1. (a) Let R and B denote the sets of red and blue terminal blocks, respectively. For each $r_i \in R$, we associate a non-negative scalar value l_{r_i}, representing the length of the wire emanating from terminal r_i. Analogously, for each $b_j \in B$, we associate a non-negative wire length l_{b_j}. Then define d_{ij} to be the Euclidean distance between terminals r_i and b_j, so that $d_{ij} = \|r_i - b_j\|_2$. The primal problem is then formulated as follows:

$$\text{maximize } -\left(\sum_{r_i \in R} l_{r_i} + \sum_{b_j \in B} l_{b_j}\right) \text{ subject to}$$
$$-\left(l_{r_i} + l_{b_j}\right) \leq -d_{ij} \quad \forall r_i \in R \land \forall b_j \in B$$
$$l_{r_i} \geq 0 \quad \forall r_i \in R$$
$$l_{b_j} \geq 0 \quad \forall b_j \in B$$

(b) To derive the dual formulation, we introduce non-negative Lagrangian multipliers $y_{ij} \geq 0$ corresponding to each primal constraint $l_{r_i} + l_{b_j} \geq d_{ij}$ to obtain:

$$\text{minimize } \sum_{r_i \in R} \sum_{b_j \in B} -d_{ij} \times y_{ij} \text{ subject to}$$
$$-\sum_{r_i \in R} y_{ij} \geq -1 \quad \forall b_j \in B$$
$$-\sum_{b_j \in B} y_{ij} \geq -1 \quad \forall r_i \in R$$
$$y_{ij} \geq 0 \quad \forall r_i \in R \land \forall b_j \in B$$

2. Let $\mathcal{P}$ denote the following linear program in n variables: $\mathcal{P} = \min\{c^T x \mid Ax \geq b, x \in \mathbb{R}^n\}$. Then, by asymmetric duality, the *primal* $\mathcal{P}$ corresponds to the *dual* $\mathcal{D} = \max\{b^T y \mid A^T y = c, y \geq 0\}$ in m variables. Suppose this *dual* is feasible with maximum value z. Then, the following linear program $\mathcal{G} = \{x \mid Ax \geq b, c^T x \leq z\}$ is feasible only if $\mathcal{P}$ is feasible with value at most z. This observation follows from the fact that $\min(c^T x) \leq c^T x \leq z$ and the existence of an optimum $\min(c^T x) > z$ in $\mathcal{P}$ violates the second linear constraint of $\mathcal{G}$; thereby, making it infeasible.

Now, if we partition the interval $[-M, +M]$ with a neighborhood around z of the form $(z - \varepsilon, z + \varepsilon)$ with $\varepsilon > 0$, then the size of the search space for the optimal value of $\mathcal{P}$ is precisely M/ε. This is because the interval $[-M, +M]$ has length/measure $2M$ and a neighborhood of the form $(z - \varepsilon, z + \varepsilon)$ has measure 2ε. So partitioning $2M$ with 2ε produces precisely M/ε neighborhoods—each of size 2ε—where the optimal value of $\mathcal{P}$ lands up to an error of $\pm\varepsilon$.

Thus, the tentative solution here is to execute binary search on these M/ε intervals, and check for feasibility on the extremities $z - \varepsilon$ and $z + \varepsilon$ using the oracle. In other words, guess $z = 0$ to be the optimal, then if $\mathcal{G}$ is feasible at $z - \varepsilon$ search left; otherwise, if $\mathcal{G}$ is infeasible at $z + \varepsilon$ search right. If none of the aforementioned conditions evaluate, i.e. $\mathcal{G}$ is infeasible at $z - \varepsilon$ and feasible at $z + \varepsilon$ (meaning that $z - \varepsilon < \min(c^T x) \leq z + \varepsilon$) report $z \pm \varepsilon$ as the optimal value of $\mathcal{P}$.

Since we have an initial problem space of size M/ε, and each time we either search left or right, we're guaranteed to consult the oracle at most $\mathcal{O}(\log(M/\varepsilon))$ times.