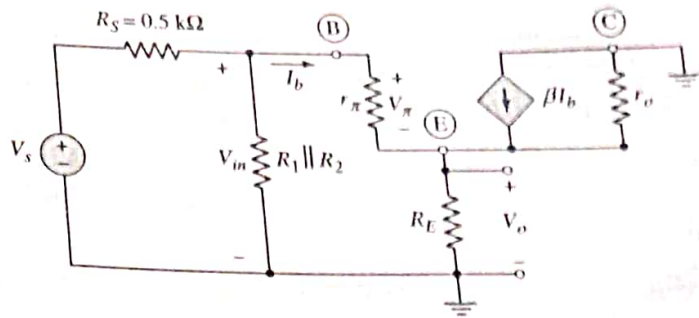


Figure 6.53 Small-signal equivalent circuit of the emitter-follower

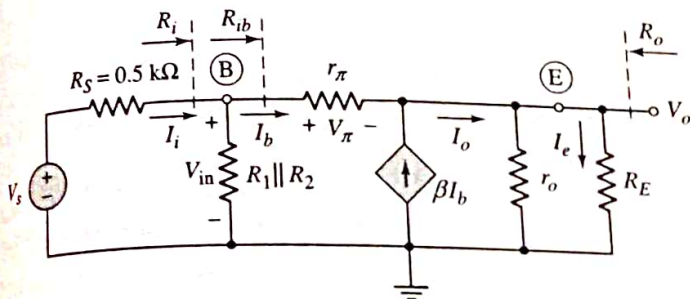


Figure 6.54 Small-signal equivalent circuit of the emitter-follower with all signal grounds connected together

Figure 6.54 shows the equivalent circuit rearranged so that all signal grounds are at the same point. We see that

$$I_o = (1 + \beta)I_b \quad (6.64)$$

so the output voltage can be written as

$$V_o = I_b(1 + \beta)(r_o \parallel R_E) \quad (6.65)$$

Writing a KVL equation around the base-emitter loop, we obtain

$$V_{in} = I_b[r_\pi + (1 + \beta)(r_o \parallel R_E)] \quad (6.66(a))$$

or

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta)(r_o \parallel R_E) \quad (6.66(b))$$

We can also write

$$V_{in} = \left( \frac{R_i}{R_i + R_S} \right) \cdot V_s \quad (6.67)$$

where  $R_i = R_1 \parallel R_2 \parallel R_{ib}$ .



Combining Equations (6.65), (6.66(b)), and (6.67), the small-signal voltage gain is

$$A_v = \frac{V_o}{V_i} = \frac{(1 + \beta)(r_e \parallel R_2)}{r_e + (1 + \beta)(r_e \parallel R_2)} \cdot \left( \frac{R_1}{R_1 + R_2} \right) \quad (6.68)$$

### EXAMPLE 6.13

**Objective:** Calculate the small-signal voltage gain of an emitter-follower circuit.

For the circuit shown in Figure 6.52, assume the transistor parameters are:  $\beta = 100$ ,  $V_{BE(on)} = 0.7$  V, and  $V_A = 80$  V.

**Solution:** The dc analysis shows that  $I_{EQ} = 0.793$  mA and  $V_{EQ} = 3.4$  V. The small-signal hybrid- $\pi$  parameters are determined to be

$$r_e = \frac{V_T/\beta}{I_{EQ}} = \frac{(0.026)(100)}{0.793} = 3.28 \text{ k}\Omega$$

$$r_{\pi} = \frac{I_{EQ}}{V_T} = \frac{0.793}{0.026} = 30.5 \text{ mA/V}$$

and

$$r_o = \frac{V_A}{I_{EQ}} = \frac{80}{0.793} \cong 100 \text{ k}\Omega$$

We may note that

$$R_{ib} = 3.28 + (101)(100 \parallel 2) = 201 \text{ k}\Omega$$

and

$$R_i = 50 \parallel 50 \parallel 201 = 22.2 \text{ k}\Omega$$

The small-signal voltage gain is then

$$A_v = \frac{(101)(100 \parallel 2)}{3.28 + (101)(100 \parallel 2)} \cdot \left( \frac{22.2}{22.2 + 0.5} \right)$$

or

$$A_v = +0.962$$

**Comment:** The magnitude of the voltage gain is slightly less than 1. An examination of Equation (6.68) shows that this is always true. Also, the voltage gain is positive, which means that the output signal voltage at the emitter is in phase with the input signal voltage. The reason for the terminology emitter-follower is now clear. The output voltage at the emitter is essentially equal to the input voltage.

At first glance, a transistor amplifier with a voltage gain essentially of 1 may not seem to be of much value. However, the input and output resistance characteristics make this circuit extremely useful in many applications as we will show in the next section.

### EXERCISE PROBLEM

**Ex 6.13:** For the circuit shown in Figure 6.52, let  $V_{CC} = 5$  V,  $\beta = 120$ ,  $V_A = 100$  V,  $R_E = 1$  k $\Omega$ ,  $V_{BE(on)} = 0.7$  V,  $R_1 = 25$  k $\Omega$ , and  $R_2 = 50$  k $\Omega$ . (a) Determine the small-signal voltage gain  $A_v = V_o/V_i$ . (b) Find the input resistance looking into the base of the transistor. (Ans. (a)  $A_v = 0.956$  (b)  $R_{ib} = 120$  k $\Omega$ )



## COMPUTER ANALYSIS EXERCISE

**P56.4:** Perform a PSpice simulation on the circuit in Figure 6.52. (a) Determine the small-signal voltage gain and (b) find the effective resistance seen by the signal source  $v_i$ .

## 6.6.2 Input and Output Impedance

## Input Resistance

The input impedance, or small-signal input resistance for low-frequency signals, of the emitter-follower is determined in the same manner as for the common-emitter circuit. For the circuit in Figure 6.52, the input resistance looking into the base is denoted  $R_{ib}$  and is indicated in the small-signal equivalent circuit shown in Figure 6.54.

The input resistance  $R_{ib}$  was given by Equation (6.66(b)) as

$$R_{ib} = r_\pi + (1 + \beta)(r_o \parallel R_E)$$

Since the emitter current is  $(1 + \beta)$  times the base current, the effective impedance in the emitter is multiplied by  $(1 + \beta)$ . We saw this same effect when an emitter resistor was included in a common-emitter circuit. This multiplication by  $(1 + \beta)$  is again called the **resistance reflection rule**. The input resistance at the base is  $r_\pi$  plus the effective resistance in the emitter multiplied by the  $(1 + \beta)$  factor. This resistance reflection rule will be used extensively throughout the remainder of the text.

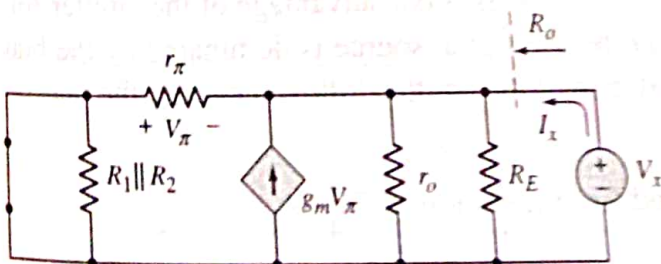
## Output Resistance

Initially, to find the output resistance of the emitter-follower circuit shown in Figure 6.52, we will assume the input signal source is ideal and that  $R_S = 0$ . The circuit shown in Figure 6.55 can be used to determine the output resistance looking back into the output terminals. The circuit is derived from the small-signal equivalent circuit shown in Figure 6.54 by setting the independent voltage source  $V_s$  equal to zero, which means that  $V_s$  acts as a short circuit. A test voltage  $V_x$  is applied to the output terminal and the resulting current is  $I_x$ . The output resistance,  $R_o$ , is given by

$$R_o = \frac{V_x}{I_x}$$

In this case, the control voltage  $V_\pi$  is not zero, but is a function of the applied test voltage. From Figure 6.55, we see that  $V_\pi = -V_x$ . Summing currents at the output node, we have

$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi}$$



**Figure 6.55** Small-signal equivalent circuit of the emitter-follower used to determine the output resistance. resistance  $R_S$  is assumed to be zero (an ideal signal source).

Since  $V_x = -V_x$ , Equation (6.70) can be written as

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_m + \frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{r_\pi} \quad (6.71)$$

or the output resistance is given by

$$R_o = \frac{1}{g_m} \parallel R_E \parallel r_o \parallel r_\pi \quad (6.72)$$

The output resistance may also be written in a slightly different form. Equation (6.71) can be written in the form

$$\frac{1}{R_o} = \left( g_m + \frac{1}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} = \left( \frac{1 + \beta}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} \quad (6.73)$$

or the output resistance can be written in the form

$$R_o = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_o \quad (6.74)$$

Equation (6.74) says that the output resistance looking back into the output terminals is the effective resistance in the emitter,  $R_E \parallel r_o$ , in parallel with the resistance looking back into the emitter. In turn, the resistance looking into the emitter is the total resistance in the base circuit divided by  $(1 + \beta)$ . This is an important result and is called the **inverse resistance reflection rule** and is the inverse of the reflection rule looking to the base.

### EXAMPLE 6.14

**Objective:** Calculate the input and output resistance of the emitter-follower circuit shown in Figure 6.52. Assume  $R_S = 0$ .

The small-signal parameters, as determined in Example 6.13, are  $r_\pi = 3.28 \text{ k}\Omega$ ,  $\beta = 100$ , and  $r_o = 100 \text{ k}\Omega$ .

**Solution (Input Resistance):** The input resistance looking into the base was determined in Example 6.13 as

$$R_{ib} = r_\pi + (1 + \beta)(r_o \parallel R_E) = 3.28 + (101)(100 \parallel 2) = 201 \text{ k}\Omega$$

and the input resistance seen by the signal source  $R_i$  is

$$R_i = R_1 \parallel R_2 \parallel R_{ib} = 50 \parallel 50 \parallel 201 = 22.2 \text{ k}\Omega$$

**Comment:** The input resistance of the emitter-follower looking into the base is substantially larger than that of the simple common-emitter circuit because of the  $(1 + \beta)$  factor. This is one advantage of the emitter-follower circuit. However, in this case, the input resistance seen by the signal source is dominated by the bias resistors  $R_1$  and  $R_2$ . To take advantage of the large input resistance of the emitter-follower circuit, the bias resistors must be designed to be much larger.

**Solution (Output Resistance):** The output resistance is found from Equation (6.74) as

$$R_o = \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \parallel r_o = \left( \frac{3.28}{101} \right) \parallel 2 \parallel 100$$



$$R_o = 0.0325 \parallel 2 \parallel 100 = 0.0320 \text{ k}\Omega \rightarrow 32.0 \Omega$$

The output resistance is dominated by the first term that has  $(1 + \beta)$  in the denominator.

**Comment:** The emitter-follower circuit is sometimes referred to as an impedance transformer, since the input impedance is large and the output impedance is small. The very low output resistance makes the emitter-follower act almost like an ideal voltage source, so the output is not loaded down when used to drive another load. Because of this, the emitter-follower is often used as the output stage of a multistage amplifier.

### EXERCISE PROBLEM

**Ex 6.14:** Consider the circuit and transistor parameters described in Exercise Ex6.13 for the circuit shown in Figure 6.52. For the case of  $R_S = 0$ , determine the output resistance looking into the output terminals. (Ans. 11.1  $\Omega$ )

We can determine the output resistance of the emitter-follower circuit taking into account a nonzero source resistance. The circuit in Figure 6.56 is derived from the small-signal equivalent circuit shown in Figure 6.54 and can be used to find  $R_o$ . The independent source  $V_s$  is set equal to zero and a test voltage  $V_x$  is applied to the output terminals. Again, the control voltage  $V_\pi$  is not zero, but is a function of the test voltage. Summing currents at the output node, we have

$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 \parallel R_2 \parallel R_S} \quad (6.75)$$

The control voltage can be written in terms of the test voltage by a voltage divider equation as

$$V_\pi = -\left(\frac{r_\pi}{r_\pi + R_1 \parallel R_2 \parallel R_S}\right) \cdot V_x \quad (6.76)$$

Equation (6.75) can then be written as

$$I_x = \left(\frac{g_m r_\pi}{r_\pi + R_1 \parallel R_2 \parallel R_S}\right) \cdot V_x + \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 \parallel R_2 \parallel R_S} \quad (6.77)$$

Noting that  $g_m r_\pi = \beta$ , we find

$$\frac{I_x}{V_x} = \frac{1}{R_o} = \frac{1 + \beta}{r_\pi + R_1 \parallel R_2 \parallel R_S} + \frac{1}{R_E} + \frac{1}{r_o} \quad (6.78)$$

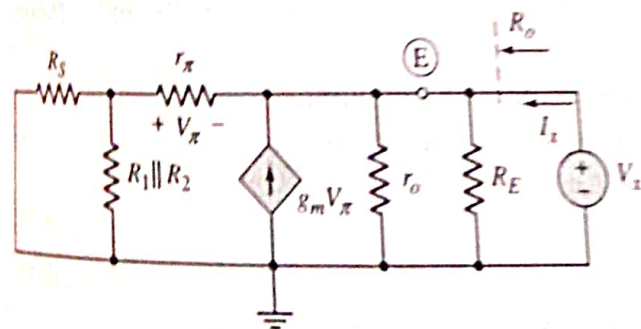


Figure 6.56 Small-signal equivalent circuit of the emitter-follower used to determine the output resistance including the effect of the source resistance  $R_S$

or

$$R_o = \left( \frac{r_o + R_1 \parallel R_2 \parallel R_s}{1 + \beta} \right) \parallel R_E \parallel r_o$$

In this case, the source resistance and bias resistances contribute to the output resistance.

(6.79)

### 6.8.3 Small-Signal Current Gain

We can determine the small-signal current gain of an emitter-follower by using the input resistance and the concept of current dividers. For the small-signal emitter-follower equivalent circuit shown in Figure 6.54, the small-signal current gain is defined as

$$A_i = \frac{I_e}{I_i}$$

(6.80)

where  $I_e$  and  $I_i$  are the output and input current phasors.

Using a current divider equation, we can write the base current in terms of the input current, as follows:

$$I_b = \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) I_i$$

(6.81)

Since  $g_m V_{\pi} = \beta I_b$ , then,

$$I_o = (1 + \beta) I_b = (1 + \beta) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) I_i$$

(6.82)

Writing the load current in terms of  $I_o$  produces

$$I_e = \left( \frac{r_o}{r_o + R_E} \right) I_o$$

(6.83)

Combining Equations (6.82) and (6.83), we obtain the small-signal current gain, as follows:

$$A_i = \frac{I_e}{I_i} = (1 + \beta) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) \left( \frac{r_o}{r_o + R_E} \right)$$

(6.84)

If we assume that  $R_1 \parallel R_2 \gg R_{ib}$  and  $r_o \gg R_E$ , then

$$A_i \cong (1 + \beta)$$

(6.85)

which is the current gain of the transistor.

Although the small-signal voltage gain of the emitter follower is slightly less than 1, the small-signal current gain is normally greater than 1. Therefore, the emitter-follower circuit produces a small-signal power gain.

Although we did not explicitly calculate a current gain in the common-emitter circuit previously, the analysis is the same as that for the emitter-follower and in general the current gain is also greater than unity.

## DESIGN EXAMPLE 6.15

**Objective:** To design an emitter-follower amplifier to meet an output resistance specification.