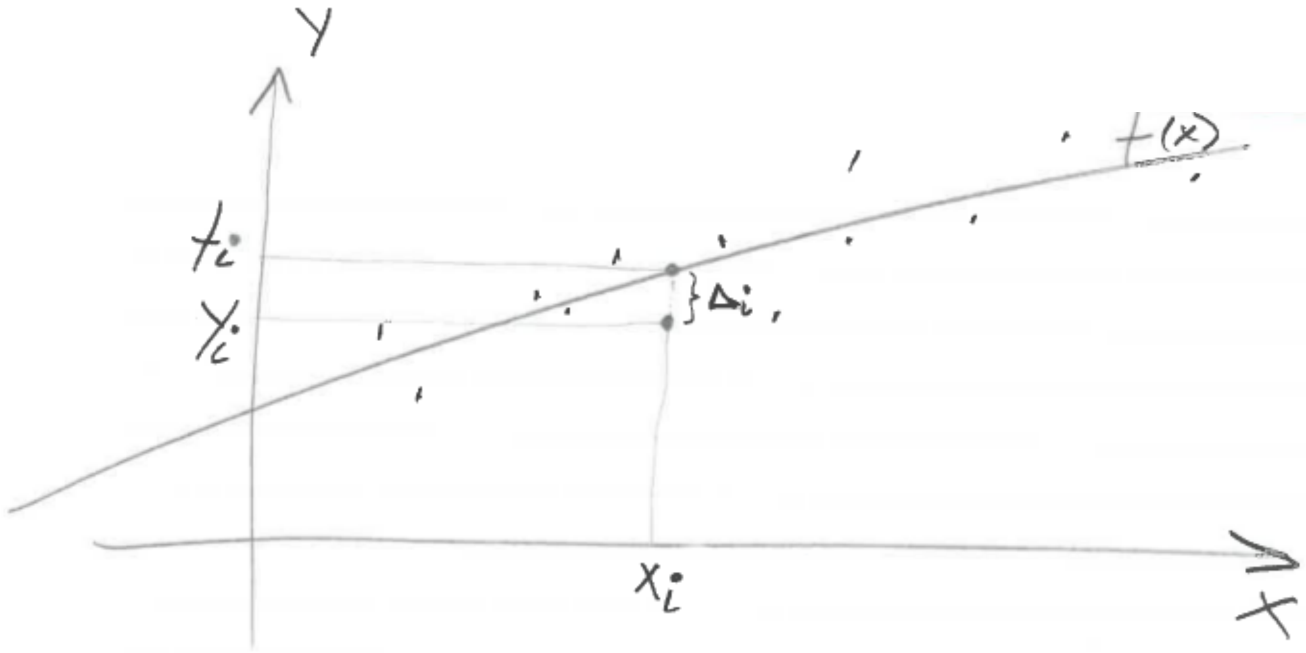

$$f(x) = a + bx$$

$$t_i = f(x_i) = a + bx_i$$

$$\Delta_i = t_i - y_i = a + bx_i - y_i$$

$$\chi^2 = \sum_i \Delta_i^2 = \sum_i (a + bx_i - y_i)^2$$

$$\chi^2 = \sum_i (a^2 + b^2 x_i^2 + y_i^2 + 2abx_i - 2ay_i - 2bx_iy_i)$$

$$\chi^2 = Na^2 + b^2 \sum_i x_i^2 + \sum_i y_i^2 + 2ab \sum_i x_i - 2a\sum_i y_i - 2b\sum_i x_iy_i$$

$$\partial_a \chi^2 = 2Na + 2b\sum_i x_i - 2\sum_i y_i \equiv 0$$

$$\partial_b \chi^2 = 2b\sum_i x_i^2 + 2a\sum_i x_i \quad x_iy_i \equiv 0$$

$$N = \sum_i 1$$

$$Na + b\sum_i x_i = \sum y_i \quad / : N$$

$$\bar{x} = \frac{1}{N}\sum x_i \qquad \bar{y} = \frac{1}{N}\sum_i y_i$$

$$a + b\bar{x} = \bar{y} \qquad \boxed{a = \bar{y} - b\bar{x}}$$

$$b\sum_i x_i^2 + a\sum_i x_i - \sum x_i y_i = 0$$

$$b\sum x_i^2 + N\bar{x}(\bar{y} - b\bar{x}) - \sum x_i y_i = 0$$

$$b\sum x_i^2 + N\bar{x}\bar{y} - N\bar{x}^2 b - \sum x_i y_i = 0$$

$$b\left(\sum x_i^2 - N\bar{x}^2\right) = \sum x_i y_i - N\bar{x}\bar{y}$$

$$b = \frac{\sum x_i y_i - N\bar{x}\bar{y}}{\sum x_i^2 - N\bar{x}^2} = \frac{\sum x_i y_i - N\bar{x}\bar{y} - N\bar{x}\bar{y} + N\bar{x}\bar{y}}{\sum x_i^2 - N\bar{x}^2 - N\bar{x}^2 + N\bar{x}^2}$$

$$b = \frac{\sum x_i y_i - \bar{y} \sum x_i - \bar{x} \sum y_i + \bar{x}\bar{y} \sum_i}{\sum x_i^2 - 2\bar{x} \sum x_i + \bar{x}^2 \sum_i}$$

$$b = \frac{\sum x_i (y_i - \bar{y}) - \sum_i \bar{x} (y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$\boxed{b = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}}$$

$$\sigma_b^2 = \frac{1}{N-2} \, \frac{\frac{1}{N}\sum(a+bx_i-y_i)^2}{\frac{1}{N}\sum x_i^2 - \overline{x}^2} \quad \begin{matrix} / \cdot N \\ / \cdot N \end{matrix}$$

$$\sigma_b^2 = \frac{1}{N-2} \, \frac{\sum_i (a+bx_i-y_i)^2}{\sum x_i^2 - N\overline{x}^2}$$

$$\sigma_b = \sqrt{\frac{1}{N-2} \, \frac{\sum_i (a+bx_i-y_i)^2}{\sum_i (x_i-\overline{x})^2}}$$

$$\sigma_a = \sigma_b \sqrt{\frac{1}{N} \sum_i x_i^2}$$

$$\chi^2 = \sum_i \frac{(a + b x_i - y_i)^2}{\sigma_i^2}$$

$$\chi^2 = \sum_i \frac{1}{\sigma_i^2}\left(a^2 + b^2 x_i^2 + y_i^2 + 2ab x_i - 2a y_i - 2b x_i\right)$$

$$\chi^2 = a^2 \sum_i \frac{1}{\sigma_i^2} + b^2 \sum_i \frac{x_i^2}{\sigma_i^2} + \sum \frac{y_i^2}{\sigma_i^2} + 2ab \sum \frac{x_i}{\sigma_i^2} - 2a \sum \frac{y_i}{\sigma_i^2} - 2b \sum \frac{x_i y_i}{\sigma_i^2}$$

$$\partial_a \chi^2 = \cancel{2}a \sum \frac{1}{\sigma_i^2} + \cancel{2}b \sum \frac{x_i}{\sigma_i^2} - \cancel{2} \sum \frac{y_i}{\sigma_i^2} \equiv 0$$

$$\partial_b \chi^2 = \cancel{2}b \sum \frac{x_i^2}{\sigma_i^2} + \cancel{2}a \sum \frac{x_i}{\sigma_i^2} - \cancel{2} \sum \frac{x_i y_i}{\sigma_i^2} \equiv 0$$

$$\overline{X} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$

$$\overline{Y} = \frac{\sum \frac{y_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$

$$a + b\bar{x} = \bar{y}$$

$$\boxed{a = \bar{y} - b\bar{x}}$$

$$b\sum_i \frac{x_i^2}{\sigma_i^2} + a\sum \frac{x_i}{\sigma_i^2} - \sum \frac{x_i y_i}{\sigma_i^2} = 0$$

$$b\sum_i \frac{x_i^2}{\sigma_i^2} + (\bar{y} - b\bar{x})\bar{x}\sum_i \frac{1}{\sigma_i^2} - \sum_i \frac{x_i y_i}{\sigma_i^2} = 0$$

$$b\sum_i \frac{x_i^2}{\sigma_i^2} + \bar{x}\bar{y}\cdot\sum \frac{1}{\sigma_i^2} - \bar{x}^2\sum \frac{1}{\sigma_i^2}\, b - \sum \frac{x_i y_i}{\sigma_i^2} = 0$$

$$b\left(\sum_i \frac{x_i^2}{\sigma_i^2} - \bar{x}^2\cdot\sum \frac{1}{\sigma_i^2}\right) = \sum \frac{x_i y_i}{\sigma_i^2} - \bar{x}\bar{y}\sum \frac{1}{\sigma_i^2}$$

$$b = \frac{\sum \frac{x_i y_i}{\sigma_i^2} - \bar{x}\bar{y}\sum \frac{1}{\sigma_i^2} - \bar{x}\bar{y}\sum \frac{1}{\sigma_i^2} + \bar{x}\bar{y}\sum \frac{1}{\sigma_i^2}}{\sum_i \frac{x_i^2}{\sigma_i^2} - \bar{x}^2\sum \frac{1}{\sigma_i^2} - \bar{x}^2\sum \frac{1}{\sigma_i^2} + \bar{x}^2\sum \frac{1}{\sigma_i^2}}$$

$$\boxed{b = \frac{\sum_i \frac{1}{\sigma_i^2}(x_i-\bar{x})(y_i-\bar{y})}{\sum_i \frac{1}{\sigma_i^2}(x_i-\bar{x})^2}}$$

$$\sigma_b = \sqrt{\frac{1}{N-2}\,\frac{\sum_i \frac{1}{\sigma_i^2}(a+bx_i-y_i)^2}{\sum_i \frac{1}{\sigma_i^2}(x_i-\bar{x})^2}}$$

$$\sigma_a = \sigma_b\sqrt{\frac{\sum_i \frac{x_i^2}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}}}$$