Answers to Chapter 8 Exercises

Review and practice exercises

■ **8.1. Bertrand in the real world.** The Bertrand model of price competition suggests that, under a given set of conditions, firms make zero economic profits even if there are only two firms. However, there are many instances of industries with a small number of competitors where firms appear to earn more than zero economic profits.

Give an example of an industry dominated by a couple of firms where profits are significant. Explain why the predictions of the Bertrand model are not borne out.

Answer: Many potential examples, including P&G and Unilever, Pepsi and Coke, Christie's and Sotheby's, Dell and Lenovo, breakfast cereals, air travel (?), vodka. Idealized conditions which may fail:

- Infinite supply ability (implicit in constant marginal cost assumption). In fact, capacity limitations may dull the incentives to undercut. This may apply to airlines in routes when capacity is lower than demand.

- Perfect substitutes. In practice, firms may create brands to distinguish themselves and thus soften price competition. This is probably the most important point in the P&G/Unilever, Pepsi/Coke, vodka, breakfast cereal examples.

- Identical marginal costs. Frequently, one of the firms is more efficient than the other. This may apply to Dell in the desktop computer market and possibly to Southwest Airlines in air travel.

- One-time competition. Repeated interaction opens the scope for implicit or explicit collusion. Christie's and Sotheby's may be a good example.

■ **8.2. Bertrand and zero profits.** According to Bertrand's theory, price competition drives firms' profits down to zero even if there are only two competitors in the market. Why don't we observe this in practice very often?

Answer: See the discussion starting on page 5. Some of the ways out of the so-called "Bertrand trap" include product differentiation, dynamic competition, asymmetric costs, and capacity constraints.

■ **8.3. E-commerce.**   E-commerce represents an increasing fraction of economic trans-
actions in many different industries. Does e-commerce create a Bertrand trap? What is
special about e-commerce (and, more generally, the new economy) that makes the Bertrand
trap a dangerous trap? How can e-commerce firms avoid the trap?

Answer: In many ways, e-commerce makes sellers look more alike, that is, reduces the
degree of product differentiation. In this sense, it makes firms more prone to fall in the
Bertrand trap. Moreover, unlike brick-and-mortar stores, capacity constraints tend to be
less important at online stores — because, for example, they can pool the inventories at
various physical locations — which further increases the chance of a Bertrand trap.
   Having said that, there are ways in which an online company can differentiate itself. For
example, Amazon.com, by means of a successful branding strategy and a superior search
engine, has managed to carve out a considerable market share of relatively loyal consumers.

■ **8.4. Price discrimination.** A monopolist is generally better off by price-discriminating.
What about a duopolist? Consider the special case when Market A is Firm 1's "strong"
market, whereas market B is firm 2's "strong" market.

Answer: There may be situations where duopolists are strictly worse off by their ability to
price discriminate. Suppose that Firm 1 has a strong following in market A, whereas Firm
2 has a strong following in market B. If firms must set the same price in both markets, they
have good reasons not to price very aggressively: each firm sets a high price and enjoys
a large market share in its "home" market. In fact, undercutting the rival in its home
market would imply a large cost in terms of home market margin for a relatively low gain.
If however the firms can set different prices in each market, then there is less of a reason
for Firm 1 not to go after Firm 2's home market more aggressively. This may end up in a
prisoner's dilemma type of outcome: firms are worse off by pricing aggressively even though
firms would be better off if both priced less aggressively.[13]

■ **8.5. Cournot model assumptions.**   Three criticisms are frequently raised against the
use of the Cournot oligopoly model: (i) firms normally choose prices, not quantities; (ii)
firms don't normally take their decisions simultaneously; (iii) firms are frequently ignorant
of their rivals' costs; in fact, they do not use the notion of Nash equilibrium when making
their strategic decisions. How would you respond to these criticisms? (Hint: in addition to
this chapter, you may want to refer to Chapter 7.)

Answer:

1. As explained at the end of Section 8.1, if firms are capacity constrained, then price
   competition "looks like" like quantity competition.

2. If there are significant information lags, then sequential decisions "look like" simulta-
   neous decisions. See Chapter 7.

3. At the end of Section 8.2, we discussed an argument for the relevance of Nash equi-
   librium which only requires each firm to know its own profit function.

■ **8.6. Cournot vs. Bertrand.** Which model (Cournot, Bertrand) would you think provides

a better approximation to each of the following industries: oil refining, internet access, insurance. Why?

Answer: Capacity constraints seem relatively more important in oil refining and relatively less important in insurance. Given the discussion in Section 8.3, one would be inclined to select the Cournot model for oil refining and the Bertrand model for insurance. Internet access is an intermediate case between the previous two other examples.

■ **8.7. ByeByeCold.** You are currently the sole seller of ByeByeCold, a revolutionary drug that almost instantly eliminates cold symptoms. Although the production cost is only $.10 per dose, you sell ByeByeCold for $1.39 per dose, for a total profit of $900m a year. You are currently considering licensing ByeByeCold to a second producer. Neither you nor your competitor have any significant capacity constraints. One of your managers suggested, since the firm would be sharing the market with a competitor, it would be appropriate to charge a flat fee that covers half the current profits plus a generous margin; the value of $700m was suggested. An alternative proposal would be to set a royalty fee of $.50 per dose. What is your opinion?

Answer: If you license ByeByeCold for a flat fee, you will be competing with another firm selling the same product and with a similar marginal cost. Except for the possibility of collusion, this would imply approximately zero profits for both firms. It follows that half the current profits would not be sufficient to compensate for the profit loss from licensing. In fact, there exists no licensing contract that would be profitable for both parties.

Why do firms ever license, then? One possibility is that the second firm is more efficient in production so that there are gains from bringing it on board. Also, there may be reasons why production by a second firm increases the size of the market. If these were true, then it is possible that a profitable licensing arrangement can be made.

■ **8.8. French generics manufacturer.** Consider the last problem in Section 8.4. Suppose that a retailing campaign costing € 80m is expected to increase demand by 40%. Suppose also that the current rupee/euro exchange rate is 50 INR/€. Should the French firm go ahead with the campaign? One macroeconomics expert tells you that "it is likely that the rupee will appreciate in the near future." How would this influence your decision?

Answer: As we saw earlier, the French company's profit when the exchange rate is 50 is $\pi_1 = $ € 250 m. The advertising campaign would increase demand by 40%. Since price remains constant, it would also increase profits by 40%, that is by an amount equal to 40% × 250 m = € 100 m. This is more than cost ($80m), so the French company should go ahead with the campaign. The possibility of the rupee appreciating only makes things better: it raises the Indian firm's cost in €. which means the French firm can charge a higher price. Not only does this increase the French firm's profit, it also increases the increase in profit resulting from the retailing campaign.

■ **8.9. Karmania automobiles.** There are two auto producers in Karmania, F1 and F2. The cars they produce are essentially identical. The market inverse demand curve is given by $p = a - b\,Q$, where p is price (in thousands of dollars); Q market output (in thousands

of units); and a and b are parameters. It is estimated that $a = 25$ and $b = .1$. Both F1 and F2 have a marginal cost of 10 thousand dollars per car.

Competition in the Karmania auto market works as follows. At the beginning of each year, both firms simultaneously and independently decide how many cars to produce. Then the market price adjusts so that supply equals demand.

(a) Determine F1's best response mapping.

Answer: F1's profit is given by

$$\pi_1 = p\,q_1 - c\,q_1 = \left(25 - .1\left(q_1 + q_2\right)\right)q_1 - 10\,q_1$$

Taking the derivative with respect to q_1 and equating to zero, we get

$$25 - .1\left(q_1 + q_2\right) - .1\,q_1 - 10 = 0$$

which leads to

$$q_1 = 75 - \frac{1}{2}\,q_2$$

which gives F1's best response mapping.

(b) Determine the equilibrium of the game played between F1 and F2.

Answer: Since F1 and F2 have the same marginal cost, in equilibrium $q_1 = q_2 = q$. Substituting in the best-response mapping derived above, we get

$$q = 75 - \frac{1}{2}\,q$$

or simply

$$q = \frac{2}{3}\,75 = 50$$

Regarding price, we have
$$p = 25 - .1 \times (50 + 50) = 15$$

(c) Suppose that an increase in incomes shifts demand to $p = 28 - 0.1\,Q$. What do you expect will happen to price and the number of cars sold?

Answer: The new first-order condition for F1 is given by

$$28 - .1\left(q_1 + q_2\right) - .1\,q_1 - 10 = 0$$

leading to

$$q_1 = 90 - \frac{1}{2}\,q_2$$

Again, $q_1 = q_2 = q$ implies $q = 90 - \frac{1}{2}\,q$, $\left(1 + \frac{1}{2}\right)q = 90$, or simply

$$q = \frac{2}{3}\,90 = 60$$

Finally,
$$p = 28 - .1 \times (60 + 60) = 16$$

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We conclude that the increase in demand leads to an increase in price from 15 to 16 and in output from 50 to 60.

■ **8.10. Ethanol.** In the ethanol industry, each firm chooses what output to produce and price is determined by aggregate output. Market demand is given by $Q = 1500 - 2\,p$, where Q is in million tons and p in \$/ton. There are two producers and their marginal costs are constant and given by $c_1 = 340$, $c_2 = 420$ (both in \$/ton).

(a) Determine equilibrium price, output and market shares.

Answer: Given the nature of industry competition, the Cournot model provides a good approximation for firm behavior. Firm 1's profit function is given by

$$q_1 = (a - b\,Q)\,q_1 - c_1\,q_1$$

Keeping in mind that $Q = q_1 + 1_2$ (and thus $dQ/dq_1 = 1$), the first-order condition for profit maximization becomes

$$a - b\,Q - b\,q_1 - c_1 = 0 \tag{8.7}$$

Likewise, for Firm 2 we have

$$a - b\,Q - b\,q_2 - c_2 = 0$$

Adding up these two equations gives

$$a - 2\,b\,Q - b\,q_2 - b\,q_1 - c_1 - c_2 = 0$$

Since $Q = q_1 + q_2$, this simplifies to

$$Q = \frac{a - c_1 - c_2}{3\,b} \tag{8.8}$$

Going back to (8.7), we get

$$q_1 = \frac{a - c_1}{b} - Q$$

Substituting (8.8) for Q, we get

$$q_1 = \frac{a - c_1}{b} - \frac{a - c_1 - c_2}{3\,b} = \frac{3\,a - 3\,c_1 - a + c_1 + c_2}{3\,b} = \frac{2\,a + c_2 - 2\,c_1}{3\,b}$$

More generally,

$$q_i = \frac{2\,a + c_j - 2\,c_i}{3\,b}$$

Since $Q = 1500 - 2\,p$, $p = 750 - Q/2$, and thus we have $a = 750$, $b = .5$, $c_1 = 340$ and $c_2 = 420$. It follows that

$$q_1 = 327 \text{ million tons}$$
$$q_2 = 167 \text{ million tons}$$
$$s_1 = 66\%$$
$$s_2 = 34\%$$
$$p = 503 \text{ \$/ton}$$
$$\pi_1 = 53.36 \text{ \$billion}$$
$$\pi_2 = 13.89 \text{ \$billion}$$

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Firm 2 is currently considering two possible strategies: (a) a public opinion campaign that would cost \$1.15 billion and shift the demand curve to $Q = 1520 - 2\,p$; (b) a capital investment of \$4.9 billion that would reduce marginal cost c_2 to 400 \$/ton.

> (b) Are investments (a) and (b) worthwhile in isolation? Are they worth-
> while if taken together? Justify your answer.

Answer: The shift in the demand curve corresponds to a new value of a, which is now 760. All other parameter values remain constant. Recomputing the equilibrium, we now get $\pi_2 = 15.02$, an increase of \$1.13 billion. This does not compensate the \$1.15 billion investment. Recomputing profit with the lower marginal cost $c_2 = 400$ (and the initial value of $a = 750$), we get $\pi_2 = 18.69$, an increase of \$4.8 billion. This does not compensate the \$4.9 billion investment.

Now consider the *joint* investment strategy: at a cost \$1.15 + \$4.9 = \$6.05 billion, parameter a becomes 760, whereas c_2 becomes 400. Recomputing the equilibrium, we get $\pi_2 = 20$, a gain of \$6.11 billion — greater than the \$6.05 billion investment.

We conclude that the whole is greater than the sum of the parts: the increase in profits from expanding demand and reducing marginal costs is greater than the sum of the partial increases. The idea is that, with a greater demand, the benefit from lower costs is greater: a one dollar decrease in marginal cost applies to a larger quantity. Conversely, an increase in demand is worth more the greater the margin the firm sells for; and a lower marginal cost implies a higher margin.

■ **8.11. Natural gas.** Suppose there are only two natural gas producers in Kabralkstan. In each period, firms determine how much natural gas to sell; market price is then determined by total demand and total supply. Marginal cost is given by 77 for Firm 1 and 74 for Firm 2. Currently, Firms 1 and 2 are producing 170 and 200, respectively, whereas market price is 94.

By making an important discovery in the process of hydraulic fracturing (or "fracking"), Firm 2 managed to cut its marginal cost from 74 to 68.

> (a) What impact do you expect Firm 2's cost reduction to have on its
> market share?

Answer: Suppose that demand is linear: $p = a - b\,Q$. Equilibrium p and Q are given by

$$p = \frac{a + c_1 + c_2}{3}$$

$$Q = \frac{2\,a - c_1 - c_2}{3\,b}$$

From the first equation I get

$$a = 3\,p - c_1 - c_2 = 131$$

From the second equation I get

$$b = \frac{2\,a - c_1 - c_2}{3\,Q} = .1$$

Firm i's equilibrium output is given by

$$q_i = \frac{a + c_j - 2\,c_i}{3\,b}$$

and so firm i's market share is given by

$$s_i = \frac{a + c_j - 2\,c_i}{2\,a - c_1 - c_2}$$

It follows that, initially, Firm 2's market share is given by

$$s_2 = \frac{131 + 77 - 2 \times 74}{2 \times 131 - 77 - 74} \approx 54\%$$

With a lower cost, Firm 2's market share becomes

$$s_2 = \frac{131 + 77 - 2 \times 68}{2 \times 131 - 77 - 68} \approx 62\%$$

Some studies suggest that Firm 2's new production process may not be environmentally sound.

(b) How much would Firm 1 be willing to pay in support of a campaign to (successfully) prevent Firm 2 from using its new fracking process?

Answer: Firm i's equilibrium profit is given by

$$\widehat{\pi}_i = \frac{1}{b} \left(\frac{a + c_j - 2\,c_i}{3} \right)^2$$

It follows that, if Firm 2's cost is high, then Firm 1's profit is given by

$$\widehat{\pi}_1 = \frac{1}{b} \left(\frac{131 + 74 - 2 \times 77}{3} \right)^2 = 2890$$

If Firm 2's cost is low, then Firm 1's profit is given by

$$\widehat{\widehat{\pi}}_1 = \frac{1}{b} \left(\frac{131 + 68 - 2 \times 77}{3} \right)^2 = 2250$$

It follows that Firm 1 would be willing to pay up to $2890 - 2250 = 640$ to prevent Firm 2 from lowering its cost.

Challenging exercises

■ **8.12. Wolframium.**   Suppose there are two producers of wolframium in the world. Wolframium is a homogenous product. Producers set prices simultaneously and capacity constraints are not binding at the current levels of world demand. Both producers have a marginal cost of \$900 per metric tonne. One producer is located in the US, the other one in Mexico. Demand for wolframium is exclusively found in the US It is estimated that, at $p = \$1000$, world demand for woframium is 130 thousand metric tonnes per year, and that demand elasticity is $\epsilon = -.5$.

(a) Suppose the government imposes an import tax of 20% on wolframium imports. What are equilibrium price and profits?

Answer: The US producer sets a price of $\$1080 = 900\,(1+20\%)$ (or infinitesimally smaller) and receives a profit of $D(1080)\,(1080-900) = 180\,D(1080)$, where $D(p)$ is the demand curve. Given the information on demand and elasticity, we have

$$-.5 = \frac{\log D(1080) - \log 130}{\log 1080 - \log 1000}$$

and so

$$\log D(1080) = \log 130 - .5\,(\log 1080 - \log 1000) = 4.829$$

From this we estimate that $D(1080) = \exp(4.829) = 125.1$. Finally, we conclude that the US firm's profit is given by $180 \times 125.1 = 20736$, or $\$20.7\,\mathrm{bn}$. The Mexican firm sets a price of 1080 and earns zero profit.

Alternatively, we can estimate demand at $p = 1080$ by using the percent variation method:

$$D(1080) = 120\left(1 - .5\,\frac{1080 - 1000}{1000}\right) = 115.2$$

    (b) Suppose a third producer enters the wolframium industry. It is located in China and has a marginal cost of $\$600$ per metric tonne. What impact does this have on the equilibrium prices and profits?

Answer: The US producer now sets a price of $\$900$. The Chinese firm sets a price of $\$750$ (or infinitesimally smaller), so that price after import taxes is $\$900$, beating the US firm. The American firm makes zero profit, the Chinese company earns a profit of $D(900)\,(750-600) = 150\,D(900)$, where $D(p)$ is the demand curve.

An estimate of $D(900)$ is given by

$$D(900) = 120\left(1 - .5\,\frac{900 - 1000}{1000}\right) = 126.$$

We conclude the Chinese producer earns a profit of 150×126, or $\$18.9\,\mathrm{bn}$.

■ **8.13. Shipbuilding.** The world shipbuilding industry is dominated by three countries/ regions: Japan, Europe and China. Demand for ships is given by $p = a - b\,Q$, where b has been estimated by industry participants to equal 0.42. Before 2006, the world quarterly production of ships was 19 bulk carriers per quarter. The average price of a bulk carrier was US$17.8 million. Country market shares were as follows: China 24%, Europe 8%, Japan 68%.

    (a) Assuming that the industry is well described by a Cournot game played between countries, estimate each country's production marginal cost before 2006.

Answer: First we estimate the inverse demand intercept by setting

$$a = p + b\,Q = 17.8 + 0.42 \times 19 = 25.8$$

Next we use the first order condition to get

$$c_i = a - \beta Q\left(2\,s_i - \sum_{j \neq i} s_j\right)$$

which implies

$$c_{\text{China}} = 15.9$$
$$c_{\text{Europe}} = 17.2$$
$$c_{\text{Japan}} = 12.4$$

In 2006, China introduced a government plan to guide the development of its shipbuilding industry. After 2006, the number of Chinese shipyards increased dramatically. The same happened with China's ship production rate: its market share jumped to 50%, while Europe's dropped to 5% and Japan's tp 45%. World Trade Organization (WTO) agreements prohibit government industrial subsidies. Thus, complaints by WTO members led to an investigation to find out if the Chinese government subsidized shipbuilding and if so by how much.

      (b) Suppose that a production subsidy of z implies a decrease in China's marginal cost from c to $c - z$. Use the pre- and post-2006 data to estimate z.

Answer: By repeating the same process as in the previous question, we get

$$c_{\text{China}} = 13.8$$

so that $z = 15.9 - 13.8 = $ US$2.1 million, which in turn represents 13% of cost.

      (c) Compute consumer surplus and profits by country. Who was hurt and who gained from China's production subsidies?[14]

■ **8.14. Strategic trade policy.** Suppose a given country's domestic market is supplied by two firms competing a la Cournot: firm 1, a domestic firm, and firm 2, a foreign firm. Demand is given by $p = a - Q$, where Q is total output, and marginal costs by c_1 and c_2, where we assume $c_i < a$ $(i = 1, 2)$. Suppose that the domestic government levies an import tariff t to be paid by firm 2 for every unit sold in the domestic market.

      (a) Determine the equilibrium values of q_i for a given value of t.

Answer: Variable profit functions are given by

$$\pi_1 = (a - q_1 - q_2 - c_1)\, q_1$$
$$\pi_2 = (a - q_1 - q_2 - c_2 - t)\, q_2$$

From the first-order conditions, we get the best response mappings

$$q_1^* = \tfrac{1}{2}\,(a - c_1) - \tfrac{1}{2}\, q_2$$
$$q_2^* = \tfrac{1}{2}\,(a - c_1 - t) - \tfrac{1}{2}\, q_1$$

Solving the system of best responses, we get

$$\widehat{q}_1 = \tfrac{1}{3}\,(a + c_2 + t - 2\, c_1)$$
$$\widehat{q}_2 = \tfrac{1}{3}\,(a + c_1 - 2\, c_2 - 2\, t)$$

(b) Show that a small import tariff increases domestic welfare, where the latter is defined as the sum of consumer surplus, the domestic firm's profit and import revenues.

Answer: Equilibrium consumer surplus is given by

$$\widehat{S} = \tfrac{1}{2}\,(a - p)^2$$

The domestic firm's equilibrium profit is given by

$$\widehat{\pi}_1 = \tfrac{1}{9}\,(a + c_2 + t - 2\,c_1)^2$$

Import tariff revenues are simply $t\,q_2$, or $t\,\widehat{t}_2$ in equilibrium. Adding up the three terms, we conclude that domestic welfare is given by

$$
\begin{aligned}
\widehat{W}_d &= \widehat{S} + \widehat{\pi}_1 + t\,\widehat{q}_2 \\
&= \tfrac{1}{2}\,(a - p)^2 + \tfrac{1}{9}\,(a + c_2 + t - 2\,c_1)^2 + \tfrac{t}{3}\,(a + c_1 - 2\,c_2 - 2\,t)
\end{aligned}
$$

Taking the derivative with respect to t and *then* setting $t = 0$, we get

$$\left.\frac{d\widehat{W}_d}{dt}\right|_{t=0} = \tfrac{1}{3}\,(a - c_2)$$

which is positive, given our assumption that $c_i < a$.

(c) Show that, the more efficient the foreign firm, the greater the increase in domestic welfare from an import tariff. Discuss.

Answer: From the previous answer, we see that, the lower c_2, the greater $d\widehat{W}_d/dt$. The main reason is that, the lower c_2, the greater q_2 is. This implies that the import tariff t applies to a larger quantity. In the limit, if $c_2 = a$, the foreign firm's imports are equal to zero, in which case the import tariff has no effect.

(d) Show that, if c_1 is not very different from c_2, then a small import tariff decreases world welfare, where the latter is defined as the sum of consumer surplus, and the profits of the domestic and foreign firms (notice that import duties are a transfer and thus do not enter world welfare calculations).

Answer: The foreign firm's equilibrium profit is given by

$$\widehat{\pi}_2 = \tfrac{1}{9}\,(a + c_1 - 2\,c_2 - 2\,t)^2$$

This implies that world welfare is given by

$$
\begin{aligned}
\widehat{W}_t &= \widehat{S} + \widehat{\pi}_1 + \widehat{\pi}_2 \\
&= \tfrac{1}{2}\,(a - p)^2 + \tfrac{1}{9}\,(a + c_2 + t - 2\,c_1)^2 + \tfrac{1}{9}\,(a + c_1 - 2\,c_1 - 2\,t)^2
\end{aligned}
$$

Taking the derivative with respect to t and *then* setting $t = 0$, we get

$$\left.\frac{d\widehat{W}_t}{dt}\right|_{t=0,\,c_2=c_1} = -\tfrac{4}{9}\,(a - c_1)$$

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which is negative, given our assumption that $c_i < a$.

      (e) In light of the above results, what can be an important role of the World Trade Organization (WTO)?

Answer: The above results suggest that trade policy may have the nature of a prisoner's dilemma (cf Chapter 7): for each country individually, raising tariffs is a dominant strategy, but the world as a whole is worse off when countries raise tariffs. For this reason, a world organization that attempts to collectively reduce import tariffs may contribute to an increase in world welfare. Long live the WTO!

■ **8.15. Cournot with n asymmetric firms.** Consider an industry with n output setting firms, each with constant marginal cost c_i and fixed cost F_i. Market demand is given by $p = a - bQ$, where $Q = \sum_{i=1}^{n} q_i$.

      (a) Show that firm i's best-response mapping is given by $q_i^*(Q_{-i}) = \frac{a - c_i}{2b} - \frac{1}{2} Q_{-i}$, where $Q_{-i} \equiv \sum_{j \neq i} q_j$.

Answer: Firm i's profit function is given by

$$\pi_i = (a - bQ)\, q_i - c_i\, q_i - F_i$$

The first-order condition for profit maximization is given by

$$a - bQ - b\, q_i - c_i = 0 \tag{8.9}$$

Since $Q = \sum_{i=1}^{n} q_i$, this can be re-written as

$$a - b \sum_{j \neq i} q_j - 2b\, q_i - c_i = 0$$

or simply

$$q_i^*(q_j) = \frac{a - c - i}{2b} - \frac{1}{2} \sum_{j \neq i} q_j$$

      (b) Show that, in equilibrium, total output is given by $\widehat{Q} = \left(n\, a - \sum_{i=1}^{n} c_i\right) / \left(b\, (n+1)\right)$. (Hint: add up all n first-order conditions for profit maximization.)

Answer: Adding up all equations (8.9), we get

$$n\, a - n\, b\, Q - b \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} c_i = 0$$

Since $Q = \sum_{i=1}^{n} q_i$, this reduces to

$$n\, a - (n+1)\, b\, Q - \sum_{i=1}^{n} c_i$$

or simply

$$Q = \frac{n\, a - \sum_{i=1}^{n} c_i}{b\, (n+1)} \tag{8.10}$$

(c) Show that equilibrium price is given by $\hat{p} = \left(a + \sum_{i=1}^{n} c_i\right)/(n+1)$

Answer: Substituting for Q in the demand curve, we get

$$
\begin{aligned}
p &= a - bQ \\
&= a - \frac{na - \sum_{i=1}^{n} c_i}{n+1} \\
&= \frac{(n+1)a - na + \sum_{i=1}^{n} c_i}{n+1} \\
&= \frac{a + \sum_{i=1}^{n} c_i}{n+1}
\end{aligned}
\tag{8.11}
$$

(d) Show that, in equilibrium, firm i's output level is given by
$\hat{q}_i = \left(a - n c_i + \sum_{j \neq i} c_j\right)/(b(n+1))$

Answer: Substituting the expression for Q in (8.9) and solving with respect to q_i, we get

$$
\begin{aligned}
q_i &= \frac{1}{b}(a - bQ - c_i) \\
&= \frac{1}{b}\left(a - \frac{na - \sum_{i=1}^{n} c_i}{n+1} - c_i\right) \\
&= \frac{(n+1)(a - c_i) - na + \sum_{i=1}^{n} c_i}{b(n+1)} \\
&= \frac{a - n c_i + \sum_{j \neq i} c_j}{b(n+1)}
\end{aligned}
$$

(e) Show that, in equilibrium, firm i's profit is given by
$\hat{\pi}_i = \left(a - n c_i + \sum_{j \neq i} c_j\right)^2/(n+1)^2/b - F_i$

Answer: The profit function can be re-written as

$$
\pi_i = (a - bQ - c_i)q_i - F_i
$$

Moreover, the first-order condition can be rewritten as

$$
a - bQ - c_i = b q_i
$$

Hence, *at the equilibrium point*, we have

$$
\hat{\pi}_i = b q_i^2 - F_i = \frac{1}{b}\left(\frac{a - n c_i + \sum_{j \neq i} c_j}{n+1}\right)^2 - F_i
$$

(f) Show that, in equilibrium, consumer surplus is given by
$CS = \frac{1}{2b}\left(na - \sum_{i=1}^{n} c_i\right)^2/(n+1)^2$

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Answer: Consumer surplus is given by $CS = \frac{1}{2}(a - Q)Q$. Substituting (8.10) for Q

$$
\begin{aligned}
CS &= \frac{1}{2}(a - p)Q \\
&= \frac{1}{2}\big(a - (a - bQ)\big)Q \\
&= \frac{b}{2}Q^2 \\
&= \frac{1}{2b}\left(\frac{na - \sum_{i=1}^{n} c_i}{n + 1}\right)^2
\end{aligned}
$$

■ **8.16. Elasticity rule (reprise).** Show that the elasticity rule derived in Chapter 3, that is $(p - MC)/MC = -1/\epsilon$, holds under Cournot competition with linear demand and costs, where MC is firm i's marginal cost and ϵ its demand elasticity (not the market elasticity).

Answer: From the definition of demand elasticity, I can write

$$
-\frac{1}{\epsilon_i} = -\frac{dp}{dQ}\frac{q_i}{p}
$$

Therefore, the elasticity rule may be re-written as

$$
p - c_i = -\frac{dp}{dQ}q_i \tag{8.12}
$$

Let the inverse demand curve be given by $p = a - bQ$. From Exercise 8.8.15,

$$
p = \frac{a + \sum_{i=1}^{n} c_i}{n + 1}
$$

which implies

$$
p - c_i = \frac{a - (n + 1)c_i + \sum_{i=1}^{n} c_i}{n + 1} \tag{8.13}
$$

Also from Exercise 8.8.15,

$$
q_i = \frac{a - n c_i + \sum_{j \neq i} c_j}{b(n + 1)}
$$

which I can re-write as

$$
q_i = \frac{a - (n + 1)c_i + \sum_{j=1}^{n} c_j}{b(n + 1)} \tag{8.14}
$$

Substituting (8.13) for $p - c_i$, (8.14) for q_i, and b for $-\frac{dp}{dQ}$ in (8.12), we obtain an equality. Bingo!

■ **8.17. Efficiency loss under Cournot.** Consider a market where two firms simultaneous set quantities of a homogeneous product with demand given by $Q = 37.5 - P/4$. Each firm has constant marginal cost equal to 30.

(a) Determine equilibrium output and price.

Answer: Duopolist i's profit is given by

$$\pi_i = q_i\, p(Q) - C(q_i) = q_i\left(150 - 4\left(q_i + q_j\right)\right) - 30\,q_i$$

The first order condition for profit maximization is given by:

$$150 - 4\left(q_i + q_j\right) - 4\,q_i - 30 = 0$$

By symmetry, we have $q_i = q_j = q$. Solving the above equation, we then get $q = 10$. Moreover, $p = 150 - 8\,q = 70$.

      (b) Compute the efficiency loss as a percentage of the efficiency loss under monopoly.

Answer: The monopoly profit function is given by

$$\pi_m = Q\,p(Q) - C(Q_i) = Q\left(150 - 4\,Q\right) - 30\,Q$$

The first order condition for profit maximization is given by:

$$150 - 8\,Q - 30 = 0$$

Solving with respect to Q we get $Q = 15$, and then $p = 90$.

    Under perfect competition the prevailing price would be given by marginal cost: $p = 30$; total quantity would be $Q = 30$ and welfare

$$W = CS = \frac{\left(p(0) - p\right)\,Q}{2} = 1800$$

Under duopoly, total welfare is given by:

$$W_d = 2\,\pi + CS = 2\,q\,(p - c) + \left(p(0) - p\right)\,q = 1600$$

Under monopoly, total welfare is given by

$$W_m = \pi + CS = (p - c)\,Q + \frac{\left(p(0) - p\right)\,Q}{2} = 1350$$

Finally, the duopoly efficiency loss as a percentage of the monopoly efficiency loss is given by

$$EL = \frac{1800 - 1600}{1800 - 1350} = 44.\underline{4}\%$$

■ **8.18. Equilibrium price under Cournot.** Show analytically that equilibrium price under Cournot is greater than price under perfect competition but lower than monopoly price.

Answer: In a Cournot oligopoly, firm i's profit is given by $\pi_i = q_i\,P(Q) - C(q_i)$, where Q is total output. The first-order condition for profit maximization is given by

$$P(Q) + q_i\,\frac{dP}{dq_i} - MC = 0 \tag{8.15}$$

The first-order condition for a monopolist is given by

$$P(Q) + Q\frac{dP}{dQ} - MC = 0 \tag{8.16}$$

Finally, under perfect competition we have

$$P(Q) - MC = 0$$

Notice that $dP/dq_i = dP/dQ < 0$. Consider the case of oligopoly and suppose that price is equal to monopoly price. Monopoly price is such that the (8.16) holds exactly. The only difference between (8.15) and (8.16) is that the latter has Q instead of q_i. Since $Q > q_i$, it follows that, for p equal to monopoly price, the left-hand side of (8.15) is positive. Finally, if it is positive, each firm has an incentive to increase output, which results in a lower price.

By a similar argument we can also show that price under Cournot competition is greater than marginal cost.

See also Figure 8.6 for a graphical derivation of the same result.

■ **8.19. Cournot with increasing marginal cost.** Consider a duopoly for a homogenous product with demand $Q = 10 - P/2$. Each firm's cost function is given by $C = 10 + q(q+1)$.

(a) Determine the values of the Cournot equilibrium.

Answer: Duopolist i's profit is given by

$$\pi_i = q_i\, p(Q) - C(q_i) = q_i \left(20 - 2\,(q_i + q_j)\right) - 10 - q_i\,(q_i + 1)$$

The first order condition for profit maximization is given by:

$$20 - 2\,(q_i + q_j) - 2\,q_i - 2\,q_i - 1 = 0$$

The problem of duopolist j is symmetric, therefore we have $q_i = q_j = 2.375$ and $p = 10.5$.

(b) Re-compute the equilibrium values assuming that one of the firms — say, firm 2 — has a cost function given by $C = 10 + q(q+1)$.

Answer: The first-order conditions are now given by The first order condition for profit maximization is given by:

$$20 - 2\,(q_1 + q_2) - 2\,q_1 - 2\,q_1 - 1 = 0$$
$$20 - 2\,(q_i + q_1) - 2\,q_2 - 2\,q_2 - 2 = 0$$

Solving this system we get $q_1 = 2.4375, q_2 = 2.1875$

■ **8.20. Cement.** Two firms compete (a la Cournot) in the cement market. Demand for cement is given by $Q = 450 - 2\,P$. Firm 1's marginal cost is constant at 50, firm 2's at 40. A technological innovation allows firms to reduce marginal cost by 6.

(a) How much would each firm be willing to pay for the innovation if it were the only competitor to acquire it?

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Answer: We saw in class that, in an asymmetric Cournot equilibrium,

$$\pi_1 = \frac{1}{b}\left(\frac{a + c_2 - 2c_1}{3}\right)^2$$

By the same token,

$$\pi_2 = \frac{1}{b}\left(\frac{a + c_1 - 2c_2}{3}\right)^2$$

In the present case, we have $c_1 = 50$, $c_2 = 40$, $a = 225$, and $b = \frac{1}{2}$. (Recall that a and b are the coefficients of the inverse demand curve, which in this case is given by $p = 225 - Q/2$.) It follows that, in the initial equilibrium, $\pi_1 = 6050$ and $\pi_2 = 8450$.

A reduction in c_1 by 6 would take firm 1' cost down to 44. Recomputing the equilibrium, we have $\pi_1 = 6962$ and $\pi_2 = 7938$. It follows that firm 1 would be willing to pay $6962 - 6050 = 912$ for the innovation.

A reduction in c_2 by 6 would take firm 2's cost down to 34. Recomputing the equilibrium (with c_1 set at its initial level), we have $\pi_1 = 5618$ and $\pi_2 = 8622$. It follows that firm 2 would be willing to pay $8622 - 8450 = 1072$ for the innovation. Notice that firm 2 is willing to pay more for the new technology than firm 1.

Suppose the innovation costs 600. Consider a "metagame" where firms first simultaneously decide whether to acquire the innovation and then compete a la Cournot with whatever marginal cost results form the first stage.

      (b) What is the equilibrium of the 2×2 game played by firms at the technology choice stage?

Answer: First, we need to consider a fourth possibility not considered before: both firms acquire the new technology, so that $c_1 = 44$ and $c_2 = 34$. In this case, profits are given by $\pi_1 = 6498$ and $\pi_2 = 8978$. Considering all four possibilities, we construct the following payoff matrix:

		Firm 2	
		No Investment	Investment
Firm 1	No Investment	8450 / 6050	8922 / 5618
	Investment	7938 / 6362	8378 / 5898

Firm 1's best response is to make the cost-reduction investment if and only if Firm 2 does not make the investment. However, making the investment is Firm 2's dominant strategy. It follows that the unique Nash equilibrium is for Firm 2 to make the investment and for Firm 1 not to make the investment.

Applied exercises

■ **8.21. Model calibration.** Choose an industry for which you can find firm level data on

prices and market shares, and for which the Cournot model seems a good approximation.

(a) Making the necessary assumptions, estimate each firm's marginal cost and margin.

Answer: Consider for example the desktop computer industry. In 2010, global sales of Windows-based desktop computers were 351 million units, whereas average price was $605.[15] The leading market shares were as follows: HP, 17.9%; Acer, 13.9%; Dell, 12%; Lenovo, 10.9%; Asus, 5.4%; others, 39.3%. Suppose that inverse demand is given by $p = a - bQ$; that each firm has constant marginal cost; and that, from previous studies, the price elasticity of market demand is estimated to be -0.5 (at the market equilibrium level).

The price elasticity of market demand is given by

$$\epsilon = -b\,\frac{p}{Q}$$

This implies

$$b = -\epsilon\,\frac{Q}{p}$$

Given the available data on Q and p, as well as the estimate of ϵ, we estimate $b = 0.29$.

Firm i's profit function is given by

$$\pi_i = p\,q_i - c_i\,q_i$$

The first-order condition for firm i's profit maximization is given by

$$p - b\,q_i - c_i = 0$$

which implies

$$c_i = p - b\,s_i\,Q$$

since $q_i = s_i\,Q$. Since margin is given by

$$m_i = \frac{p - c_i}{p}$$

based on the available data for s_i, p and Q, as well the estimate of parameter b, we get the following estimates of marginal costs and margins:

Firm	Unit cost ($)	Margin (%)
Hewlett-Packard	587	3.0
Acer	591	2.3
Dell	593	2.0
Lenovo	594	1.8
Asus	600	0.9

(b) Use the estimated model to run the counterfactual whereby one of the firm's cost declines by 5%.

Answer: Suppose Dell's cost declines by 5%. As I showed in the answer to Exercise 8.15, the Cournot equilibrium price is given by

$$p = \frac{a + \sum_{i=1}^{n} c_i}{n + 1}$$

Moreover, firm i's equilibrium output is given by

$$q_i = \frac{a - n\,c_i + \sum_{j \neq i} c_j}{b\,(n+1)} = \frac{a - (n+1)\,c_i + \sum_{j=1}^{n} c_j}{b\,(n+1)}$$

which in turn implies that firm i's market share is

$$s_i = \frac{a - (n+1)\,c_i + \sum_{j=1}^{n} c_j}{n\,a - \sum_{j=1}^{n} c_j}$$

In order to move any further, I need to estimate the parameter a as well as obtain the value of n. Regarding a, recall that the inverse demand curve, $p = a - b\,Q$, implies that

$$a = p + b\,Q$$

Based on the available data as well as the estimate for b, I estimate that $a = 707$ (rounding to the dollar).

Regarding the value of n, I have the market shares for the leading 5 firms. This leaves a total 39.3% share of the market unaccounted for. One possible assumption is that all of the remaining firms are as big as the smallest firm for which I have data. This implies an estimate of 39.3/5.4=7.27 firms. Rounding to the next highest integer I get 8, or a total of 13 firms. (Note that the number of firm cannot be smaller than this. Why?)

We are now ready to proceed with the "comparative statics" calculations. A 5% decrease in Dell's cost implies that the new value of c_D is 563.4. The estimated values of p and s_D before and after the cost decrease are as follows.

	Data	Initial cost	Lower cost
Price	605	604.8	602.4
Dell's market share	12.0	11.7	37.9