Q1. (a) Let x^+ denote the locations of the blocks with positive polarity, and let x^- denote the location of the blocks with negative polarity. Also, let y_{ij} represent a non-negative decision variable representing the length of the wire connecting terminal blocks x_i^+ and x_j^-. Using the ℓ_2 norm, we define the linear program as follows:

$$\text{minimize} \sum_{i,j} y_{ij} \text{ subject to}$$
$$y_{ij} \geq \|x_i^+ - x_j^-\|_2$$
$$y_{ij} \geq 0$$

i.e.

$$\text{maximize} \sum_{i,j} -y_{ij} \text{ subject to}$$
$$-y_{ij} \leq -\|x_i^+ - x_j^-\|_2$$
$$y_{ij} \geq 0$$

(b) Hence, its corresponding dual becomes:

$$\text{minimize} \sum_{i,j} -\|x_i^+ - x_j^-\|_2 \, z_{ij} \text{ subject to}$$
$$-z_{ij} \geq -1$$

Q2. Let $\boldsymbol{\mathcal{P}}$ denote the following linear program in n variables: $\boldsymbol{\mathcal{P}} = \min\{c^T x \mid Ax \geq b, x \in \mathbb{R}^n\}$. Then, by asymmetric duality, the *primal* $\boldsymbol{\mathcal{P}}$ corresponds to the *dual* $\boldsymbol{\mathcal{D}} = \max\{b^T y \mid A^T y = c, y \geq 0\}$ in m variables. Suppose this *dual* is feasible with maximum value z. Then, the following linear program $\boldsymbol{\mathcal{G}} = \{x \mid Ax \geq b, c^T x \leq z\}$ is feasible only if $\boldsymbol{\mathcal{P}}$ is feasible with value at most z. This follows from the fact that $\min(c^T x) \leq c^T x \leq z$ and the existence of an optimum $\min(c^T x) > z$ in $\boldsymbol{\mathcal{P}}$ violates the second linear constraint of $\boldsymbol{\mathcal{G}}$; thereby, making it infeasible.

Now, if we partition the interval $[-M, +M]$ with a neighborhood around z of the form $(z-\varepsilon, z+\varepsilon)$ with $\varepsilon > 0$, then the size of the search space for the optimal value of $\boldsymbol{\mathcal{P}}$ is precisely M/ε. This is because the interval $[-M, +M]$ has length/measure $2M$ and a neighborhood of the form $(z-\varepsilon, z+\varepsilon)$ has measure 2ε. So partitioning $2M$ with 2ε produces precisely M/ε neighborhoods—each of size 2ε—where the optimal value of $\boldsymbol{\mathcal{P}}$ lands up to an error of $\pm\varepsilon$.

Thus, the tentative solution here is to execute binary search on these M/ε intervals, and check for feasibility on the extremities $z-\varepsilon$ and $z+\varepsilon$ using the **oracle**. In other words, guess $z = 0$ to be the optimal, then if $\boldsymbol{\mathcal{G}}$ is feasible at $z-\varepsilon$ search left; otherwise, if $\mathcal{G}$ is infeasible at $z+\varepsilon$ search right. If none of the aforementioned conditions evaluate, i.e. $\boldsymbol{\mathcal{G}}$ is infeasible at $z-\varepsilon$ and feasible at $z+\varepsilon$ (meaning that $z-\varepsilon < \min(c^T x) \leq z+\varepsilon$) report $z \pm \varepsilon$ as the optimal value of $\boldsymbol{\mathcal{P}}$.

Since we have an initial problem space of size M/ε, and each time we either search left or right, we're guaranteed to consult the **oracle** at most $O(\log(M/\varepsilon))$ times.