HUL320B-odd sem 2022: Topics in IO
Problem Set 1

1. There are n consumers, with positive wealth levels $W_i, i = 1, \ldots, n$. All of them have identical utility functions over 2 goods, x and y, where x is rice, and y is the other good, given by

$$u(x, y) = x^a y^b$$

where a and b are given positive real numbers. They maximize utility given prices p_x, p_y.
(a) Obtain the expression for aggregate demand for rice.

There are m farmers. Each of them has 1 hectare of land under rice cultivation and an identical production function:

$$f(K, L) = AK^\alpha L^\beta$$

where α and β are positive and $\alpha + \beta < 1$.
(b) Obtain the cost function $C(y)$ for each firm, if $w > 0$ and $r > 0$ are the given wage and capital rental rates. Obtain the profit-maximizing output.
(c) Obtain the aggregate or industry supply function.
(d) Obtain the equilibrium price and quantity, as a function of the parameters of this problem, $a, b, W, \alpha, \beta, m, n, w, r, A$, where all consumers have identical income or wealth level W.
(e) Suppose $a = 1/4, b = 3/4, W = \$1000, \alpha = \beta = 1/3, m = 40 \times 10^6, n = 1.4 \times 10^9, w = r = 1$. Except for w and r, these numbers are meant to crudely capture some ballpark figures for India. Income W is taken to be about half of our per capita income.

What value of A would result in the equilibrium rice price being around Rupees 40 per kg? What is the equilibrium output for this A?

(2) (a) A monopoly faces the demand $D(p) = 1 - p$ and has cost function $C(q) = cq$, where c is the constant unit cost of production, and $0 < c < 1$. Calculate the monopoly price p^m and monopoply output q^m (as a function of c), the monopoly markup $\frac{p^m - c}{p^m}$, and the price elasticity of demand at the point (q^m, p^m) of the demand curve.
(b) Suppose $c = 0$. What is the elasticity of demand at (q^m, p^m)?
(c) What is the deadweight loss under monopoly when $c = 0$?

(3) Show that a monopolist will not produce at a point (q, p) on a demand curve such that the price elasticity of demand is less than 1.

(4) A market served by a monopoly has consumers with 2 kinds of preferences. Type i's utility from buying quantity q and paying T for this quantity is

$$u_i = \theta_i v(q) - T, \ i = 1, 2$$

where $v(q) = \frac{1-(1-q)^2}{2}$, the monopoly's constant unit cost of production is c, and $\theta_2 > \theta_1 > c > 0$.
(a) If type i can buy the good at price p for each unit of output, solve his or her utility maximization problem to work out her optimal demand $D_i(p)$ as a function of p.
(b) Suppose the 2 types of consumers were in separate markets that the monopolist could sell to separately, and charge different two-part tariffs of the form $T(q) = A + pq$, what price p and 'entry fee or premium' A would it charge in the two markets?
(c) Suppose both types of consumers populate a common market, the monopolist does not know who is who, but knows that type 1 consumers are in proportion λ (which is 'high enough' so the monopolist wants to sell to both types of consumers). The monopolist can simply charge a unit price p to maximize $pD(p) - cD(p)$, where

$$D(p) = \lambda D_1(p) + (1 - \lambda) D_2(p)$$

Work out the monopoly price and quantity.
(d) The monopolist is permitted to choose a price scheme $T(q) = A + pq$. What will be the profit maximizing p and A?