ETH zürich

computer graphics laboratory

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Visual Computing
Exercise 7: Light & Colors
Solution

Exercise 1) Definitions

Explain the following expressions: Luminous Flux, Luminous Intensity, Illumination, Luminance, Color Stimulus Specification

•***Luminous Flux***: *Relative spectral power density weighted by the relative spectral sensitivity integrated over the complete human visible spectrum.*
•***Luminous Intensity***: *Luminous flux emitted through a specific angle in space.*
•***Illumination***: *Luminous flux of a light source falling on a surface element.*
•***Luminance***: *Luminous Intensity of a light source, considering direction and surface normal.*
•***Color Stimulus Specification***: *Color coordinates in a color space approximating the relative spectral power density.*

Exercise 2) Color Spaces

a) How do you transform a specification in RGB into CMY?

Since CMY and RGB color spaces are complementary to each other: [C M Y] = [1 1 1] – [R G B]

b) Why were color spaces such as RGB, CMY, YIQ, and HLS specified, and in where are they being applied?

•***RGB***: *Additive color space used for color monitors. Neighboring color pixels are mixed to one color by the human eye.*
•***CMY***: *Subtractive color space used in color printers where transparent color pigments are added to a white background.*
•***YIQ***: *Is based on the psycho physical properties of the human eye. Most of the color space resolution is used for color tones, which can best be distinguished by the human eye (like skin tones). Other color tones have a lower resolution. This color space is being used in the NTSC-Norm used in television.*
•***HSV***: *"Subjective color model". Different colors can be picked in an easier way. It is being used in art etc.*
•***HLS***: *Alternative color space to HSV, which is being used for the same applications.*

c) Provide the values for a medium gray in the following color modes: RGB, CMY, YIQ and HSV.

•***RGB***: [0.5, 0.5, 0.5]
•***CMY***: [0.5, 0.5, 0.5]
•***CMYK***: e.g. [0, 0, 0, 0.5]
•***YIQ***:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.322 \\ 0.212 & -0.522 & -0.311 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$$

•***HSV***: […,0, 0.5]
•***HLS***: […,0.5 0]

Exercise 3) RGB-colorspace and white point calibration

a) Name one advantage and one disadvantage of the *RGB*-colorspace. Furthermore, list one color-space each, which does not have this advantage or disadvantage.

Advantage and colorspace not having this advantage:
- Useful for monitors and displays (in contrary to Lab, Luv, HLS, HSV, CMYK...).

Disadvantage and colorspace not having this disadvantage:
- Color mixing is not intuitive (in contrary to HLS, HSV).
- Does not allow meaningful color distance measurements (in contrary to Lab, Luv).
- Not useful for printing (in contrary to CYMK).
- Not all visible colors can be mixed from primaries (in contrary to XYZ).

b) Evaluate the z-component of the *RGB*-base vectors.

	R	*G*	*B*
Z = 1 – x - y	0.03	0.10	0.79

c) Provide the equation system for the white point calibration (check script). Name the calibration parameters C_R, C_G and C_B.

Transformation matrix with chromaticity values of the primaries:

$$\begin{bmatrix} 0.64\ldots & 0.30\ldots & 0.15.. \\ 0.33\ldots & 0.60\ldots & 0.06\ldots \\ 0.03\ldots & 0.10\ldots & 0.79\ldots \end{bmatrix}$$

C_R, C_G and C_B as additional calibration parameters:

$$\begin{bmatrix} 0.64C_R & 0.30C_G & 0.15C_B \\ 0.33C_R & 0.60C_G & 0.06C_B \\ 0.03C_R & 0.10C_G & 0.79C_B \end{bmatrix}$$

Transformation of $(1,1,1)^T$ onto white point:

$$\begin{bmatrix} 0.64C_R & 0.30C_G & 0.15C_B \\ 0.33C_R & 0.60C_G & 0.06C_B \\ 0.03C_R & 0.10C_G & 0.79C_B \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9505 \\ 1.0000 \\ 1.0890 \end{bmatrix}$$

Equation system:

$$\begin{bmatrix} 0.64 & 0.30 & 0.15 \\ 0.33 & 0.60 & 0.06 \\ 0.03 & 0.10 & 0.79 \end{bmatrix} \begin{bmatrix} C_R \\ C_G \\ C_B \end{bmatrix} = \begin{bmatrix} 0.9505 \\ 1.0000 \\ 1.0890 \end{bmatrix}$$

d) Suppose C_R = 0.6445, C_G = 1.1919 and C_B = 1.2031 are given as a solution. Evaluate the transformation matrix from the linear colorspace *RGB* into the colorspace *XYZ*.

Transformation matrix:

$$\begin{bmatrix} 0.4125 & 0.3576 & 0.1805 \\ 0.2127 & 0.7151 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$$

Exercise 4) Colorspace transformation to PAL and NTSC

a) Provide the transformation matrix from the sRGB-colorspace into the YUV-colorspace.

First row of the transformation matrix stems from the definition of the Y-coordinate:

$$\begin{bmatrix} \boldsymbol{Y} \\ \boldsymbol{U} \\ \boldsymbol{V} \end{bmatrix} = \begin{bmatrix} \mathbf{0.2126} & \mathbf{0.7152} & \mathbf{0.0722} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \boldsymbol{R} \\ \boldsymbol{G} \\ \boldsymbol{B} \end{bmatrix}$$

CB and CR expressed by R, G and B:

CB = -0.2126 R – 0.7152 G + 0.9278 B

CR = 0.7874 R – 0.7152G - 0.0722 B

Correct scaling with 0.49 and 0.88:

U = -0.1047 R – 0.3523 G + 0.4570 B

V = 0.6907 R – 0.6274G - 0.0633 B

Resulting transformation matrix:

$$\begin{bmatrix} \mathbf{0.2126} & \mathbf{0.7152} & \mathbf{0.0722} \\ \mathbf{-0.1042} & \mathbf{-0.3504} & \mathbf{0.4546} \\ \mathbf{0.6929} & \mathbf{-0.6294} & \mathbf{-0.0635} \end{bmatrix}$$

b) Evaluate the transformation matrix for the conversion from PAL to NTSC. You do not have to evaluate trigonometric expressions.

Matrix swap coordinates:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Matrix for rotation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(33°) & -\sin(33°) \\ 0 & \sin(33°) & \cos(33°) \end{bmatrix}$$

Tranformation matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(33°) & -\sin(33°) \\ 0 & \sin(33°) & \cos(33°) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin(33°) & \cos(33°) \\ 0 & \cos(33°) & \sin(33°) \end{bmatrix}$$

c) Why is such an uneven bandwidth being used?

- To take advantage of the limited bandwidth in an optimal way, channels for which the human perception is more sensitive are getting more bandwidth.
- Based on psycho physical properties of the human visual system.

Exercise 5) Pseudo colors

Given is an algorithm, which provides a gray scale image (e.g. Mandelbrot). The intensities lie between 0 and 2^{24}-1. How would you colorcode the image in order to map the whole intensity range of the image? High intensity values should correspond to warm colors such as red, while low intensity values should correspond to cold colors such as blue. Remark: A coding in gray colors in true-color-systems allows only 256 samples and is therefore not sufficient for most applications.

A conversion into the HSV-colorspace would be appropriate, since it could directly be mapped to the H (Hue) component.

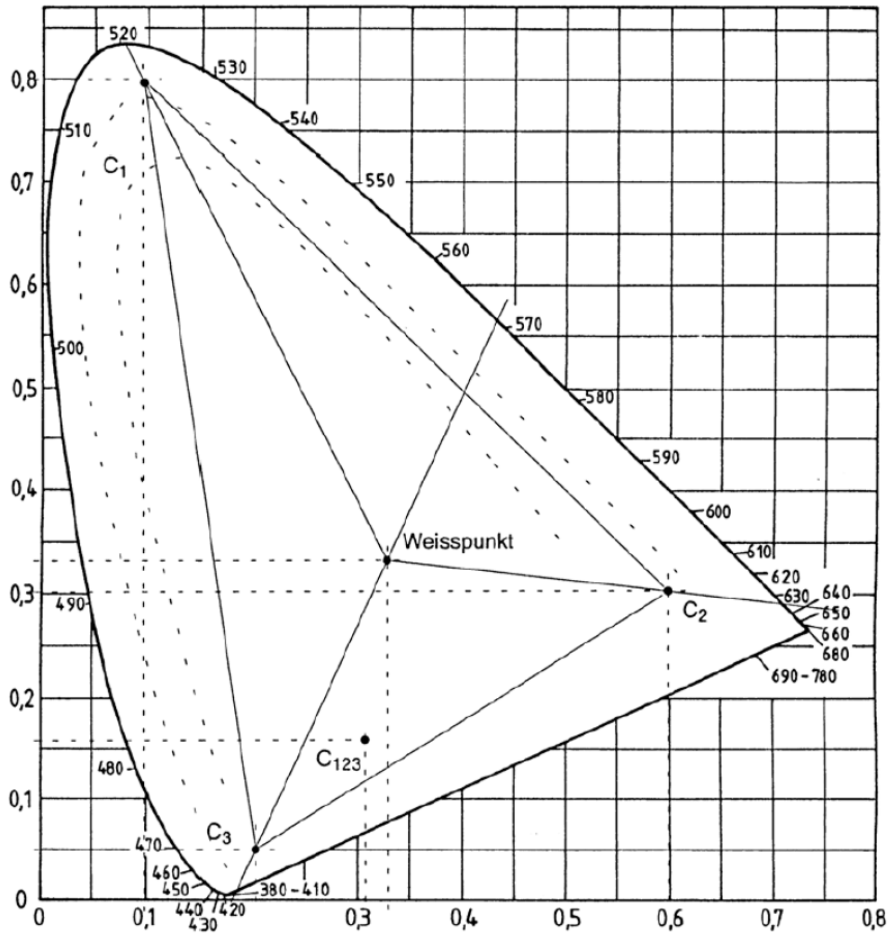
Exercise 6) CIE-Chart

a) Which properties does a mixed color in the CIE-Chart have to its primaries?

The mixed color lies in the convex hull spanned by the primaries.

b) Which meaning does the connection between 770nm and 380nm have in this chart?

It is called the purple line. Colors lying on the purple line are not spectral colors. They are created by mixing the two spectral colors red (at 770nm) and blue (at 380nm).



c) The figure below shows a CIE-Chart. Add following primaries into the chart.

	x	*y*	*Y*
C1	0.1	0.8	12
C2	0.6	0.3	26
C3	0.2	0.05	10

Solution: see values in figure above.

d) Determine the dominant wavelengths λ_1, λ_2 and λ_3 of the 3 primaries.

Straight containing white point and primary intersected with the border:
λ_1*: 520 nm*
λ_2*: 635 nm*
λ_3*: not 568 nm, meaning all complementary wavelengths to 568 nm.*

e) For each primary, draw the isoline of constant saturation passing through it into the chart.

Note that lines from the white point through the primaries are always gaining on saturation and therefore the isolines have to cross those.

f) Determine the primary C_{123}, which is the sum (in the XYZ-space) of the 3 primaries C_1, C_2 and C_3. Add it to the chart.

$y = \frac{Y}{X+Y+Z}$ (1) *and* $x = \frac{X}{X+Y+Z}$ (2).

From the first relation (1) follows: $X + Y + Z = \frac{Y}{y}$ (3)*, and from this including (2):*
$X = x\frac{Y}{y}$ (4).
Then, from (3) and (4) follows: $Z = \frac{Y}{y} - x\frac{Y}{y} - Y$.
So, X and Z can be determined for the three primaries C1, C2 and C3:
C1: X1 = 1.5 Z1=1.5
C2: X2=52 Z2=8.666
C3: X3=40 Z3=150

From this follows:
X123 = X1 + X2 + X3 = 93.5
Y123 = Y1 + Y2 + Y3 = 48
Z123 = Z1 + Z2 + Z3 = 160.1666

Inserting into (1) and (2) provides mixed primary x=0.308, y=0.159, Y=48.

g) Can all spectral colors with full saturation be mixed from three linearly independent primaries?

By mixing 3 linearly independent primaries in the CIE-Chart all colors inside and on the border of the spanning triangle can be created. Since the CIE-Chart is horseshoe shaped it is never possible to create another spectral color by mixing three spectral colors.