

Haskell 102

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- ▶ 101: basic skills
 - ▶ Reading function types
 - ▶ Pattern matching
 - ▶ Data structures
- ▶ 102: first project
 - ▶ Genericity
 - ▶ IO and do notation
 - ▶ Build a game!



- ▶ 101 Recap
- ▶ Remaining obstacles
- ▶ Typeclasses overview
- ▶ Common examples
- ▶ Advanced syntax



- ▶ 101 Recap
- ▶ Remaining obstacles
- ▶ Typeclasses overview
- ▶ Typeclasses
- ▶ Typeclasses
- ▶ Typeclasses
- ▶ Typeclasses...



- ▶ Haskell 101
- ▶ download the codelab
- ▶ apt-get install haskell-platform



Curried functions, partial application

f :: Int → Int → [Int]

Curried functions, partial application

f :: Int → (Int → [Int])

f :: Int → Int → [Int]

Curried functions, partial application

f :: Int → (Int → [Int])

f :: Int → Int → [Int]

f 1 :: Int → [Int]

Curried functions, partial application

```
f      :: Int → ( Int → [Int] )  
f      ::           Int → Int → [Int]  
f 1    ::           Int → [Int]  
  
(f 1) 2  ::           [Int]
```

Curried functions, partial application

```
f      :: Int → ( Int → [Int] )  
f      :: Int → Int → [Int]  
f 1    :: Int → [Int]  
f 1 2   :: [Int]  
(f 1) 2 :: [Int]
```

Type constructors

```
-- enum  
data Bool  = False | True  
data Color = Red | Green | Blue
```

Type constructors

```
-- enum
data Bool = False | True
data Color = Red | Green | Blue

-- struct
data Point = Point { x :: Double, y :: Double }
```

Type constructors

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-- enum
data Bool = False | True
data Color = Red | Green | Blue

-- struct
data Point = Point { x :: Double, y :: Double }

-- a bit more interesting
data Minutes = Minutes Int
data Maybe a = Nothing | Just a
data List a = Nil | Cell a (List a)
```

Type constructors

```
-- enum
data Bool = False | True
data Color = Red | Green | Blue

-- struct
data Point = Point { x :: Double, y :: Double }

-- a bit more interesting
data Minutes = Minutes Int
data Maybe a = Nothing | Just a
data [a] = [] | (a:[a])
```

“Deconstructors” and pattern matching

```
not :: Bool → Bool  
not True  = False  
not False = True
```

“Deconstructors” and pattern matching

```
not :: Bool → Bool  
not True  = False  
not False = True
```

```
magnitude :: Point → Double  
magnitude (Point x y) = sqrt $ x^2 + y^2
```

“Deconstructors” and pattern matching

```
not :: Bool → Bool  
not True  = False  
not False = True
```

```
magnitude :: Point → Double  
magnitude (Point x y) = sqrt $ x^2 + y^2
```

```
length :: [a] → Int  
length []      = 0  
length (_:xs) = 1 + length xs
```

We know

- ▶ how to read function types
- ▶ how to declare new types
- ▶ how to do pattern matching

Our types are limited

```
$ ghci  
>
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue  
>
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue  
> let mycolor = Red
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue  
> let mycolor = Red  
>
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue  
> let mycolor = Red  
> mycolor
```



Our types are limited

```
$ ghci
> data Color = Red | Green | Blue
> let mycolor = Red
> mycolor
    No instance for (Show Color)
      arising from a use of `print'
```



Our types are limited

```
$ ghci  
> data Color = Red | Green | Blue  
> let mycolor = Red  
>
```



Our types are limited

```
$ ghci
> data Color = Red | Green | Blue
> let mycolor = Red
> mycolor == Red
```



Our types are limited

```
$ ghci
> data Color = Red | Green | Blue
> let mycolor = Red
> mycolor == Red
    No instance for (Eq Color)
      arising from a use of `=='
```



Type parameters constraints?

```
sumI :: [Int] → Int  
sumI = foldl' (+) 0
```

```
sumD :: [Double] → Double  
sumD = foldl' (+) 0
```

Type parameters constraints?

```
sumI :: [Int] → Int  
sumI = foldl' (+) 0
```

```
sumD :: [Double] → Double  
sumD = foldl' (+) 0
```

```
sum :: [a] → a  
sum = foldl' (+) 0
```

Type parameters constraints?

```
sumI :: [Int] → Int  
sumI = foldl' (+) 0
```

```
sumD :: [Double] → Double  
sumD = foldl' (+) 0
```

```
sum :: [a] → a  
sum = foldl' (+) 0
```

No instance for (Num a)
arising from a use of `(+)'



Cascading context

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber



Cascading context

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber

getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId =
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
  Nothing   → Nothing
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of
```

```
  Nothing   → Nothing
```

```
  Just user →
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
  Nothing   → Nothing  
  Just user → case getNextOfKin user of
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
  Nothing   → Nothing
```

```
  Just user → case getNextOfKin user of  
    Nothing       → Nothing
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
    Nothing   → Nothing
```

```
    Just user → case getNextOfKin user of
```

```
        Nothing       → Nothing
```

```
        Just nextOfKinId →
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
    Nothing   → Nothing
```

```
    Just user → case getNextOfKin user of  
        Nothing       → Nothing
```

```
        Just nextOfKinId → case getUser nextOfKinId of
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of
  Nothing   → Nothing
  Just user → case getNextOfKin user of
    Nothing           → Nothing
    Just nextOfKinId → case getUser nextOfKinId of
      Nothing       → Nothing
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
    Nothing   → Nothing
```

```
    Just user → case getNextOfKin user of
```

```
        Nothing       → Nothing
```

```
        Just nextOfKinId → case getUser nextOfKinId of
```

```
            Nothing       → Nothing
```

```
            Just nextOfKin →
```



Cascading context

```
getUser      :: Id    → Maybe User
```

```
getNextOfKin :: User → Maybe Id
```

```
getPhoneNumber :: User → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
```

```
getNextOfKinPhoneNumber userId = case getUser userId of  
    Nothing → Nothing
```

```
    Just user → case getNextOfKin user of
```

```
        Nothing → Nothing
```

```
        Just nextOfKinId → case getUser nextOfKinId of
```

```
            Nothing → Nothing
```

```
            Just nextOfKin → getPhoneNumber nextOfKin
```



- ▶ Can't apply regular functions on IO values



- ▶ Can't apply regular functions on IO values

```
$ ghci  
>
```



- ▶ Can't apply regular functions on IO values

```
$ ghci  
> let name = getLine -- IO String
```



- ▶ Can't apply regular functions on IO values

```
$ ghci  
> let name = getLine -- IO String  
>
```



- ▶ Can't apply regular functions on IO values

```
$ ghci
> let name = getLine -- IO String
> putStrLn $ "Hello " ++ name ++ "!"
```



► Can't apply regular functions on IO values

```
$ ghci
> let name = getLine -- IO String
> putStrLn $ "Hello " ++ name ++ "!"
Expected type: `String'
Actual type: `IO String'
In second argument of (++)
```



- ▶ Can't apply regular functions on IO values
- ▶ Can't get values out of IO



- ▶ Can't apply regular functions on IO values
- ▶ Can't get values out of IO

IO is impure



- ▶ Can't apply regular functions on IO values
- ▶ Can't get values out of IO
- ▶ Can't pattern match on IO



- ▶ Can't apply regular functions on IO values
- ▶ Can't get values out of IO
- ▶ Can't pattern match on IO

IO's implementation
details are hidden



We don't know

- ▶ how to extend our data types
- ▶ how to express type constraints
- ▶ how to chain contextual functions
- ▶ how to use IO

Typeclasses

```
class Show a where  
    show :: a → String
```



Typeclasses

```
class Show a where  
    show :: a → String  
  
data Color = Red | Green | Blue
```



Typeclasses

```
class Show a where
    show :: a → String

data Color = Red | Green | Blue

instance Show Color where
    show Red    = "Red"
    show Green  = "Green"
    show Blue   = "Blue"
```



Constraints

```
show :: Show a => a → String
```



Constraints

```
show :: Show a => a → String  
sum   :: Num a    => [a] → a
```



Constraints



```
show :: Show a => a → String
sum  :: Num a   => [a] → a
(==) :: Eq a    => a → a → Bool
```

Constraints



```
show :: Show a => a → String
sum  :: Num a    => [a] → a
(==) :: Eq a     => a → a → Bool
```

```
instance Show (Maybe a) where
```

Constraints



```
show :: Show a => a → String  
sum  :: Num a   => [a] → a  
(==) :: Eq a    => a → a → Bool
```

```
instance Show (Maybe a) where  
  show Nothing = "Nothing"
```

Constraints



```
show :: Show a => a → String
sum  :: Num a   => [a] → a
(==) :: Eq a    => a → a → Bool

instance Show (Maybe a) where
    show Nothing  = "Nothing"
    show (Just x) = "Just " ++ show x
```

Constraints



```
show :: Show a => a -> String
sum  :: Num a    => [a] -> a
(==) :: Eq a     => a -> a -> Bool

instance Show a => Show (Maybe a) where
  show Nothing  = "Nothing"
  show (Just x) = "Just " ++ show x
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
> show Blue
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
> show Blue  
"Blue"
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
> show Blue
"Blue"
> read "Green" :: Color
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
> show Blue
"Blue"
> read "Green" :: Color
Green
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
class Eq a where
  (==) :: a → a → Bool
  (/=) :: a → a → Bool
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
class Eq a where
  (==) :: a → a → Bool
  (/=) :: a → a → Bool
  a == b = not $ a /= b
  a /= b = not $ a == b
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
instance Eq Color where
    Red    == Red    = True
    Green == Green  = True
    Blue   == Blue   = True
    -      == -      = False
```

Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
class Eq a => Ord a where
    compare :: a → a → Ordering
    (≤)      :: a → a → Bool
    (≥)      :: a → a → Bool
    (<)      :: a → a → Bool
    (>)      :: a → a → Bool
    max     :: a → a → a
    min     :: a → a → a
```



Show Read Eq Ord Bounded Enum

```
data Color = Red | Green | Blue
```

```
class Eq a => Ord a where
    compare :: a → a → Ordering
    (≤)      :: a → a → Bool
```



Show Read Eq Ord **Bounded** Enum

```
data Color = Red | Green | Blue
```

```
class Bounded a where
    minBound :: a
    maxBound :: a
```



Show Read Eq Ord Bounded **Enum**

```
data Color = Red | Green | Blue
```

```
class Enum a where
    succ          :: a → a
    pred          :: a → a
    toEnum        :: Int → a
    fromEnum      :: a → Int
    enumFrom     :: a → [a]
    enumFromThen :: a → a →
    enumFromTo   :: a → a →
    enumFromThenTo :: a → a →
```



Deriving

```
data Color = Red | Green | Blue
```

Deriving



```
data Color = Red | Green | Blue  
deriving (Show,  
          Read,  
          Eq,  
          Ord,  
          Bounded,  
          Enum)
```



We still don't know

- ▶ how to extend our data types
- ▶ how to express type constraints
- ▶ how to chain contextual functions
- ▶ how to use IO

Contexts / wrappers

A → value

Contexts / wrappers

A → value

Maybe A → optional value

Contexts / wrappers

A → value

Maybe A → optional value

List A → repeated value

Contexts / wrappers

A → value

Maybe A → optional value

List A → repeated value

IO A → impure value

Contexts / wrappers

$A \rightarrow$ value

Maybe A \rightarrow optional value

List A \rightarrow repeated value

IO A \rightarrow impure value

C A \rightarrow “contextual” value

- ▶ Wrapping is trivial



- ▶ Wrapping is trivial

```
wrap x = [x]
```



- ▶ Wrapping is trivial

```
wrap x = [x]  
wrap x = Just x
```



- ▶ Wrapping is trivial

```
wrap x = [x]  
wrap x = Just x  
wrap x = return x
```



Similarities

- ▶ Wrapping is trivial
- ▶ Unwrapping is non-trivial / destructive / impossible



- ▶ Wrapping is trivial
- ▶ Unwrapping is non-trivial / destructive / impossible

unwrap `Nothing` = ???



- ▶ Wrapping is trivial
- ▶ Unwrapping is non-trivial / destructive / impossible

unwrap `Nothing` = ???
unwrap `[1,2,3]` = ???



- ▶ Wrapping is trivial
- ▶ Unwrapping is non-trivial / destructive / impossible

```
unwrap Nothing = ???  
unwrap [1,2,3] = ???  
unwrap getLine = NOPE
```



- ▶ Wrapping is trivial
- ▶ Unwrapping is non-trivial / destructive / impossible
- ▶ What we want: to apply functions **in the context**



Three standard functions to deal with contexts

Three standard functions to deal with contexts

fmap :: $(a \rightarrow b) \rightarrow c\ a \rightarrow c\ b$



Three standard functions to deal with contexts

`fmap :: (a → b) → c a → c b`

`ap :: c (a → b) → c a → c b`



Three standard functions to deal with contexts

fmap :: $(a \rightarrow b) \rightarrow c\ a \rightarrow c\ b$

ap :: $c\ (a \rightarrow b) \rightarrow c\ a \rightarrow c\ b$

bind :: $(a \rightarrow c\ b) \rightarrow c\ a \rightarrow c\ b$



- ▶ fmap is like map...

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`map :: (a → b) → [a] → [b]`

- ▶ fmap is like map...
- ▶ but generalised to all contexts

`fmap :: (a → b) → c a → c b`

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] =
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) =
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing    = Nothing
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing    = Nothing
fmap length getLine =
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing    = Nothing
fmap length getLine = -\_(\$)_/-
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing    = Nothing
fmap length getLine = -\_(\_)_/- -- IO Int
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts
- ▶ and it also has an operator version

```
fmap show [1, 2, 3] = ["1", "2", "3"]
fmap show (Just 42) = Just "42"
fmap show Nothing    = Nothing
fmap length getLine = -\_(\_)_/- -- IO Int
```

- ▶ fmap is like map...
- ▶ but generalised to all contexts
- ▶ and it also has an operator version

```
show <$> [1, 2, 3] = ["1", "2", "3"]
show <$> (Just 42) = Just "42"
show <$> Nothing    = Nothing
length <$> getLine = -\_(\:)_/- -- IO Int
```

- ▶ what about functions with several arguments?

- ▶ what about functions with several arguments?

```
fmap (+) (Just 3) =
```

- ▶ what about functions with several arguments?

```
fmap (+) (Just 3) = Just (3+)
```

- ▶ what about functions with several arguments?

```
fmap (+) (Just 3) = Just (3+)
Just (3+) :: Maybe (Int → Int)
```

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!

`ap :: c (a → b) → c a → c b`

`fmap (+) (Just 3) = Just (3+)`
`Just (3+) :: Maybe (Int → Int)`

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!

```
ap :: c (a → b) → c a → c b
```

```
fmap (+) (Just 3) = Just (3+)
ap (Just (3+)) (Just 39) =
```

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!

`ap :: c (a → b) → c a → c b`

`fmap (+) (Just 3) = Just (3+)`

`ap (Just (3+)) (Just 39) = Just 42`

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

`ap :: c (a → b) → c a → c b`

`fmap (+) (Just 3) = Just (3+)`

`ap (Just (3+)) (Just 39) = Just 42`

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

```
ap :: c (a → b) → c a → c b
```

```
fmap (+) (Just 3) = Just (3+)
Just (3+) <*> Just 39 = Just 42
```

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

```
ap :: c (a → b) → c a → c b
```

```
(+) <$> Just 3 = Just (3+)
```

```
Just (3+) <*> Just 39 = Just 42
```

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

`ap :: c (a → b) → c a → c b`

(+)

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

```
ap :: c (a → b) → c a → c b
```

```
(+) <$> Just 3
```

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

`ap :: c (a → b) → c a → c b`

`(+) <$> Just 3 <*> Just 39`

ap(ply)

- ▶ what about functions with several arguments?
- ▶ apply to the rescue!
- ▶ also defined as an operator

```
ap :: c (a → b) → c a → c b
```

```
(+) <$> Just 3 <*> Just 39 = Just 42
```

- ▶ what about chaining functions?

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```
div2 :: Int → Maybe Int
```

- ▶ what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div2 42 = Just 21
```

► what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div2 42 = Just 21
```

```
div2 21 = Nothing
```

- ▶ what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div4 :: Int → Maybe Int
```

- ▶ what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div4 :: Int → Maybe Int
div4 x =
```

► what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div4 :: Int → Maybe Int
div4 x = let y = div2 x
          in
```

► what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div4 :: Int → Maybe Int
div4 x = let y = div2 x
         in fmap div2 y
```

► what about chaining functions?

```
div2 :: Int → Maybe Int
```

```
div4 :: Int → Maybe Int
```

```
div4 x = let y = div2 x -- Maybe Int  
         in fmap div2 y -- Maybe (Maybe Int)
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!

```
div2 :: Int → Maybe Int
bind :: (a → c b) → c a → c b
```

```
div4 :: Int → Maybe Int
div4 x = let y = div2 x -- Maybe Int
         in fmap div2 y -- Maybe (Maybe Int)
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!

```
div2 :: Int → Maybe Int
bind :: (a → c b) → c a → c b
```

```
div4 :: Int → Maybe Int
div4 x = let y = div2 x -- Maybe Int
         in bind div2 y -- Maybe Int
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!

```
div2 :: Int → Maybe Int  
bind :: (a → c b) → c a → c b
```

```
div4 :: Int → Maybe Int  
div4 x = bind div2 $ div2 x
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!
- ▶ of course, also has an operator

```
div2 :: Int → Maybe Int  
(>>=) :: c a → (a → c b) → c b
```

```
div4 :: Int → Maybe Int  
div4 x = bind div2 $ div2 x
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!
- ▶ of course, also has an operator

```
div2 :: Int → Maybe Int  
(>>=) :: c a → (a → c b) → c b
```

```
div4 :: Int → Maybe Int  
div4 x = div2 x >>= div2
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!
- ▶ of course, also has an operator

```
div2 :: Int → Maybe Int  
(>>=) :: c a → (a → c b) → c b
```

```
div8 :: Int → Maybe Int  
div8 x = div2 x >>= div2 >>= div2
```

- ▶ what about chaining functions?
- ▶ bind to the rescue!
- ▶ of course, also has an operator

```
div2 :: Int → Maybe Int  
(>>=) :: c a → (a → c b) → c b
```

```
div16 :: Int → Maybe Int  
div16 x = div2 x >>= div2 >>= div2 >>= div2
```

bind - continued

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber



bind - continued

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber

getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber



bind - continued

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber

bind :: (a → c b) → c a → c b

getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber



bind - continued

getUser :: Id → Maybe User

getNextOfKin :: User → Maybe Id

getPhoneNumber :: User → Maybe PhoneNumber

(>>=) :: c a → (a → c b) → c b

getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber



bind - continued

```
getUser      :: Id    → Maybe User
getNextOfKin :: User → Maybe Id
getPhoneNumber :: User → Maybe PhoneNumber
(>>=)       :: c a → (a → c b) → c b
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
getNextOfKinPhoneNumber uid =
```



bind - continued

```
getUser      :: Id    → Maybe User
getNextOfKin :: User → Maybe Id
getPhoneNumber :: User → Maybe PhoneNumber
(>>=)       :: c a → (a → c b) → c b
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
getNextOfKinPhoneNumber uid =
    getUser uid      -- Maybe User
```



bind - continued

```
getUser      :: Id    → Maybe User
getNextOfKin :: User → Maybe Id
getPhoneNumber :: User → Maybe PhoneNumber
(>>=)       :: c a → (a → c b) → c b
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
getNextOfKinPhoneNumber uid =
    getUser uid      -- Maybe User
  >>= getNextOfKin   -- Maybe Id
```



bind - continued

```
getUser      :: Id    → Maybe User
getNextOfKin :: User → Maybe Id
getPhoneNumber :: User → Maybe PhoneNumber
(>>=)       :: c a → (a → c b) → c b
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
getNextOfKinPhoneNumber uid =
    getUser uid      -- Maybe User
  >>= getNextOfKin   -- Maybe Id
  >>= getUser        -- Maybe User
```



bind - continued

```
getUser      :: Id    → Maybe User
getNextOfKin :: User → Maybe Id
getPhoneNumber :: User → Maybe PhoneNumber
(>>=)       :: c a → (a → c b) → c b
```

```
getNextOfKinPhoneNumber :: Id → Maybe PhoneNumber
getNextOfKinPhoneNumber uid =
    getUser uid      -- Maybe User
  >>= getNextOfKin   -- Maybe Id
  >>= getUser        -- Maybe User
  >>= getPhoneNumber
```



The typeclasses

```
fmap :: (a → b) → c a → c b  
ap   :: c (a → b) → c a → c b  
bind :: (a → c b) → c a → c b
```

The typeclasses

```
fmap :: (a → b) → c a → c b  
ap   :: c (a → b) → c a → c b  
bind :: (a → c b) → c a → c b
```

But what about the typeclasses?

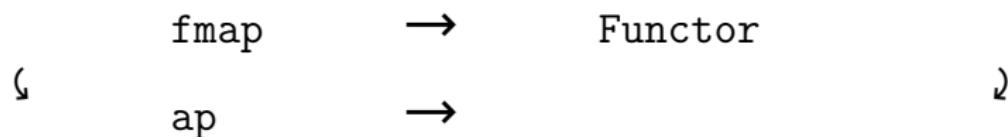
The typeclasses

fmap →

The typeclasses

fmap → Functor

The typeclasses



The typeclasses

fmap	→	Functor
ap	→	Applicative

The typeclasses

fmap	→	Functor
ap	→	Applicative
bind	→	

MONAD



The typeclasses

fmap	→	Functor
ap	→	Applicative
bind	→	Monad

We still don't know

- ▶ how to extend our data types
- ▶ how to express type constraints
- ▶ how to chain contextual functions
- ▶ how to use IO

```
class Applicative m ⇒ Monad m where
    return :: a → m a
    (=>)   :: m a → (a → m b) → m b
```

The basic building block



```
class Applicative m ⇒ Monad m where
    return :: a → m a
    (">>=")   :: m a → (a → m b) → m b

    (">>")     :: m a → m b → m b
    (>=")     :: (a → m b) → (b → m c) → (a → m c)
```

Powerful abstractions!



```
class Applicative m ⇒ Monad m where
    return :: a → m a
    (">>=") :: m a → (a → m b) → m b

    (">>") :: m a → m b → m b
    (>=") :: (a → m b) → (b → m c) → (a → m c)
sequence :: (Traversable t) ⇒ t (m a) → m (t a)
```

...that could use some sugar



Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>=
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
        getLastName
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
        getLastName >>=
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
        getLastName >>= λ lastname →
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main          :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
        getLastName >>= λ lastname →
            greetUser firstname lastname
```

Do notation

```
getFirstName :: IO String
getLastName  :: IO String
greetUser    :: String → String → IO ()
main        :: IO ()
```

```
main =
  getFirstName      firstname
  getLastName       lastname
  greetUser firstname lastname
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main         :: IO ()
```

```
main =
    getFirstName →      firstname
    getLastName  →      lastname
    greetUser   firstname lastname
```

Do notation

```
getFirstName :: IO String
getLastName   :: IO String
greetUser     :: String → String → IO ()
main         :: IO ()
```

```
main =
    getFirstName >>= λ firstname →
        getLastName >>= λ lastname →
            greetUser firstname lastname
```

```
main =
```

Do notation

```
getFirstName :: IO String
getLastName  :: IO String
greetUser    :: String → String → IO ()
main         :: IO ()
```

```
main =
  getFirstName >>= λ firstname →
    getLastName >>= λ lastname →
      greetUser firstname lastname
```

```
main = do
```

Do notation

```
getFirstName :: IO String
getLastName  :: IO String
greetUser    :: String → String → IO ()
main        :: IO ()
```

```
main =
  getFirstName >>= λ firstname →
    getLastName >>= λ lastname →
      greetUser firstname lastname
```

```
main = do
  firstname ← getFirstName
```

Do notation

```
getFirstName :: IO String
getLastName  :: IO String
greetUser    :: String → String → IO ()
main        :: IO ()
```

```
main =
  getFirstName >>= λ firstname →
    getLastName >>= λ lastname →
      greetUser firstname lastname
```

```
main = do
  firstname ← getFirstName
  lastname  ← getLastName
```

Do notation

```
getFirstName :: IO String
getLastName  :: IO String
greetUser    :: String → String → IO ()
main        :: IO ()
```

```
main =
  getFirstName >>= λ firstname →
    getLastName >>= λ lastname →
      greetUser firstname lastname
```

```
main = do
  firstname ← getFirstName
  lastname  ← getLastName
  greetUser firstname lastname
```

- ▶ typeclasses to extend our data types
- ▶ typeclasses to express type constraints
- ▶ monadic operators to chain contextual functions
- ▶ do notation to use I0



- ▶ adit.io (functors, applicatives, and monads in pictures)
- ▶ dev.stephendiehl.com/hask/ (what I wish I knew)