Tutorial - 1

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$n_1 \rightarrow$ no. of molecule in daughter cell 1

$$P(n_1 N) = \frac{N!}{n_1!\,(N-n_1)!}\, p^{n_1} q^{N-n_1}$$

where, $p =$ probability of going to daughter 1

$q = 1 - p =$ probability of going to daughter 2

so, $p+q = 1$ and $N \rightarrow$ total molecule

Here, $$\langle n_1 \rangle = p \frac{\partial}{\partial p} \sum_{n_1=0}^{N} \frac{N!}{n_1!\,(N-n_1)!}\, p^{n_1} q^{N-n_1}$$

$$= p \frac{\partial}{\partial p} (p+q)^N$$

(By using binomial theorem)

$$(p+q)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!\,k!}\, p^{n-k} b^{k}$$

so, $\langle n_1 \rangle = p \times N (p+q)^{N-1}$ $(\because p+q = 1)$

so, $\langle n_1 \rangle = pN(1)^{N-1} = pN$ (proved)

and, $\langle n_1 \rangle^2 = (pN)^2 = p^2 N^2$

$$\langle n_1^2 \rangle = p\frac{\partial}{\partial p}\, p\frac{\partial}{\partial p} \sum_{n_1=0}^{N} \frac{N!}{n_1!\,(N-n_1)!}\, p^{n_1} q^{N-n_1}$$

$$= p\frac{\partial}{\partial p}\left(p\frac{\partial}{\partial p} \sum_{n_1=0}^{N} p\right)$$

$$= \cancel{p\frac{\partial}{\partial p}(pN}$$

$$= p\frac{\partial}{\partial p}\left(p\frac{\partial}{\partial p}(p+q)^N\right)$$

$$= p\frac{\partial}{\partial p}\left(pN(p+q)^{N-1}\right)$$

$$= pN(p+q)^{N-1} + p^2N(N-1)(p+q)^{N-2}$$

$$= pN(1)^{N-1} + p^2N(N-1)(1)^{N-2}$$

$$= pN + p^2N(N-1)$$

$$\langle n_1^2 \rangle = pN + p^2N^2 - p^2N$$

Here, $\langle n_1^2 \rangle - \langle n_1 \rangle^2 = pN + \cancel{p^2N^2} - p^2N - \cancel{p^2N^2}$

$$= pN - p^2N$$

$$= pN(1-p)$$

$$= pN \times q$$

So, $\langle n_1^2 \rangle - \langle n_1 \rangle^2 = Npq$ (proved)

Now, we have,

$$\langle (I_1 - I_2)^2 \rangle = \langle (2I_1 - I_{tot})^2 \rangle$$

$\because (I_1 + I_2 = I_{tot})$

taking $I = N\alpha$

$$\langle (I_1 - I_2)^2 \rangle = \langle (2N_1\alpha - I_{tot})^2 \rangle$$

$$= 4\alpha^2 \langle N_1^2 \rangle - 4 I_{tot}\alpha \langle N_1 \rangle + I_{tot}^2$$

Considering random partitioning, $p = q = \frac{1}{2}$

so, $\langle N_1 \rangle = N/2$

$$\langle N_1^2 \rangle = \frac{N}{4} + \frac{N^2}{4}$$

$$\langle (I_1 - I_2)^2 \rangle = \alpha^2 N = \alpha I_{total}$$