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QR Decomposition-Based LS Channel Estimation Algorithm in MIMO-OFDM System

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***Abstract*—It discusses the channel estimation algorithm of LS. Because of the complexity of the matrix inversion computation in LS algorithm, we use QR decomposition in LS, that is QRD-LS. Then, the complexity of LS algorithm can be reduced. From the simulation of the LS and QRD-LS algorithm. We can see that the performance of these two algorithm is similary. However, the QRD-LS algorithm has no matrix inversion. So, this algorithm can simplify the calculation.**

Keywords—multiple-input multiple-output;orthgonal frequency division multiplexing;channel estimaiton

I. INTRODUCTION

With the development of the communication system. People demand the higher quality for communication. Multiple-input Multiple-output and Orthgonal Frequency Division Multiplexing are the two important techniques[1]. The combination of these two techniques is an inevitable trend. The difficult of the MIMO-OFDM system is channel estimation, especially to wireless channel. If we want to receive the correct signal at receiver, there must be some better channel estimation algorithms. Channel estimation can be categorized into two classes, one is training-based algorithm[2][3], which includes LS and MMSE algorithms. Another is blind estimation[4], the generally algorithm is subspace method. Due to the disadvantages of blind estimaiton , we focus on training-based channel estimation. We focus on LS algorithm in this paper,to reduce the complexity, we use QR decomposition on LS. In Section II, we briefly describe an MIMO-OFDM system. In Section III, we present the channel model. In Section IV, we introduce the LS algorithm in MIMO-OFDM systems. In Section V, we discuss the QRD-LS algorithm. In Section VI, we give the simulation results of these two algorithms. In section VII, we give the conclusion.

II. MIMO-OFDM SYSTEM MODEL

When the transmitter of MIMO system has M_t antennas, receiver has M_r antennas, the channel matrix $\mathbf{H}$ of receiver and transmitter is

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_t} \\ h_{21} & h_{22} & \cdots & h_{2M_t} \\ \cdots & \cdots & \cdots & \cdots \\ h_{M_r1} & h_{M_r2} & \cdots & h_{M_rM_t} \end{bmatrix}$$

Where h_{ji} is the channel fading coefficient from transmitter antenna i to receiver antenna j ,it is Rayleigh fading ,and independents with each other.

The OFDM system is shown in Fig. 1. Where, the blocks on the top row correspond to components in the transmitter and the bottom row to the receiver. The input data stream is modulated using regular modulation techniques such as phase shift keying (PSK)or quadrature amplitude modulation (QAM). The modulation signal $X(n)$ (n =0,1,..., N , where N is the number of subcarriers) is converted into parallel signals and passed to the IFFT block. The IFFT operation modulates the parallel signals onto orthogonal subcarriers as a group. The narrowband signals outputted are $x(k)$ (k =0,1,..., N), where

$$x(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi kn/N}, \quad 0 \le k \le N-1$$

When an OFDM signal passes through the channel it will experience ISI and interchannel interference (ICI). The ISI arises from channel delay spread and ICI is caused by the loss of orthogonality of the subcarriers due to the frequency response of the channel. In order to eliminate the effects of ISI and ICI a cyclic prefix (CP) of length N_{cp} , where N_{cp} is greater than the channel order, is appended to the beginning of the signal. The total length of the signal becomes $N + N_{cp}$. In the cyclic prefix extension, the end portion of the signal will be copied and appended to the beginning of the signal. Repeating the last elements at the beginning converts the linear convolution of the channel into circular convolution, thereby preserving the orthogonality of the subcarriers. At the receiver, the inverse operations are performed to recover the transmitted bits.

A typical MIMO-OFDM system[5] is depicted in Fig. 2.

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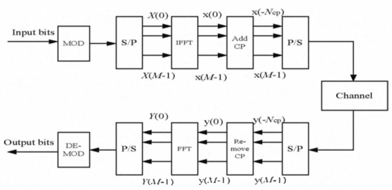

Figure 1. OFDM system model

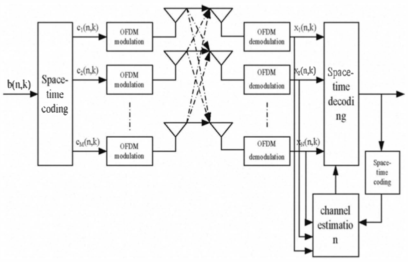

Figure 2. MIMO-OFDM system model

The MIMO-OFDM system consits M_t transmit antennas and M_r receive antennas. At time t ,a block of binary input data stream is modulated, and then passed through MIMO encoder to produce M_t data streams. Each of the M_t data streams is grouped into blocks of N symbols, and then OFDM modulated for transmission across the MIMO channels. The received signal at each antenna will be a summation of all the signals from the multiple paths plus the noise. The noise process is additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2. It is assumed that the signal and noise are independent of each other, which is a common assumption made in literature.

III. CHANNEL MODEL

The channel considered in this paper is a frequency selective Rayleigh fading channel. The radio channel can be modeled as a linear filter with a time varying impulse response[6]. The impulse response for a fading multipath channel is modeled as

$$h(\tau,t)=\sum_{i=1}^{L}a_i(t)\delta(\tau-\tau_i(t)) \tag{1}$$

where $a_i(t)$ is the complex amplitude, $\tau_i(t)$ is the delay of the i th path and L is the length of the channel. The frequency response at time t is

$$H(t,f)=\int h(\tau,t)e^{-j2\pi ft}d\tau \tag{2}$$

In an OFDM system, the channel frequency response can be expressed as

$$H[n,k]=\sum_{l=0}^{L-1}h[n,l]e^{-j2\pi kl/K} \tag{3}$$

where K is the number of tones of the OFDM block.

In general, wireless communication channels are time-varying either due to the motion of the transmitter or receiver, or the change in the wireless environment.

However, in indoor environments under lower mobility scenarios the channel variations are slow and can be ignored for the duration of a packet. Then (3) can be rewritten as

$$H[k]=\sum_{l=0}^{L-1}h[l]e^{-j2\pi kl/K} \tag{4}$$

IV. LS ALGORITHM IN MIMO-OFDM SYSTEM

Using the MIMO-OFDM system model, we will develop the channel estimator for MIMO-OFDM. We will assume a 2-by-2 MIMO-OFDM system to illustrate the channel estimation. The received signal at the j th antenna for the k th subcarrier in expanded form is defined as:

$$Y_k^{(j)}[n]=H_k^{(j,1)}S_k^{(1)}[n]+H_k^{(j,2)}S_k^{(2)}[n]+V_k^{(j)}$$
$$k=0,1,...,N-1 \tag{5}$$

The above equation is under determined. There are two unknown elements $H_k^{(j,1)}$ and $H_k^{(j,2)}$ from different channels, however the unknowns cannot be solved with just one equation. In a 2-by-2 system, two samples of the received signal, $Y_k^{(j)}[n]$ and $Y_k^{(j)}[n+1]$ are required to estimate $H_k^{(j,1)}$ and $H_k^{(j,2)}$. If we study one OFDM block of N subcarriers, the received signal at the j th antenna is represented by the following equation

$$\begin{bmatrix} Y_1^j(n) \\ Y_2^j(n) \\ \vdots \\ Y_N^j(n) \end{bmatrix} = \begin{bmatrix} S_1^1(n) & 0 & \cdots & 0 & \cdots & S_1^{M_t}(n) & 0 & \cdots & 0 \\ 0 & S_2^1(n) & & \vdots & \cdots & 0 & S_2^{M_t}(n) & & \vdots \\ \vdots & & \ddots & 0 & \cdots & \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & S_N^1(n) & \cdots & 0 & \cdots & 0 & S_N^{M_t}(n) \end{bmatrix} \begin{bmatrix} H_1^{(1,j)} \\ H_2^{(1,j)} \\ \vdots \\ H_N^{(1,j)} \\ \vdots \\ H_1^{(M_t,j)} \\ H_2^{(M_t,j)} \\ \vdots \\ H_N^{(M_t,j)} \end{bmatrix} + \begin{bmatrix} V_1^j(n) \\ V_2^j(n) \\ \vdots \\ V_N^j(n) \end{bmatrix} \tag{6}$$

Its vector equation is

$$\mathbf{Y}^{(j)}=X\mathbf{H}^{(j)}+\mathbf{V}^{(j)}$$

Note, the H(j) vector contains NM_t unknown elements but there are only N equations available in a block. In general, for M_t transmit antennas we need to collect a minimum of M_t blocks to solve for the channel unknowns. The complexity of the estimation problem increases significantly since the matrix size is increased by M_t -folds. Basically, we know that the CFR can be expressed in terms of the CIR through the Fourier transformation. Hence, the received signal model in (6) can be expressed in terms of the CIR. The benefit of this representation is that usually the length of the CIR is much less than the number of subcarriers of the system. In order to model the received signal in terms of the CIR we first need to express the CFR as a function of the CIR. The equation can be rewritten in vector form as

$$\mathbf{H}^{(j,i)} = \mathbf{F}\mathbf{h}^{(j,i)} \tag{7}$$

其中，

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & e^{-\frac{j2\pi(1)(1)}{N}} & e^{-\frac{j2\pi(1)(2)}{N}} & & e^{-\frac{j2\pi(1)(L-1)}{N}} \\ 1 & e^{-\frac{j2\pi(2)(1)}{N}} & e^{-\frac{j2\pi(2)(2)}{N}} & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ 1 & e^{-\frac{j2\pi(N-1)(1)}{N}} & & & e^{-\frac{j2\pi(N-1)(L-1)}{N}} \end{bmatrix}$$

is the Fourier transform matrix of size ($N \times L$), and $\mathbf{h}^{(j,i)} = [h^{(j,i)}(0), h^{(j,i)}(1), ..., h^{(j,i)}(L-1)]^H$ is the ($L \times 1$) channel impulse vector. To extend the Fourier transformation to multiple channels we need to define the following:

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{F} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{F} & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{F} \end{bmatrix}$$

is a block diagonal matrix of M_t $\mathbf{F}$ s, and $\mathbf{h}^{(j)} = [\mathbf{h}^{(j,1)}, \mathbf{h}^{(j,2)}, ..., \mathbf{h}^{(j,M_t)}]^H$ is a $M_t L \times 1$ vector. Then the alternate received signal of the *jth* antenna in terms of the CIR can be expressed as

$$\mathbf{Y}^{(j)} = \mathbf{X}\mathbf{\Phi}\mathbf{h}^{(j)} + \mathbf{V}^{(j)}$$

$$= \mathbf{W}\mathbf{h}^{(j)} + \mathbf{V}^{(j)} \tag{8}$$

In this representation, the number of elements in $\mathbf{h}^{(j)}$ to be resolved are $M_t L$. If we assume that the number of subcarriers is N in an OFDM block, and N > $M_t L$, then only one OFDM is required to solve for $\mathbf{h}^{(j)}$.

Using the received signal model in (8) the LS estimate is expressed as:

$$\hat{\mathbf{h}}_{LS}^{(j)} = (\mathbf{W}^H\mathbf{W})^{-1}\mathbf{W}^H\mathbf{Y}^{(j)} \tag{9}$$

Where, $(\mathbf{W}^H\mathbf{W})^{-1}$ has the dimensions of $(M_t L \times M_t L)$, which is still full rank if $M_t L \leq N$.

V. QRD-LS ALGORITHM

Direct computation of the LS solution involves a matrix inversion, which is high complexity. We use QR decomposition(QRD) avoid explicit inversions and are more robust. QR decomposition is an orthogonal matrix triangularization technique that reduces a full rank matrix into a simpler form. QRD-LS algorithm is robust than LS algorithm. Consider a matrix $\mathbf{A}$ of size ($m \times n$), then the QR decomposition is defined as

$$\mathbf{A} = \mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{Q}$ is a ($m \times m$) unitary matrix, $\mathbf{R}$ is a ($n \times n$) upper triangular matrix and $\mathbf{0}$ is a null matrix. A unitary matrix is one that satisfies the following condition

$$\mathbf{I} = \mathbf{Q}^H\mathbf{Q} \tag{11}$$

To apply QRD to the problem of channel estimation we recall the MIMO-OFDM system model

$$\mathbf{Y} = \mathbf{W}\mathbf{h} + \mathbf{V} \tag{12}$$

The LS estimation solution is found by minimizing the norm square of error function $\varepsilon = \mathbf{Y} - \mathbf{W}\hat{\mathbf{h}}$ to give the estimate

$$\hat{\mathbf{h}} = (\mathbf{W}^H\mathbf{W})^{-1}\mathbf{W}^H\mathbf{Y} \tag{13}$$

The inversion of $\mathbf{W}^H\mathbf{W}$ of size (LNt × LNt) is high in complexity and will significantly increase when the size of $\mathbf{W}$ increases, which is dependent on the channel length or the number of transmit antennas. To avoid the matrix inversion we can directly apply QR decomposition to the error equation as follows

$$\mathbf{W}\hat{\mathbf{h}} = \mathbf{Y}$$

$$\mathbf{Q}\mathbf{R}\hat{\mathbf{h}} = \mathbf{Y} \tag{14}$$

$$\mathbf{R}\hat{\mathbf{h}} = \mathbf{Q}^H\mathbf{Y}$$

Note, $\mathbf{R}$ is an upper triangular matrix, hence $\hat{\mathbf{h}}$ can be solved through back-substitution. In summary, the solution is obtained through the following steps:

1. QR decompose $\mathbf{W}$ into QR
2. Premultiple $\mathbf{Y}$ by $\mathbf{Q}^H$
3. Solve for $\hat{\mathbf{h}}$ by back-substitution

VI. SIMULATION RESULTS

System parameters as follows: Number of Subcarriers, N=64; FFT Length=64; QPSK Modulation; Cyclic Prefix, N_{cp} =16; Transmit antennas, M_t =2; Receive antennas, M_r =2; Max Time Delay=25ns; Sampling Frequency=80MHz.

The channel has L paths where the amplitude of each path varies independently according to the Rayleigh distribution with an exponential power delay profile, and can be represented as

$$h_l = \mathbf{N}(0,1/2\sigma_l^2) + j\mathbf{N}(0,1/2\sigma_l^2),$$
$$l = 0,1,\ldots,L-1$$

where $\sigma_l^2 = (1-e^{-\frac{Ts}{Trms}})e^{-\frac{lTs}{Trms}}$, $\mathbf{N}(0,1/2\sigma_l^2)$ is a zero mean Gaussian random variable with variance $1/2\sigma_l^2$. T_{rms} is the max time delay, T_s is the sampling period. $(1-e^{-\frac{Ts}{Trms}})$ is chosen such that the channel gain is unity, i.e. $\left(\sum_{l=0}^{L-1}\sigma_l^2 = 1\right)$. where L is the length of the channel,and it is approximated by $\left(L = \dfrac{10T_{rms}}{T_s}\right)$.

In addition, the channel assumed to be quasi-static, meaning that the channel remains relatively constant for the duration of the OFDM packet.

From the Fig. 3 and Fig. 4, we can see that the performance of the QRD-LS and LS algorithm are similary. This is same as our expection. Because we know that the essence of QRD-LS is LS algorithm. The diffrience of these two algorithms is the realization of QRD-LS via QR decomposition, however, the standard LS algorithm via matrix inversion. From the following figures, we can see that the BER and MSE of QRD-LS are similary to LS, that is the estimation accuracy of the QRD-LS has not been improved. But we can take advantage of the QR decomposition to reduce the system complexity.

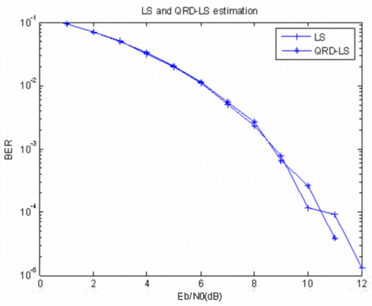

Figure 3. BER of LS and QRD-LS

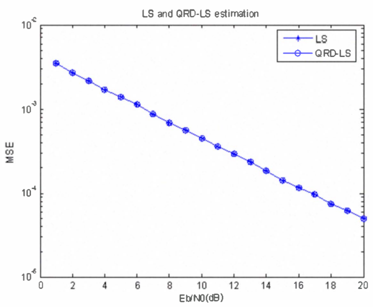

Figure 4. MSE of LS and QRD-LS

VII. CONCLUSION

We focus on LS algorithm in this paper, to reduce the complexity, we use QR decomposition on LS, that is QRD-LS algorithm. From the simulation results of LS and QRD-LS algorithm, we can see that these two algorithms have similary performance, this result is same as our expection. But these is no matrix inversion in QRD-LS algorithm. So, the QRD-LS algorithm can simplify the computation.

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REFERENCES

[1] Wenbo Wang, Kan Zheng, The OFDM techniques of broad band wireless communication, Posts&Telecom Press, Bei Jing, 2003.

[2] Y. Li, "Pilot-symbol-aided channel estimation for OFDM wireless systems", IEEE Trans. on Vechicular Tech, vol. 49, no. 4, 2000.

[3] X. Wang and K.J.R. Liu, "Channel estimation for multicarrier modulation systems using a time-frequency polynomial model", IEEE Trans. on Commun, vol. 50, no. 7, 2002.

[4] Marc C. Necker and Gordon L. Stüber, Fellow, IEEE, "Totally Blind Channel Estimation for OFDM on Fast Varying Mobile Radio Channels", IEEE Trans. on Wireless Commun, VOL. 3, NO. 5, pp. 1514-1525, september 2004.

[5] Andreas F. Molisch, Moe Z. Win, "MIMO Systems with Antenna Selection", IEEE microwave magazine, March 2004.

[6] T.S. Rappaport, Wireless Communications Principles and Practices Second Edition, New Jersey: Prentice-Hall, 2002.

[7] Kathryn Kar Ying Lo, Channel Estimation of Frequency Selective Channels for MIMO-OFDM, UNIVERSITY OF CALGARY, September, 2005.