hul 320B: Problem Set 1 Qs 4(a) – 4(c) Solutions

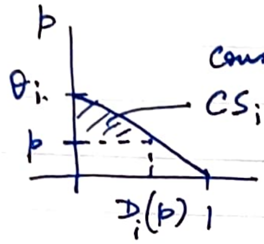
4(a). $\max_q \left[\theta_i v(q) - pq\right]$. Cons^r of Type i solves this when facing price p.

FOC: $\theta_i v'(q) = p$

Here, $p = \theta_i (1-q)$ or

$q = D_i(p) = 1 - \frac{p}{\theta_i}$. //

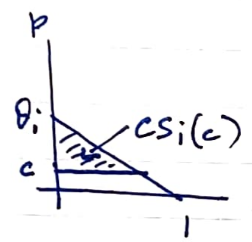
4(b).



Consumer surplus if Type i buys at price p :

$CS_i(p) = \frac{1}{2} D_i(p) (\theta_i - p)$

$= \frac{1}{2\theta_i} (\theta_i - p)^2$.



We saw that a monopolist who could charge a 2-part tariff could maximize profit by extracting all consumer surplus.

So for type i, charge

$T_i(q) = A_i + cq$,

with $A_i = CS_i(c) = \frac{1}{2\theta_i}(\theta_i - c)^2$

& unit price $p = c$ (unit marginal cost). //

4(c) $D(p) = \lambda D_1(p) + (1-\lambda) D_2(p)$

$= \lambda\left(1 - \frac{p}{\theta_1}\right) + (1-\lambda)\left(1 - \frac{p}{\theta_2}\right)$

$= 1 - \left(\frac{\lambda}{\theta_1} + \frac{1-\lambda}{\theta_2}\right) p$

So $D(p) = 1 - Kp$, where $K = \frac{\lambda}{\theta_1} + \frac{1-\lambda}{\theta_2}$.

Note : $D'(p) = -K$

Monopolist with unit cost c solves $\max_p \left[p D(p) - c D(p)\right]$

FOC) $p D'(p) + D(p) - c D'(p) = 0$

$\Rightarrow p(-K) + 1 - Kp - c(-K) = 0$

$\Rightarrow p = \frac{1 + cK}{2K}$

$q = 1 - Kp = 1 - \cancel{K}\frac{(1+cK)}{2\cancel{K}}$ //

4(d) Please do yourselves as practice for Minor.