<u>Monopoly</u>.

0: Price Elasticity of Demand

I : Why monopoly?

- Entry not free : patents
  : hard to bear sunk costs — borrowing problems
  : coercion.
  : ↑ing returns to scale
  : network effects.

Monopoly may be efficient from minimum cost point of view.

Natural Monopoly

Hard to store

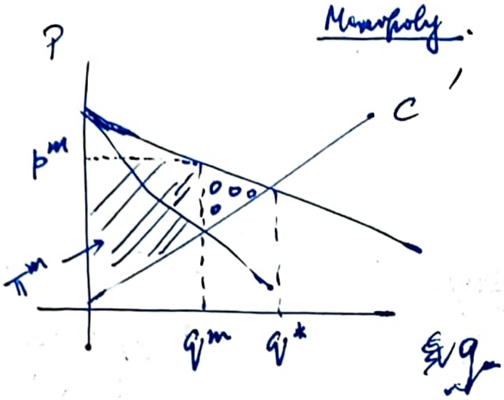
Locality important

Power, Railroads, Phone.

Network

: War of Attrition resulting in monopoly

II.



(1) $$\max_q \; P(q)q - C(q)$$

FOC: $P(q) + qP'(q) = C'(q)$.

↑q from q^m: $MC = C'(q)$.

MR : I sell for around $P(q)$, adjusted for the drop $P'(q)$ in price on all q units.

(2) - DWL.

- Can't ↑ q from q^m & charge a price lower than p^m, if price discrimination not possible.

(3). $q = D(p)$, $p = D^{-1}(q) = P(q)$.

$$\max_p \; pD(p) - C(D(p))$$

$$pD'(p) + D(p) - C'(D(p)) \cdot D'(p) = 0.$$

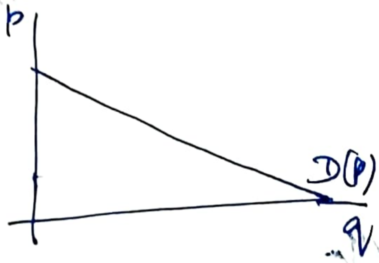
$$\Rightarrow [p - C'(D(p))] D'(p) = -D(p)$$

$$\frac{p - C'(D(p))}{p} = -\frac{D(p)}{D'(p) \cdot p} = \frac{1}{\varepsilon},$$

where $\varepsilon = -D'(p) \cdot \frac{p}{D(p)}$ is the price elasticity of demand.

Lower ε, (more <u>in</u>elastic), higher <u>markup</u>.

On price elasticity of demand.



$D(p) = A - Bp$, unit = 1 kg mango. p = Rs.

$D'(p) = -B = -10$ say:

Re 1 ↑ ⇒ 10 Kg less demanded. per kg

10 kg per Rupee.

$\hat{p}$ = Rs 1.

Suppose I measure weight in quintals (100 kg):

Then 0.1 quintal ↓ per Rupee.

So 'B' depends on units.

Unit free measure of response to price

$$\frac{\Delta D/D}{\Delta p/p} \qquad \varepsilon = \frac{\Delta D}{\Delta p}\cdot\frac{p}{D}$$

And in the limit,

$$D'(p)\cdot\frac{p}{D(p)}.$$

This is −ve.

$$\varepsilon = -D'(p)\cdot\frac{p}{D(p)}$$

$0 < \varepsilon < 1$: inelastic

$\varepsilon > 1$: elastic: more than proportionate ↓ in demand due to ↑ in price.

$$q = D(p) = p^{-\varepsilon}$$

$$D'(p) = -\varepsilon p^{-\varepsilon-1}$$

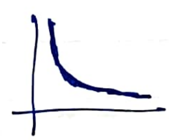
$$D'(p)\cdot\frac{p}{D(p)} = -\varepsilon p^{-\varepsilon-1}\cdot\frac{p}{p^{-\varepsilon}} = -\varepsilon p^{-\varepsilon-1}\cdot p^{1+\varepsilon} = -\varepsilon.$$

So elasticity $\equiv -D'(p)\cdot\frac{p}{D(p)} = \varepsilon$.

Alternatively, $\ln q = -\varepsilon \ln p$

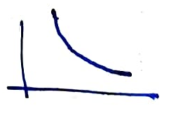
elasticity $\equiv \frac{d\ln q}{d\ln p} = -\varepsilon$.

$\varepsilon = 1$: $pq = 1$: rectangular hyperbola →



$\varepsilon = 2$: $q = \frac{1}{p^2}$: elastic →

$\varepsilon = \frac{1}{2}$ $q = \frac{1}{\sqrt{p}}$: inelastic.



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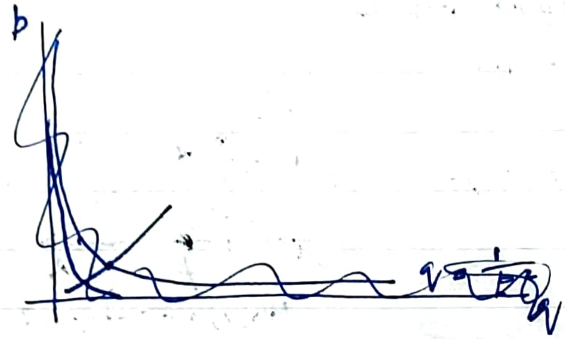
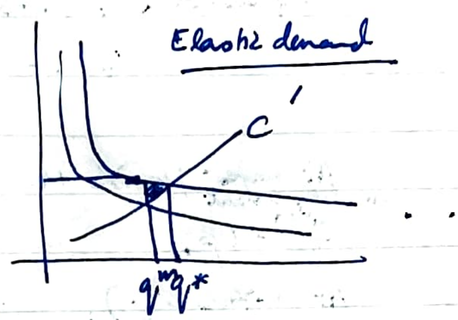
IO Topics

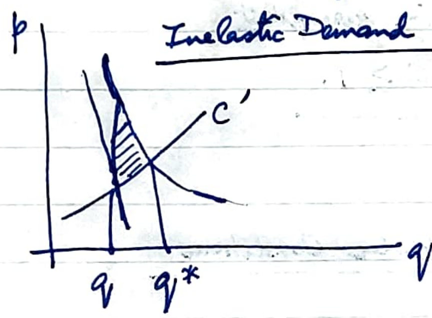
Deadweight loss under different elasticities



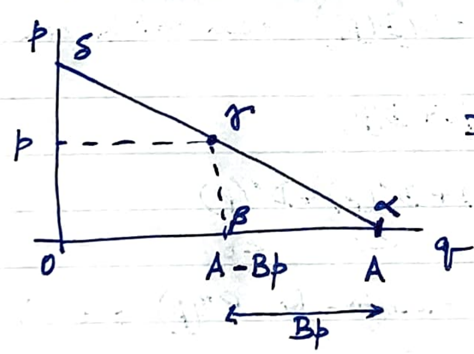
Elastic demand



Inelastic Demand



Straight line Demand



$$D(p) = A - Bp.$$

$$\varepsilon = -\frac{D'(p)\cdot p}{D(p)} = +\frac{B\cdot p}{A - Bp} = \frac{\overline{\beta A}}{\overline{O\beta}} = \frac{\overline{\alpha\gamma}}{\overline{\gamma\delta}}$$

At the midpoint of the demand curve, ∴, $\varepsilon = 1$.

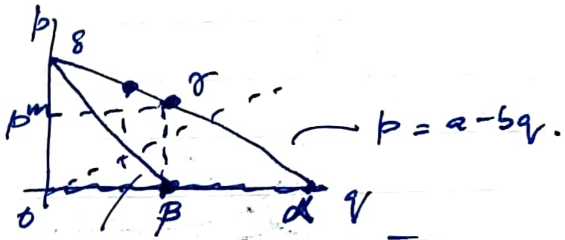
~~Close to~~ ~~Also~~ To its NorthWest, $\varepsilon > 1$; & $\to \infty$ as a long moves to zero demand, ∵ $\frac{p}{D(p)} \to \infty$.

To its SouthEast $\varepsilon < 1$. $\to 0$ as long moves to zero price.

Revenue $= \cancel{p}(a - bq)\cdot q$

Marginal Revenue $= (a - bq)\cdot 1 + q(-b)$

$= a - 2bq.$



At lowest possible $MC \equiv c'(q) = 0$, monopoly output $q^m = O\beta$.

At higher $c'(\cdot)$,

$q^m < O\beta$; The monopolist operates where $\varepsilon \geq 1$.

$MR(q) = a - 2bq$ $\therefore \dfrac{\alpha\beta}{O\beta} = 1$

$\Rightarrow \dfrac{\alpha\gamma}{\gamma\delta} = 1$

Second-Order Conditions:

Now, $\because$ price is not a given, revenue $R(p) = pD(p)$ need not be concave $\therefore$ so even if $C(D(p))$ is convex, $\pi(p) = pD(p) - C(D(p))$ need not be concave or quasiconcave.

$$R'(p) = pD'(p) + D(p)$$

$$R''(p) = pD''(p) + D'(p) + D'(p)$$

$$\lessgtr 0 \qquad < 0$$

One sufficient condition for $R''(p) \leq 0$ is $D''(p) \leq 0$ (linear or concave demand).

More generally, for $R''(p) \leq 0$, $D(p)$ must not be "too convex".

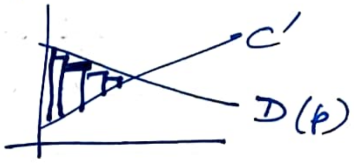
Cost Distortions

Shareholders can base reward structure of managers based on the performance of the firm relative to that of competing firms. This incentive scheme is absent in a monopoly; & can lead to slack.

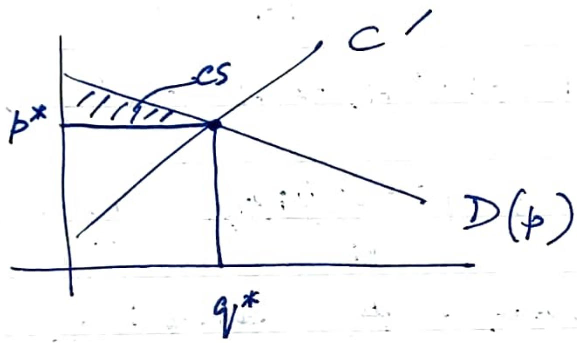
Price Discrimination

First degree or perfect.

If the monopolist can charge a difference price for each unit, then can go up all the way to WTP.



Alternatively, it can set a two-part tariff to accomplish this:



Suppose the monopolist charges p^* per unit & sells q^* units.

$$A = CS(q^*, p^*) = \int_0^{q^*} (P(q) - p^*)\, dq.$$

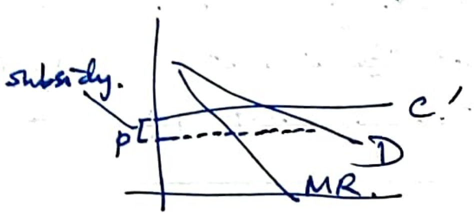
Monopolist's Contract:

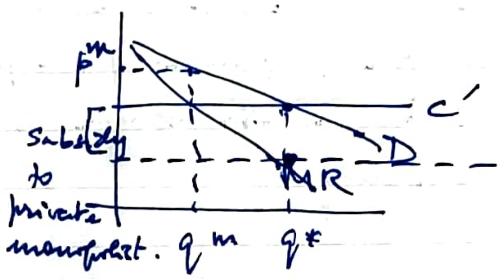
$$T(q) = \begin{cases} p^* q + A, & \text{if } q > 0 \\ 0 & \text{if } q = 0. \end{cases}$$

Sweeps up all the surplus as a participation or user fee.

Other applications: Franchise contracts, MacDonalds.

Power sector in India





Third-degree price discrimination

m different markets / locations / no arbitrage.

demographics : insured / not ; senior citizens discounts etc.

$$\text{Max} \sum_{i=1}^{m} p_i D(p_i) - C\left(\sum_{i=1}^{m} D_i(p_i)\right)$$

FOC wrt p_i : $p_i D'(p_i) + D(p_i) - c'(\quad) \cdot D_i'(p_i) = 0.$

$$\Rightarrow \frac{p_i - c'\left(\sum D_j(p_j)\right)}{p_i} = \frac{1}{\varepsilon_i}.$$

In more inelastic markets, markup is higher.

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IO Topics

Second degree price (& quality) discrimination

Setting
- People have different preferences for quantity/quality of a good.
- Firm knows this; knows what these different preferences look like, but
  - either does not know which consr has what preference
  - or cannot legally discriminate between consrs.
- General point: How to segment the market
  Can offer a menu of prices & quantities, s.t. consrs with different preferences will themselves self-select different ~~items~~ (p_i, q_i) items from the menu.

Examples:
- economy class & business class air/~~b~~ train fares
- Tatkal passports
- Youtube & Youtube Premium } Platforms.
- Amazon Prime & normal purchases
- Regular Customer offers
- TV ~~pa~~ channel packs
- Quality differentiated prices at any market place or mandi

- Why did Tata Nano fail to win over a market, even though it was targeted to a supposed consumer at the low value end of the spectrum?

Model.

Consumers are differentiated into 2 "types" by preference for quality/quantity.

$u(q, \theta_i, y) = \theta_i v(q) + y$, where y is utility from all other goods. (Quasilinear), $i = 1, 2$.

$\theta_2 > \theta_1$.

$v(0) = 0$, $v' > 0$, $v'' < 0$.

Given any quality/quantity q,

$u(q, \theta_2, y) - u(q, \theta_1, y) = (\theta_2 - \theta_1) v(q)$ is increasing in q.
$(\because (\theta_2 - \theta_1) v'(q) > 0)$.

i.e. Type θ_2 agent ~~~~ is willing to pay more for a unit increase in q.

Monopolist will charge a tariff $T(q)$ for quantity q or good of quality q.

We can think of a unit price p as a linear tariff :

$$T(q) = p \cdot q .$$

And $T(q) = A + pq$ is a two-part tariff or affine tariff.

$T(q)$ can also be nonlinear in general.

If consumer type i buys quantity/quality q at ~~\$~~ tariff $T = T(q)$, her utility

$$= \theta_i v(q) + y - T \quad ;$$

we will simply write

$$\theta_i v(q) - T .$$

~~In~~ With quasilinear utility, ~~level of y~~ optimal demand for q does not depend on y.

Let the monopolist have a unit cost of prod^n c. (for unit quantity or quality) :

We can Assume $v'(0) = \infty$, so an optimum will always exist :

or go by an example :

$$v(q) = \frac{1 - (1-q)^2}{2} \quad ;$$

$$v'(q) = 1 - q .$$

For a linear tariff $T(q) = pq$,

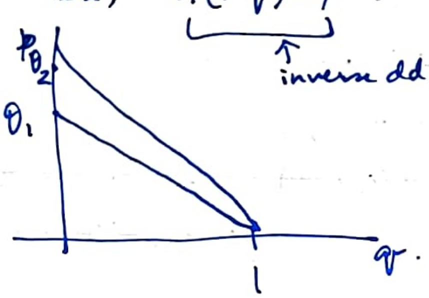
consumer maxes

$$\theta_i v(q) - pq$$

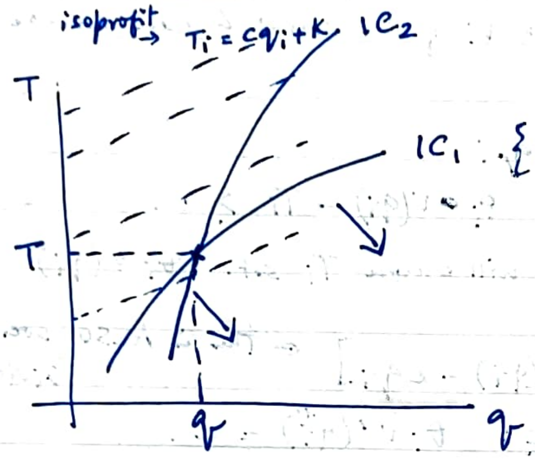
FOC : $\theta_i v'(q) = p$

Here, $\underbrace{\theta_i (1-q) = p}_{}$. Then, let $0 < c < 1$.

inverse dd curve of type i.



Indifference curves.



IC_1 : $\{(q, T) \mid \theta_1 v(q) - T = k\}$.

utility ↑es as q ↑es & or T ↓es.

Slope of IC_1 :

$T = \theta_1 v(q) - k$

$\left.\frac{dT}{dq}\right|_1 = \theta_1 v'(q)$.

Take any (q, T).

An IC_2 passes thru (q, T), steeper than IC_1,

$\because \left.\frac{dT}{dq}\right|_2 = \theta_2 v'(q) > \theta_1 v'(q)$.

Monopolist does not know any specific cons^rs preference/type but knows that $\lambda \equiv$ proportion of type 1 consumers.

~~Monopolist~~ If λ is very small, the monopolist may wish to not serve type 1 at all.

Simple example:

Complete
~~Perfect~~ information first best

Monopolist ~~can~~ knows θ_i of the cons^r & can make a personalized offer:

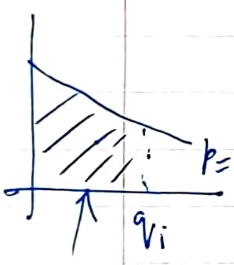
$$\max_{q_i, T_i} \; T_i - cq_i$$

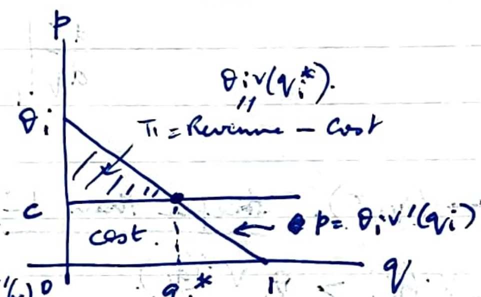
$$\text{s.t.} \quad \theta_i v(q_i) - T_i \geq 0. \qquad \text{– I.R.}$$

Given any q_i, will choose T_i s.t. $\theta_i v(q_i) - T_i = 0$.

$$\Rightarrow \max_{q_i} \left[\theta_i v(q_i) - cq_i\right]$$ ← This is ALSO social surplus: CS+PS. $(\theta_i v(q_i) - T_i + T_i - cq_i)$.

FOC: $\theta_i v'(q_i^*) = c$.





If $v(q) = \dfrac{1-(1-q)^2}{2}$,

this $\Rightarrow \theta_i(1-q_i) = c$.

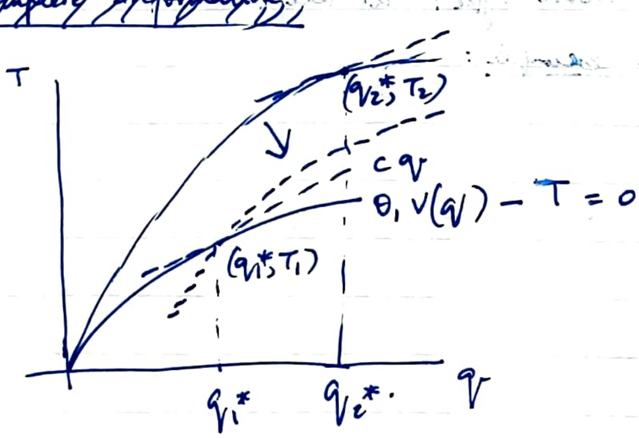
And profit $\pi = \displaystyle\int_0^{q_i^*} \left(\theta_i v'(q_i) - c\right) dq_i$

$$\pi = \theta_i v(q_i^*) - cq_i^*$$

Social optimum: Marginal social benefit $\theta_i v(q_i^*)$ equals marginal social cost c.

First-best.

Incomplete Information



$(T = \theta_1 v(q))$ || Passes thru origin.

First-best is NOT incentive compatible.

2nd best (incomplete info) Assume both types served

$$\max_{q_1, T_1, q_2, T_2} \lambda(T_1 - cq_1) + (1-\lambda)(T_2 - cq_2)$$

s.t. $\theta_1 v(q_1) - T_1 \geq 0$ — IR_1

$\theta_2 v(q_2) - T_2 \geq 0$ — IR_2

$\theta_1 v(q_1) - T_1 \geq \theta_1 v(q_2) - T_2$ — IC_1

$\theta_2 v(q_2) - T_2 \geq \theta_2 v(q_1) - T_1$ — IC_2

If (q_1, T_1), (q_2, T_2) are available, then type 2 can ~~also~~ always buy (q_1, T_1) & get

$\theta_2 v(q_1) - T_1 > \theta_1 v(q_1) - T_1$.

$\because$ IC_2 must hold,

$\theta_2 v(q_2) - T_2 \geq \theta_2 v(q_1) - T_1 > \theta_1 v(q_1) - T_1$.

So, if $\theta_1 v(q_1) - T_1 \geq 0$, ~~and~~ & IC_2 holds,

then $\theta_2 v(q_2) - T_2 \geq 0$ is automatically satisfied, with '>'.

So ignore IR_2.

This also means

$\theta_1 v(q_1) - T_1 = 0$ at π max.

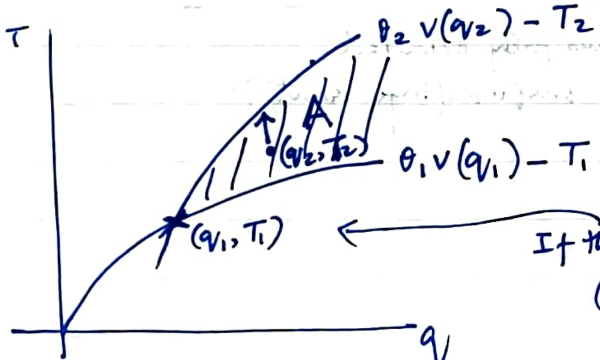
Suppose not.

Then $\theta_1 v(q_1) - T_1 > 0$ — IR_1

& $\theta_2 v(q_2) - T_2 > 0$ anyway. — IR_2

So firm can ↑ T_1 & T_2 by the same amount, slightly, say by $\varepsilon > 0$, & IC_1 & IC_2 will not be disturbed, & nor are IR_1 & IR_2 disturbed.

π ↑es. Contradicting π max.



If this is the offered (q_1, T_1), then (q_2, T_2) must be in zone A to satisfy IC_2.

Then, IC_1 is automatically satisfied with '>'.

Keeping q_2 fixed in zone A, ↑ T_2 as much as feasible.

$\Rightarrow$ IC_2 must bind.

So, $\theta_1 v(q_1) = T_1$

$$\theta_2 v(q_2) - T_2 = \theta_2 v(q_1) - T_1 = (\theta_2 - \theta_1) v(q_1).$$

: Firm de facto offers this surplus. It is the additional cost to offering any +ve q_1 & $v(q_1)$.

$$\Rightarrow T_2 = \theta_2 (v(q_2) - v(q_1)) + T_1$$
$$= \theta_2 (v(q_2) - v(q_1)) + \theta_1 v(q_1)$$
$$= \theta_2 v(q_2) - \underbrace{(\theta_2 - \theta_1) v(q_1)}_{\text{surplus that type 2 gets}}.$$

So,

$$\max_{q_1, q_2} \lambda(\theta_1 v(q_1) - c q_1) + (1-\lambda)(\theta_2 v(q_2) - c q_2 - (\theta_2 - \theta_1) v(q_1))$$

And leads to $\theta v_1'(q_1^S) > c.$

The marginal cost of providing additional q_1 is not just c: it also includes $(\theta_2 - \theta_1) v'(q_1)$, the additional surplus that must then be offered to type 2.

FOC: $\lambda(\theta_1 v'(q_1) - c) - (1-\lambda)\underbrace{(\theta_2 - \theta_1)}_{>0} v'(q_1) = 0.$ — (1).

$\theta_2 v'(q_2) = c$ — (2).

~~$\theta_1 v'(q_1) = \frac{1-\lambda}{\lambda}(\theta_2 - \theta_1) v$~~

~~$\lambda \theta_1 v'(q_1) = \lambda c$~~

$$[\lambda \theta_1 - (1-\lambda)(\theta_2 - \theta_1)] v'(q_1) = \lambda c$$

$$\theta_1 v'(q_1)\left[\lambda - (1-\lambda)\frac{(\theta_2 - \theta_1)}{\theta_1}\right] = \lambda c \qquad \lambda\theta_1 - \theta_1 + \lambda\theta_1$$

$$\theta_1 v'(q_1)\left[1 - \underbrace{\frac{1-\lambda}{\lambda} \frac{\theta_2 - \theta_1}{\theta_1}}\right] = c.$$

This is > 0, & if λ is large enough, it is < 1.

So, $\theta_1 v'(q_1) > c.$

(i) q_1^S is real low ; (ii) q_2^S is socially optimal

(iii) Type 1 surplus fully extracted.

(iv) Type 2 has surplus (info rent).

$$\frac{1-\lambda}{\lambda} \frac{(\theta_2 - \theta_1)}{\theta_1} < 1 \qquad (\lambda \text{ sufficiently high})$$

~~$(1-\lambda)(\theta_2 - \theta_1) < \lambda\theta_1$~~

$$\Leftrightarrow \frac{\theta_2 - \theta_1}{\theta_1} < \frac{\lambda}{1-\lambda}.$$

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320B IO Topics

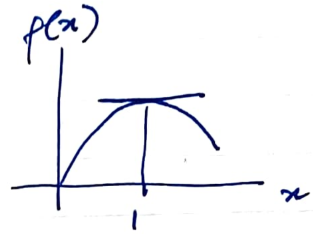
On Kuhn-Tucker Theory

Want to maximize subject to equality constraints and non-negativity constraints.

(1) $f(x) = 1-(1-x)^2$.

Maximize $f(x)$

s.t. $x \geq 0$.

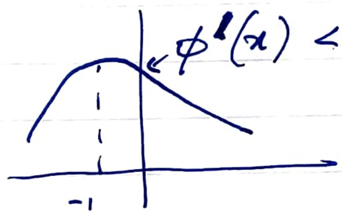


The FOC: $f'(x) = 0$ works here.

(2) $\phi(x) = -4x-(1-x)^2$.

Max $\phi(x)$

s.t. $x \geq 0$.



$\phi'(x) < 0$ at $x=0$.

The following Karush-Kuhn-Tucker condition works:

KKT FOC: $\phi'(x) \leq 0$, $x \geq 0$ with complementary slackness: i.e.

If at the max $x > 0$, then $\phi'(x) = 0$

& If at the max $\phi'(x) < 0$, then $x = 0$.

Notice that this is a more general condition that would work for Problem (1) above $\left[\text{Max } f(x) = 1-(1-x)^2 \text{ s.t. } x \geq 0\right]$ as well.

Set S, function $f : S \to \mathbb{R}$.

x^* is a max of f on S if $f(x^*) \geq f(x)$, $\forall x \in S$

strict max if $f(x^*) > f(x)$ $\forall x \neq x^*$.

Local Max :

Need a topology τ on S.

τ is a ~~d~~ collection of subsets $G \subseteq S$ s.t.

$\emptyset$ & S $\in \tau$

τ is closed under arbitrary unions & finite intersections.

$G \in \tau$, G is known as an open set.

x^* is a local max if $f(x^*) \geq f(x)$ $\forall x \in G$, where G is an open set containing x^*.

For a metric space S_d (with metric d) or a normed vectorspace,

$y \in G$ is an interior point if $B(y; \varepsilon) \subseteq G$ for some $\varepsilon > 0$.

$B(y, \varepsilon) = \{ z \in S \mid d(z, y) < \varepsilon \}$.

G is an open set if all its points are interior points.

$\to f : S \to \mathbb{R}$. $S \subseteq \mathbb{R}^n$.

Theorem 1) Suppose x^* is a local max or min in the interior of S, & suppose f is differentiable at x^*.

Then $\nabla f(x^*) = \vec{0}$.

Theorem 2) Suppose in addition that $g_i : S \to \mathbb{R}$, $i = 1, \ldots, k$, $k < n$, all continuously differentiable, & $c_i \in \mathbb{R}$, $i = 1, \ldots, k$.

Suppose x^* is a max or min on

$\mathcal{U} \cap \{ \vec{x} \in S \mid g_i(\vec{x}) = c_i, \ i = 1, \ldots, k \}$, where $\mathcal{U} \subseteq \mathbb{R}^n$ is open.

$g = (g_1, \ldots, g_k)$.

Suppose $\operatorname{rank} \left[Dg(x^*) \right]_{k \times n} = k$.

Then $\exists$ $\lambda_1^*, \ldots, \lambda_k^*$, not all zero, s.t.

$$\nabla f(x^*) + \sum_{i=1}^{k} \lambda_i^* \nabla g_i(x^*) = \vec{0}.$$

Theorem 3) Suppose x^* is a max of f on
$$\mathcal{U} \cap \{\vec{x} \in \mathbb{R}^n \mid g_i(\vec{x}) = c_i,\ i=1,\dots,k,\ \&\ x_j \geq 0,\ j=1,\dots,n\},$$

& suppose $\text{Rank}\left(Dg(x^*)_{k\times n}\right) = k$

Then $\exists\ \lambda_1^*, \dots, \lambda_k^*$ s.t.

$$\forall j=1,\dots,n: \frac{\partial f}{\partial x_j} - \sum \lambda_i^* \frac{\partial g_i}{\partial x_j} \leq 0, \quad x_j \geq 0, \text{ with complementary slackness}$$

eg 1). Max $x+y$

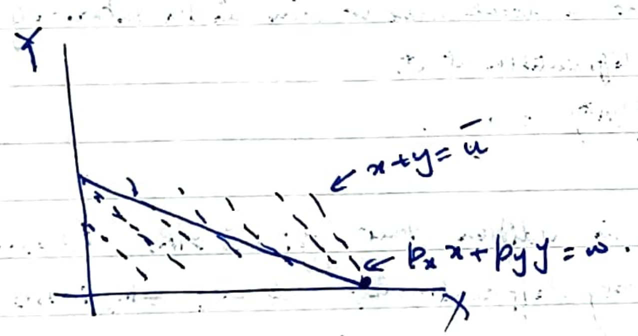
s.t. $p_x x + p_y y = w, \quad x \geq 0, y \geq 0.$

$$\mathcal{L}(x, y, \lambda) = x+y + \lambda(w - p_x x - p_y y)$$

FOC) $\frac{\partial \mathcal{L}}{\partial x} = 1 - \lambda p_x \leq 0, \quad x \geq 0$ with CS

$\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda p_y \leq 0, \quad y \geq 0$ with CS

$w - p_x x - p_y y = 0.$



$\left|\frac{p_x}{p_y}\right| < \frac{1}{1}$

If I give up 1 unit of Y, I get p_y rupees. With which I buy $\frac{p_y}{p_x}$ units of good X. Loss in utility = 1

Gain in utility = $\frac{p_y}{p_x} > 1$.

So, buy only x!

More generally with $u(x,y)$, loss in utility $= u_y \cdot 1$

& gain in utility $= \frac{p_y}{p_x} \cdot u_x$.

$$\frac{p_y}{p_x} \cdot u_x - u_y = 0 \Rightarrow \frac{u_x}{u_y} = \frac{p_x}{p_y}.$$

$$\because du = u\left(x + \frac{p_y}{p_x}, y-1\right) - u(x,y)$$

$$\simeq u_x \cdot \frac{p_y}{p_x} + u_y \cdot (-1).$$

Or if you like dy less $\Rightarrow \frac{p_y dy}{p_x}$ more for x : $u_x \cdot \frac{p_y}{p_x} dy - u_y dy = 0$

$\Rightarrow \frac{u_x}{u_y} = \frac{p_x}{p_y}$.

Back to linear utility:

I. Suppose at max $x = 0, y > 0$: $\Rightarrow y = \frac{w}{p_y}$.

& $1 - \lambda p_y = 0 \Rightarrow \lambda = \frac{1}{p_y}$.

$$1 - \lambda p_x \leq 0 \Rightarrow 1 - \frac{p_x}{p_y} \leq 0$$

$$\Rightarrow \boxed{\frac{p_x}{p_y} \geq 1}.$$

II. Suppose at max $x > 0, y = 0$: $x = \frac{w}{p_x}$.

$$\lambda = \frac{1}{p_x}.$$

$$1 - \lambda p_y \leq 0 \Rightarrow 1 - \frac{p_y}{p_x} \leq 0$$

$$\Rightarrow \boxed{\frac{p_x}{p_y} \leq 1}.$$

III. Suppose at max $x > 0, y > 0$. $\Rightarrow 1 - \lambda p_x = 0, \ 1 - \lambda p_y = 0$

$$\Rightarrow \lambda = \frac{1}{p_x} = \frac{1}{p_y}$$

$$\Rightarrow p_x = p_y.$$

We have demands :
$$x(p_x, p_y, w) = \begin{cases} \frac{w}{p_x} & \text{if } \frac{p_x}{p_y} \leq 1 \\ \frac{w - p_y y}{p_x} & \text{if } p_x = p_y \\ 0 & \text{if } \frac{p_x}{p_y} > 1. \end{cases}$$