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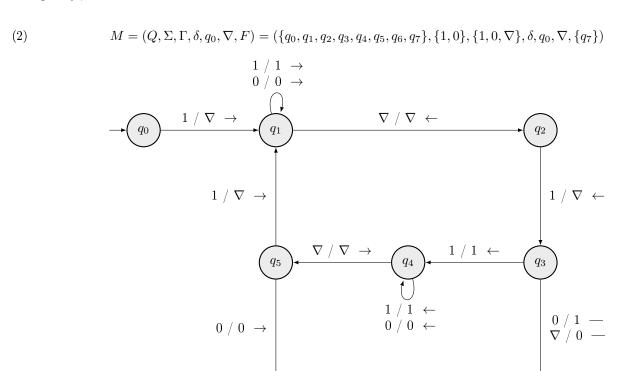
1 Maszyny Turinga obliczające funkcje w alfabecie unarnym

1.1 Zadanie 1.1

Zaprojektuj maszynę Turinga, które oblicza funkcję odejmowania ograniczonego f dla liczb naturalnych m i n w reprezentacji unarnej, czyli

(1)
$$f(m,n) = m - n = \begin{cases} m - n, & \text{jeżeli } m \ge n, \\ 0, & \text{jeżeli } m < n \end{cases}$$

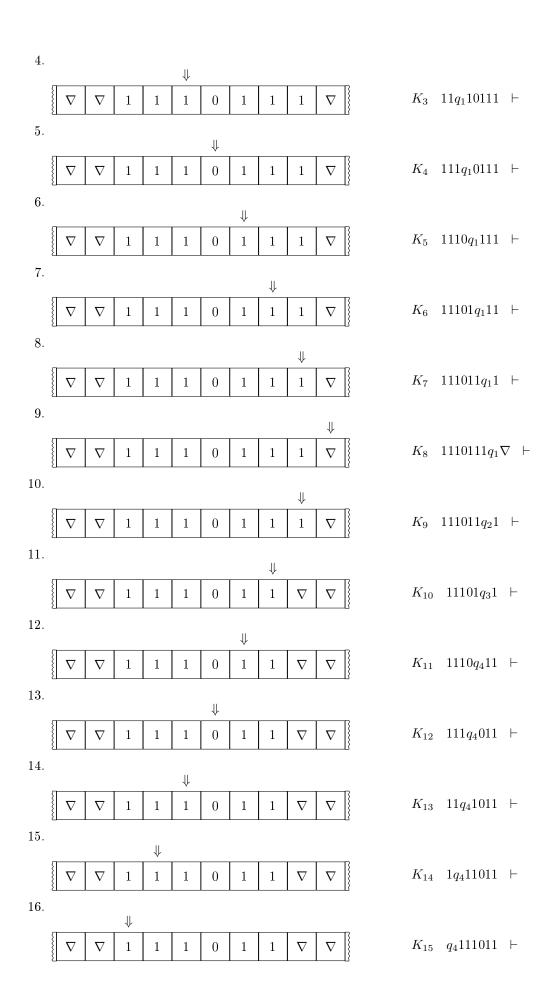
Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

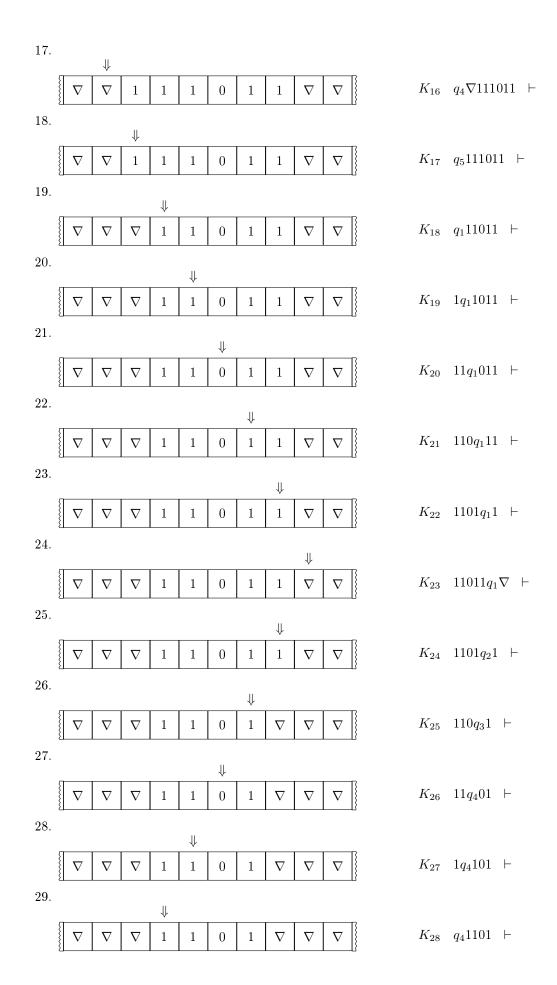


1.1.1 f(3,2)

1. $K_0 \quad q_0 1 1 1 1 0 1 1 1 \quad \vdash$ 1 1 1 1 0 1 1 1 ∇ ∇ 2. K_1 $q_11110111$ \vdash 1 0 3. $K_2 \quad 1q_1110111 \quad \vdash$ ∇ 0 1

 $\nabla \ / \ \nabla \ \leftarrow$





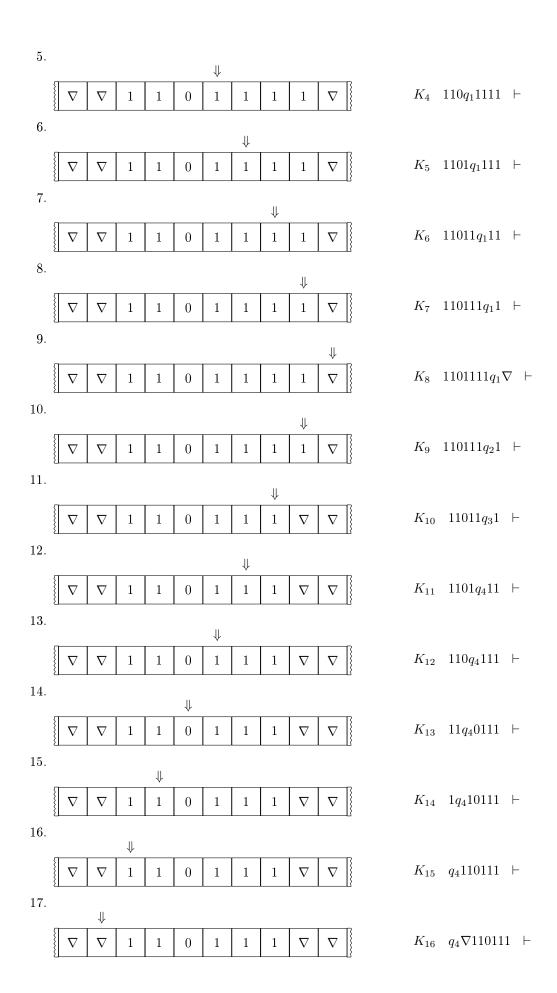
30. K_{29} $q_4\nabla 1101$ \vdash 31. K_{30} q_51101 \vdash ∇ ∇ 1 0 1 ∇ ∇ 32. K_{31} q_1101 \vdash 0 ∇ 33. K_{32} $1q_101$ \vdash ∇ ∇ 0 ∇ ∇ ∇ 34. K_{33} 10 q_1 1 \vdash 35. K_{34} $101q_1\nabla$ \vdash ∇ ∇ 0 ∇ ∇ ∇ 36. K_{35} 10 q_2 1 \vdash 37. K_{36} $1q_30$ \vdash ∇ ∇ 0 ∇ ∇ ∇ 38. K_{37} 1 q_71 1.1.2 f(2, 3)1. ∇ K_0 $q_0111011111$ \vdash ∇ 2. K_1 $q_111011111$ \vdash ∇ ∇ 1 3. $K_2 \quad 1q_1101111 \quad \vdash$

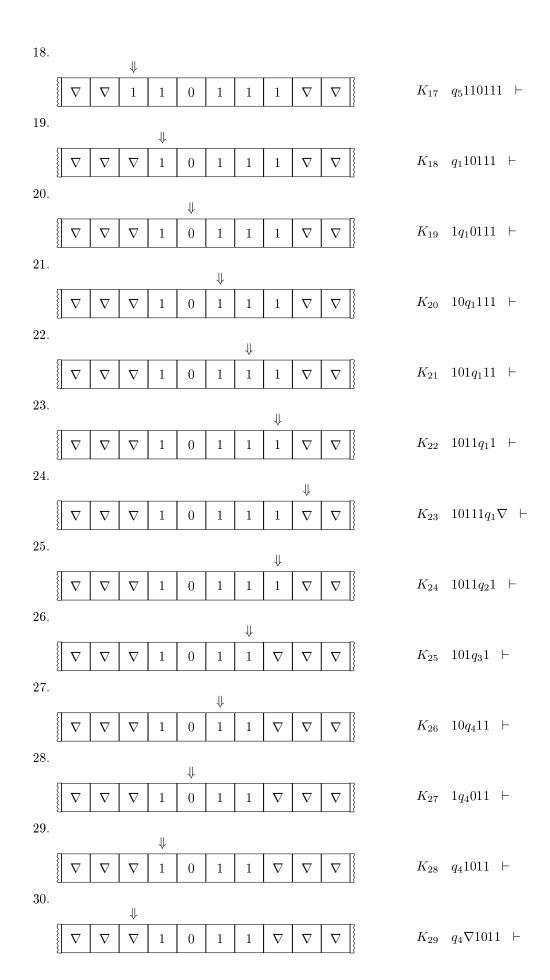
4.

 ∇

1

 K_3 11 q_1 01111 \vdash





31. K_{30} q_51011 \vdash 32. ∇ ∇ ∇ ∇ 0 1 ∇ K_{31} q_1011 \vdash 33. K_{32} $0q_111$ \vdash 34. K_{33} 01 q_1 1 \vdash ∇ ∇ 0 ∇ 35. K_{34} $011q_1\nabla$ \vdash 36. K_{35} 01 q_2 1 \vdash 0 37. $K_{36} \quad 0q_31 \quad \vdash$ 38. K_{37} q_401 \vdash 39. K_{38} $q_4\nabla 01$ \vdash 40. K_{39} q_501 \vdash 41. K_{40} $0q_61$ \vdash 42. K_{41} $0q_6\nabla$ \vdash 43.

 ∇

 K_{42} q_70

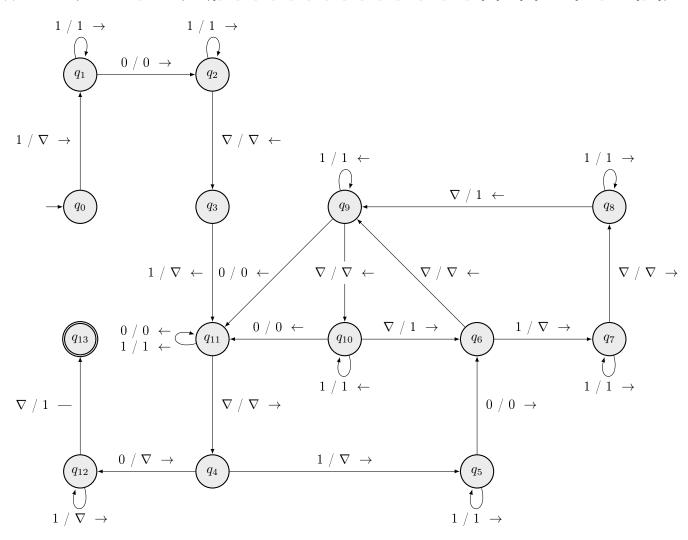
1.2 Zadanie 1.2

Zaprojektuj maszynę Turinga, które oblicza funkcję mnożenia f dla liczb naturalnych m i n w reprezentacji unarnej, czyli

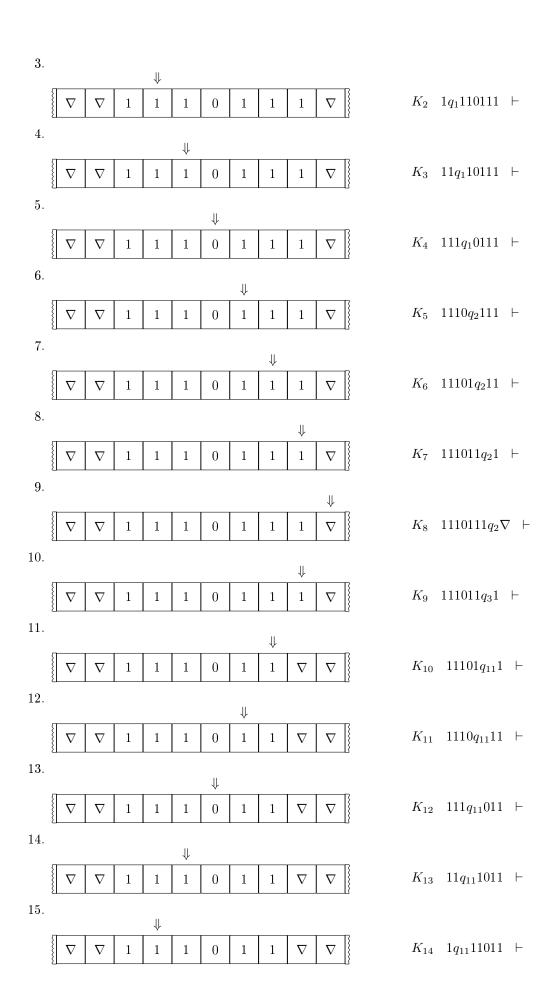
$$f(m,n) = m \cdot n$$

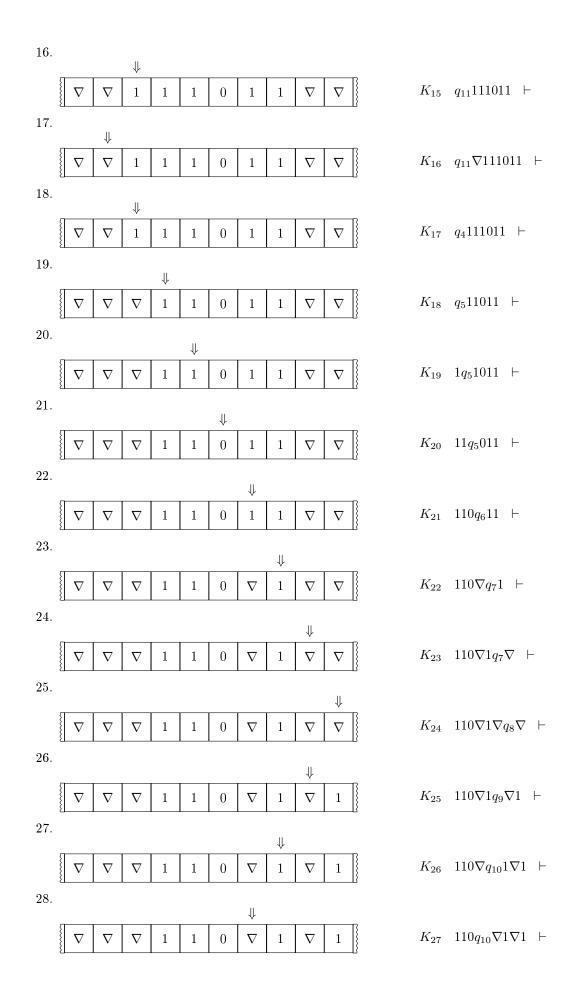
Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia, w tym pomnóż 3.2 lub 2.3 (wykonaj rysunki taśmy i zapisz konfiguracje).

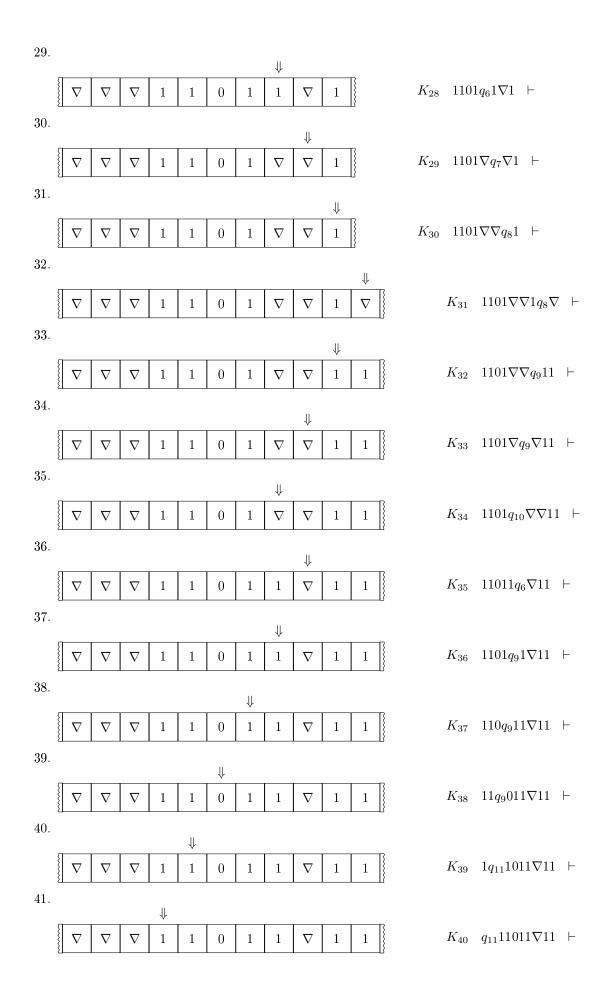
 $(4) \qquad M=(Q,\Sigma,\Gamma,\delta,q_{0},\nabla,F)=(\{q_{0},q_{1},q_{2},q_{3},q_{4},q_{5},q_{6},q_{7},q_{8},q_{9},q_{10},q_{11},q_{12},q_{13}\},\{1,0\},\{1,0,\nabla\},\delta,q_{0},\nabla,\{q_{13}\})$

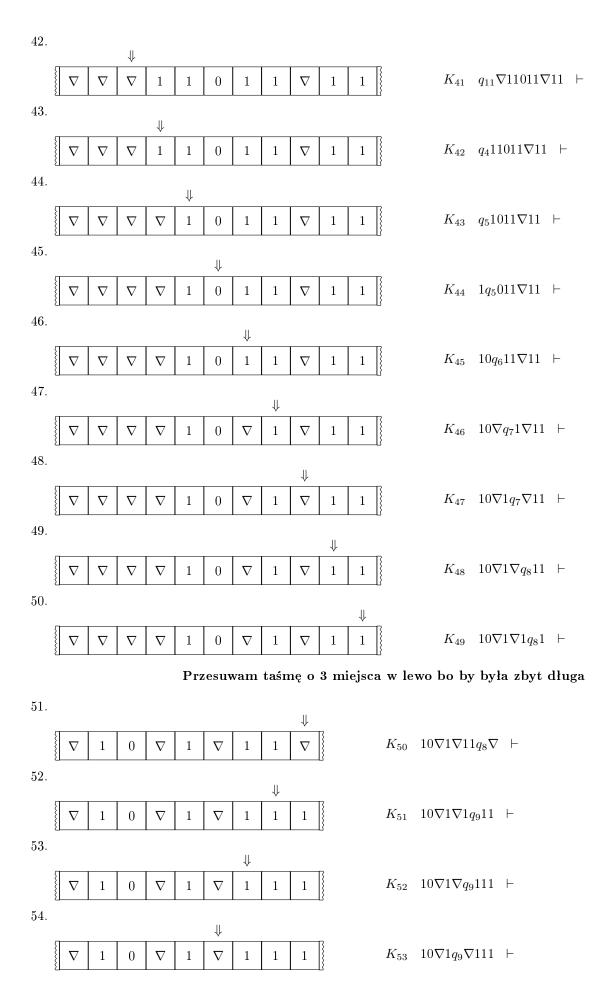


1.2.1 f(3, 2)

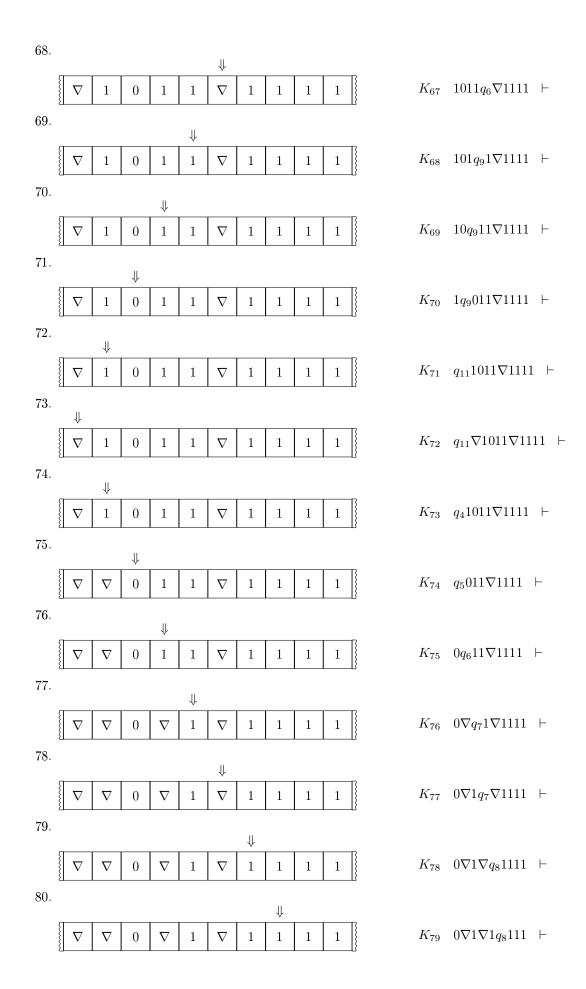


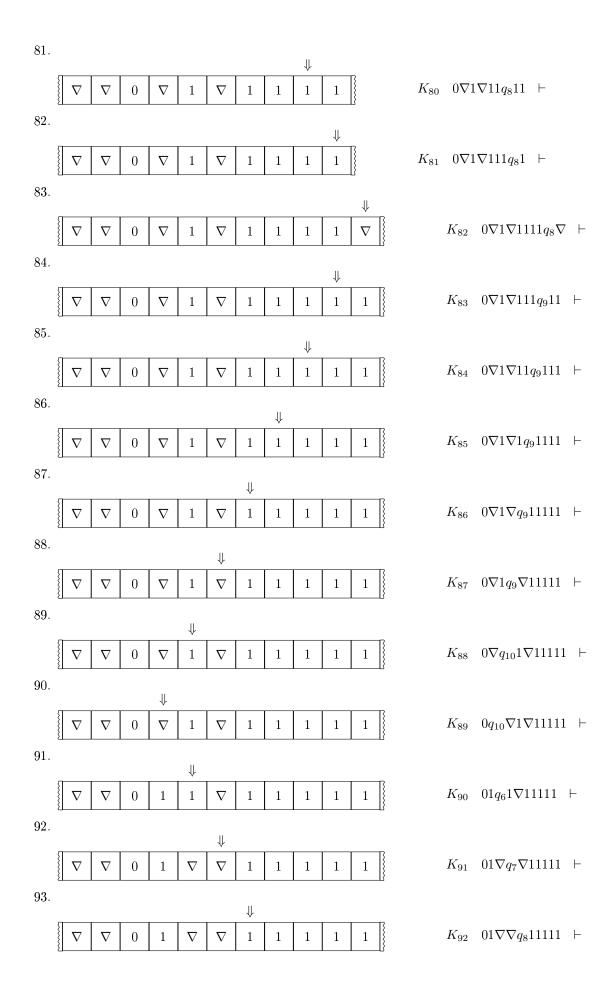


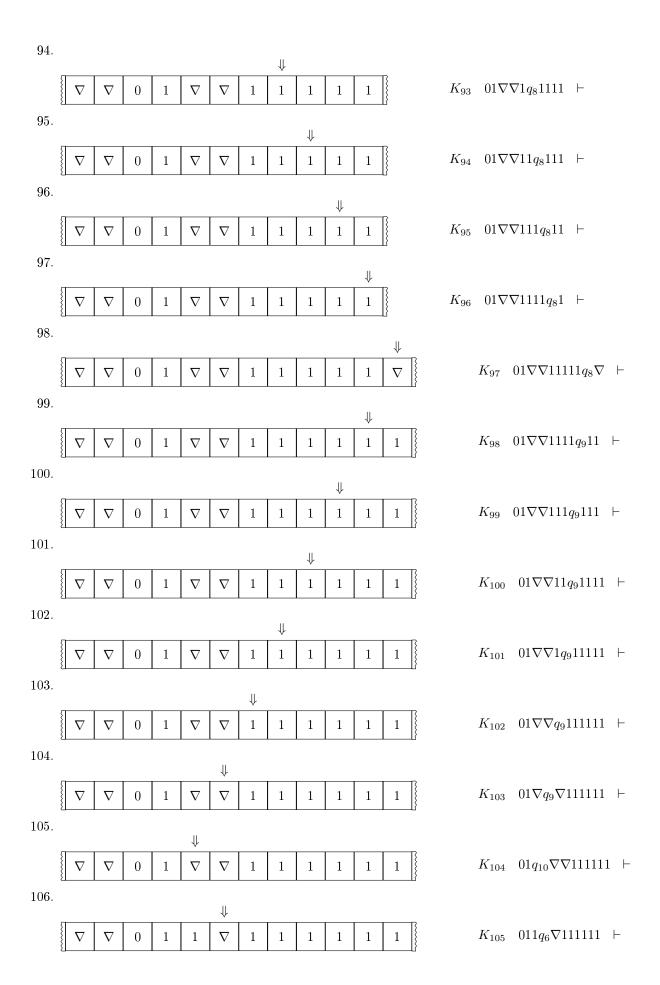


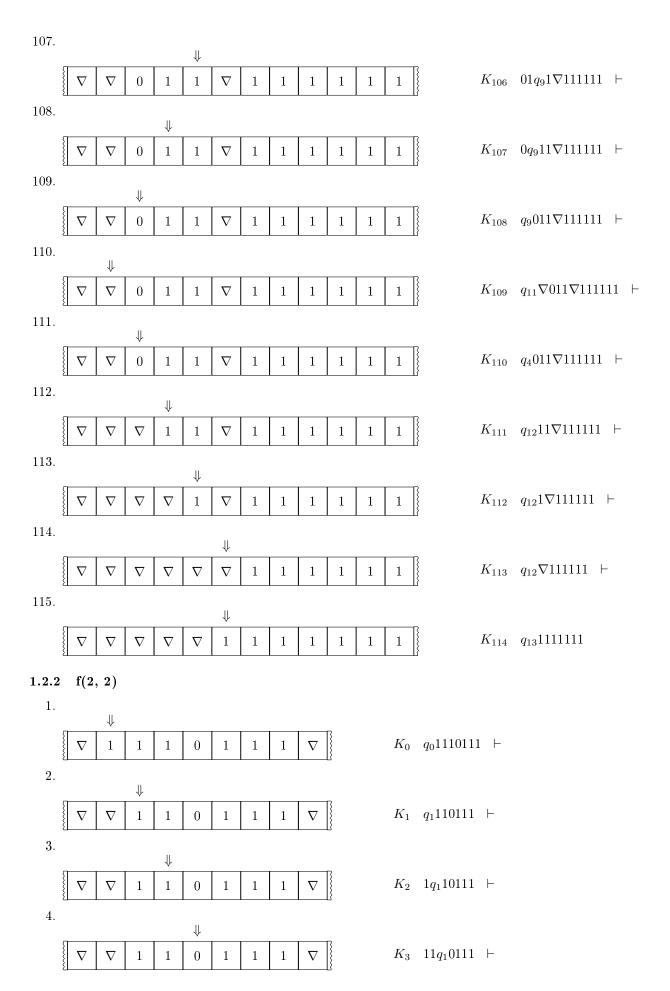


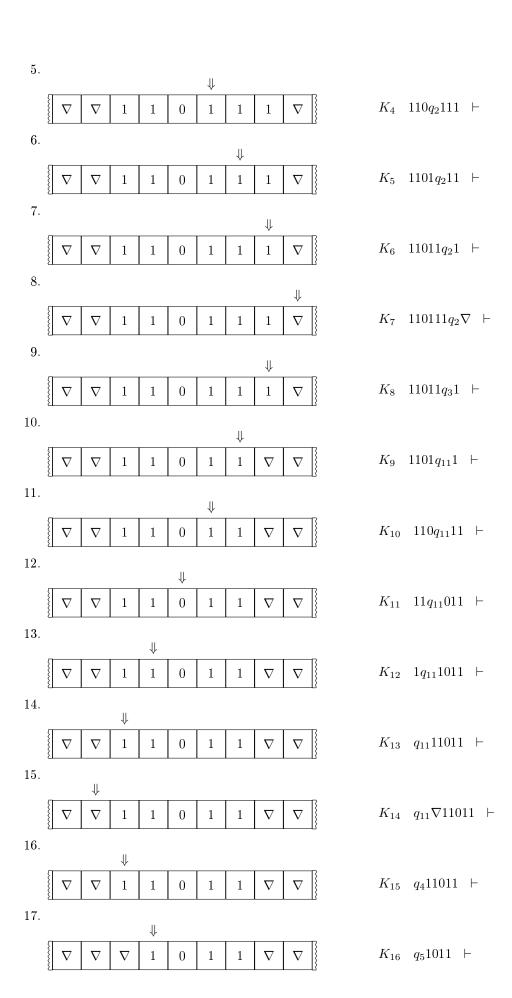
55. K_{54} $10\nabla q_{10}1\nabla 111$ \vdash ∇ 56. K_{55} $10q_{10}\nabla 1\nabla 111$ \vdash 0 ∇ 1 ∇ 1 1 1 57. K_{56} 101 q_6 1 ∇ 111 \vdash ∇ 58. K_{57} $101\nabla q_7\nabla 111$ \vdash ∇ ∇ 0 1 ∇ 1 1 59. K_{58} $101\nabla\nabla q_8111$ \vdash 60. K_{59} $101\nabla\nabla 1q_811$ \vdash 0 ∇ ∇ 1 1 1 61. K_{60} $101\nabla\nabla11q_{8}1$ \vdash 62. K_{61} $101\nabla\nabla111q_8\nabla$ \vdash 1 ∇ 63. K_{62} $101\nabla\nabla11q_{9}11$ \vdash 64. K_{63} $101\nabla\nabla 1q_9111$ \vdash ∇ 65. K_{64} $101\nabla\nabla q_91111$ \vdash ∇ 66. K_{65} $101\nabla q_9\nabla 1111$ \vdash 67. K_{66} $101q_{10}\nabla\nabla1111$ \vdash ∇ 1 1 1

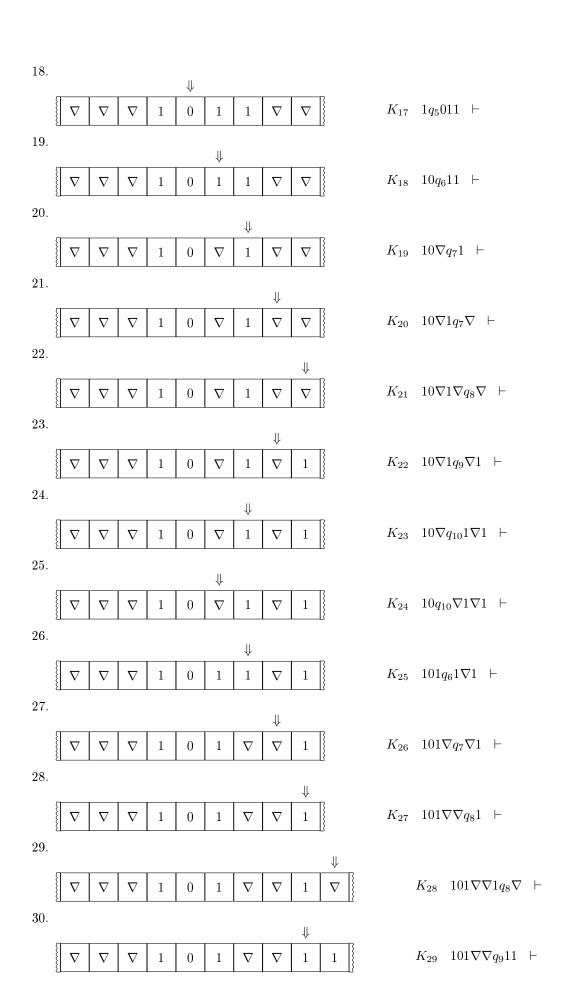


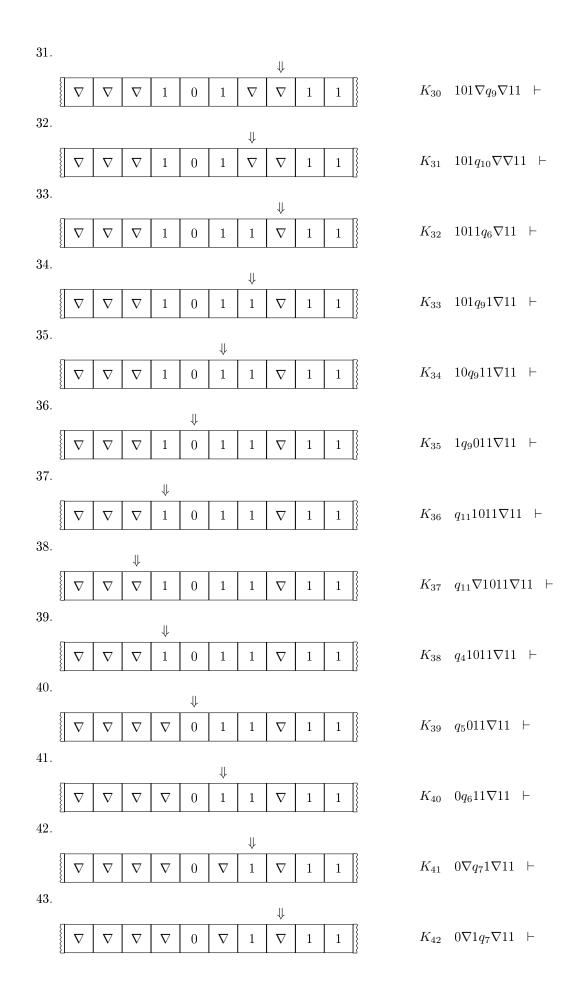


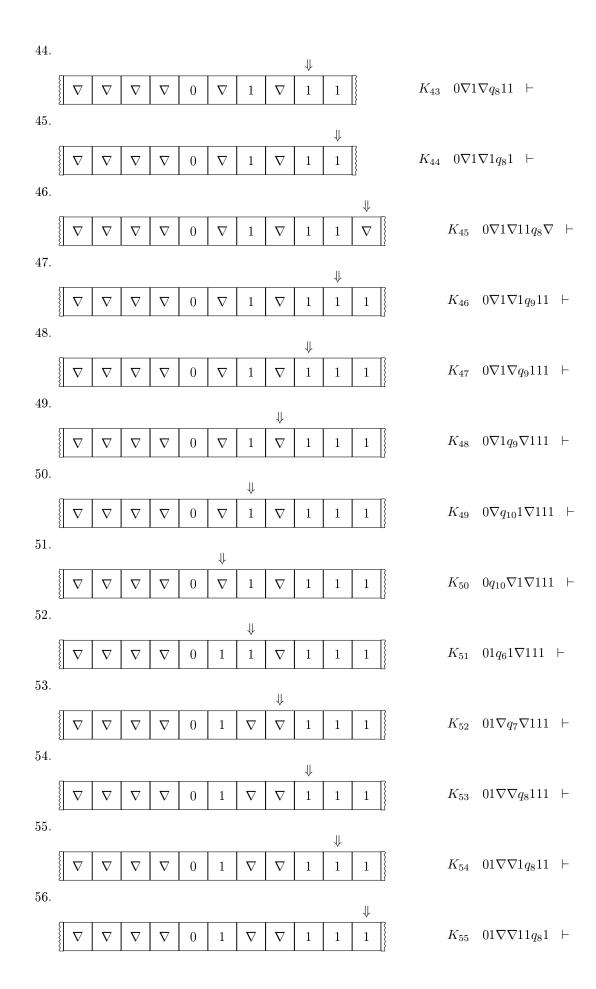


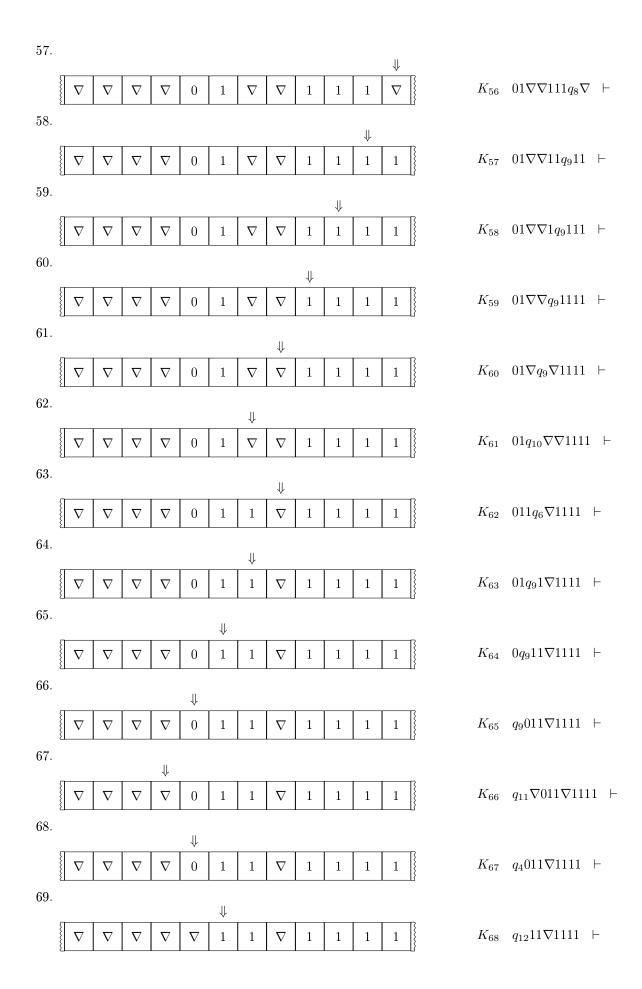
















71.

							\Downarrow								
∇	1	1	1	1	K_7	0	$q_{12}\nabla 1111$	⊢							

72.

•							\Downarrow						
∇	1	1	1	1	1	K_{71}	$q_{13}11111$						

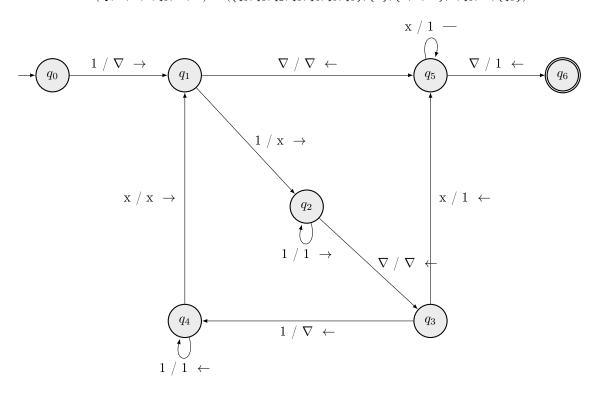
1.3 Zadanie 1.3

Zaprojektuj maszynę Turinga, które oblicza funkcję f dla liczby naturalnej n w reprezentacji unarnej, gdzie

(5)
$$f(n) = \begin{cases} \frac{n}{2}, & \text{jeżeli } n \text{ jest parzysta,} \\ \frac{n+1}{2}, & \text{jeżeli } n \text{ jest nieparzysta} \end{cases}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(6)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{1\}, \{1, x, \nabla\}, \delta, q_0, \nabla, \{q_6\})$$



1.3.1 f(3)

	\Downarrow							
∇	1	1	1	1	∇	K_0	q_011111	\vdash

11. $\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad K_{10} \quad \mathbf{x} q_{3}\mathbf{x} \vdash$

12. $\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad K_{11} \quad q_5 \ge 1 \quad \vdash \qquad \qquad K_{12} \quad q_5 \ge 1 \quad \vdash \qquad \qquad K_{13} \quad q_5 \ge 1 \quad \vdash \qquad \qquad K_{14} \quad q_5 \ge 1 \quad \vdash \qquad \qquad K_{15} \quad q_5 \ge 1 \quad \vdash \qquad K_{15} \quad q_5 = 1$

13. $\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad K_{12} \quad q_5 \nabla 11 \quad \vdash \qquad \qquad K_{12} \quad q_5 \nabla 11 \quad \vdash \qquad \qquad K_{13} \quad q_5 \nabla 11 \quad \vdash \qquad \qquad K_{14} \quad q_5 \nabla 11 \quad \vdash \qquad \qquad K_{15} \quad q_5 \nabla 11 \quad \vdash \qquad \qquad K_{15} \quad q_5 \nabla 11 \quad \vdash \qquad K_{15} \quad q_5 \quad q_5 \nabla 11 \quad \vdash \qquad K_{15} \quad q_5 \quad q_$

1.3.2 f(6)

1.



 $K_0 \quad q_0 1 1 1 1 1 1 1 1 1 \vdash$

2.



 K_1 $q_11111111$ \vdash

3.

			\Downarrow					
∇	∇	х	1	1	1	1	1	∇

 K_2 $\times q_2 111111$ \vdash

4.

				\Downarrow				
∇	∇	х	1	1	1	1	1	∇

 K_3 $\times 1q_21111$ \vdash

5.

					\Downarrow			
∇	∇	х	1	1	1	1	1	∇

 K_4 x11 q_2 111 \vdash

6.

						\Downarrow		
∇	∇	X	1	1	1	1	1	∇

 K_5 x111 q_2 11 \vdash

7.



 K_6 x1111 q_2 1 \vdash

8.



 K_7 x11111 $q_2\nabla$ \vdash

9.

								\Downarrow		
F	∇	∇	х	1	1	1	1	1	∇	~~~

 K_8 x1111 q_3 1 \vdash

10.

						\Downarrow			
∇	∇	x	1	1	1	1	∇	∇	3

 K_9 x111 q_4 1 \vdash

11.

					\Downarrow			
∇	∇	X	1	1	1	1	∇	∇

 K_{10} x11 q_4 11 \vdash

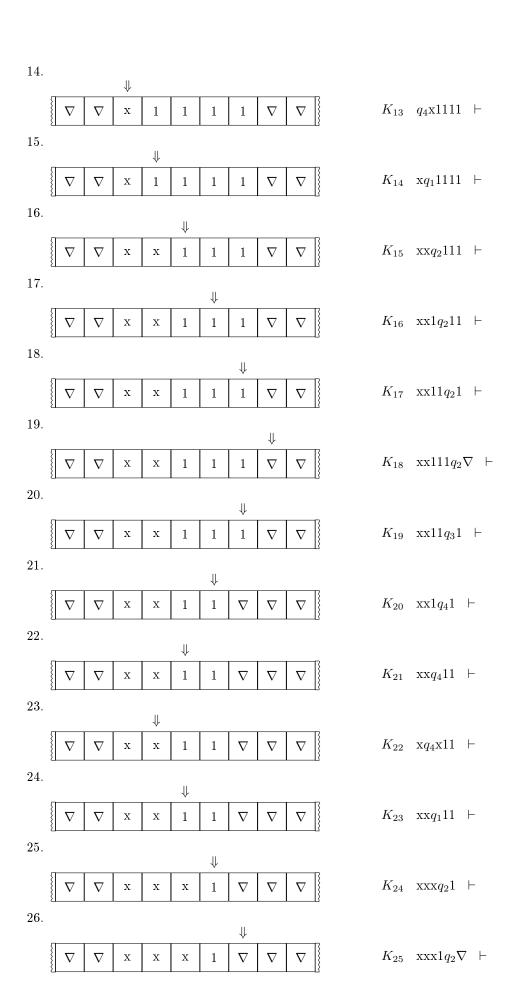
12.

				\Downarrow					
∇	∇	x	1	1	1	1	∇	∇	3

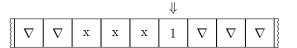
 K_{11} x1 q_4 111 \vdash

13.

 K_{12} $\times q_41111$ \vdash



27.



 K_{26} $xxxq_31$ \vdash

28.

∇	∇	x	X	X	∇	∇	∇	∇

 K_{27} xxq_4x \vdash

29.

					\Downarrow				
∇	∇	Х	X	X	∇	∇	∇	∇	

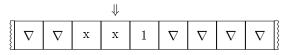
 $K_{28} \quad \mathbf{x}\mathbf{x}\mathbf{x}q_1\nabla \quad \vdash$

30.

				\Downarrow				
∇	∇	X	X	X	∇	∇	∇	∇

$$K_{29}$$
 xxq_5x \vdash

31.



$$K_{30}$$
 $\times q_5 \times 1$ \vdash

32.



$$K_{31}$$
 q_5 x11 \vdash

33.



$$K_{32}$$
 $q_5\nabla 111$ \vdash

$$K_{33}$$
 $q_6\nabla 1111$

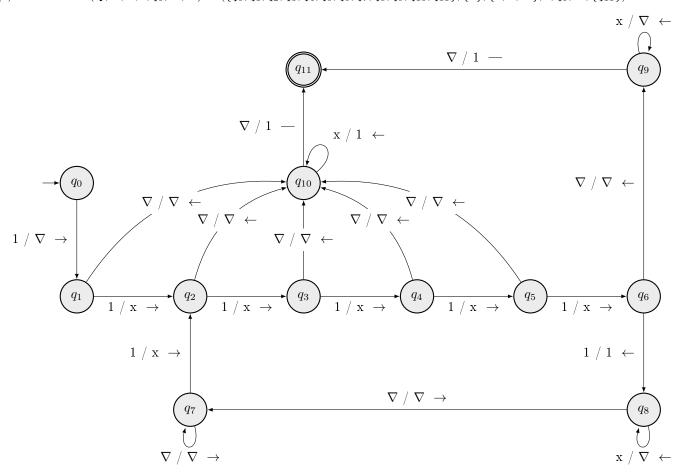
1.4 Zadanie 1.4

Zaprojektuj maszynę Turinga, które oblicza funkcję f dla liczby naturalnej n w reprezentacji unarnej, gdzie

$$f(n) = n \mod 5$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(8) M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}\}, \{1\}, \{1, x, \nabla\}, \delta, q_0, \nabla, \{q_{11}\})$$



1.4.1 f(5)

1.

	\Downarrow						
∇	1	1	1	1	1	1	$oxed{\nabla}$

$$K_0 \quad q_0 1 1 1 1 1 1 1 \quad \vdash$$

2.

		\Downarrow					
∇	∇	1	1	1	1	1	$oxed{ abla}$

$$K_1$$
 $q_1111111$ \vdash

3.

_			\Downarrow				
∇	∇	Х	1	1	1	1	∇

$$K_2$$
 $\times q_2 1111$ \vdash

				\Downarrow			
∇	∇	X	X	1	1	1	∇

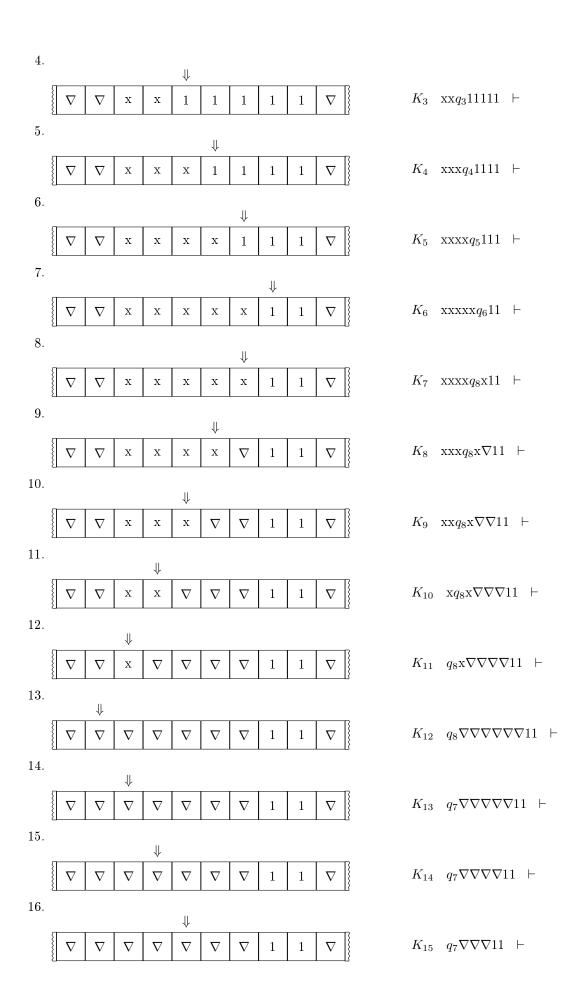
$$K_3 \quad xxq_3111 \quad \vdash$$

5. $\nabla \mid \}$ $K_4 \quad xxxq_411 \quad \vdash$ 6. $K_5 \quad xxxxq_51 \quad \vdash$ ∇ X X х X 7. ∇ $K_6 \quad \text{xxxxx} q_6 \nabla \quad \vdash$ 8. ∇ $K_7 \quad xxxxq_9x \quad \vdash$ X X X 9. $K_8 \quad \mathbf{x}\mathbf{x}\mathbf{x}q_9\mathbf{x} \quad \vdash$ 10. $K_9 \quad \mathbf{x} \mathbf{x} q_9 \mathbf{x} \quad \vdash$ X ∇ 11. K_{10} $\times q_9 \times$ \vdash 12. K_{11} $q_9\mathbf{x}$ \vdash ∇ X ∇ ∇ ∇ ∇ 13.

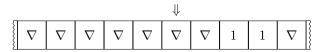
1.4.2 f(7)

1. $\begin{picture}(1,0) \put(0,0) \put(0$

 K_1 $q_111111111$ \vdash

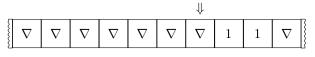


17.



 K_{16} $q_7 \nabla \nabla 11$ \vdash

18.



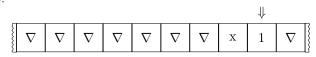
 K_{17} $q_7\nabla 11$ \vdash

19.

							\Downarrow		
∇	1	1	∇						

 K_{18} q_711 \vdash

20.



 K_{19} $\times q_2 1$ \vdash

21.



 $K_{20} \quad \mathbf{x} \mathbf{x} q_3 \nabla \quad \vdash$

22.



 K_{21} $\mathbf{x}q_{10}\mathbf{x}$ \vdash

23.



 K_{22} q_{10} x1 \vdash

24.



 K_{23} $q_{10}\nabla 11$ \vdash

25.

							\Downarrow				
لسسم	∇	∇	∇	∇	∇	∇	1	1	1	∇	00000

 K_{24} $q_{11}111$

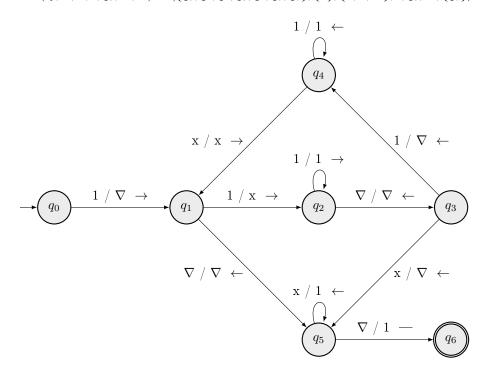
1.5 Zadanie 1.5

Zaprojektuj maszynę Turinga, które oblicza funkcję f dla liczby naturalnej n w reprezentacji unarnej, gdzie

$$f(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(10)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{1\}, \{1, x, \nabla\}, \delta, q_0, \nabla, \{q_6\})$$



1.5.1 f(5)

1.



$$K_0 \quad q_0 1 1 1 1 1 1 1 \vdash$$

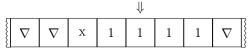
2.

$$K_1 \quad q_1 1 1 1 1 1 1 \vdash$$

3.

$$K_2 \quad \mathbf{x} q_2 \mathbf{1} \mathbf{1} \mathbf{1} \mathbf{1} \quad \vdash$$

4.



$$K_3$$
 $\times 1q_2111$ \vdash

					\Downarrow		
∇	∇	x	1	1	1	1	∇

$$K_4$$
 x11 q_2 11 \vdash

6. $\nabla \parallel$ K_5 x111 q_2 1 \vdash 7. K_6 x1111 $q_2\nabla$ \vdash ∇ X 1 1 1 8. $\nabla \mid \rangle$ K_7 x111 q_3 1 \vdash 9. K_8 x11 q_4 1 \vdash ∇ ∇ Х 1 1 ∇ 10. K_9 $x1q_411$ \vdash 11. 12.

 K_{10} $\times q_4111$ \vdash

 K_{11} q_4 x111 \vdash

13. K_{12} $\times q_1 111$ \vdash

14. K_{13} xxq_211 \vdash

15. K_{14} $xx1q_21$ \vdash

16. K_{15} xx11 $q_2\nabla$ \vdash

17. K_{16} $xx1q_31$ \vdash

18. K_{17} xxq_41 \vdash ∇

19. 20. X ∇ 21.

 K_{19} xxq_11 \vdash

 K_{18} $\times q_4 \times 1$ \vdash

 $K_{20} \quad \mathbf{x}\mathbf{x}\mathbf{x}q_2\nabla \quad \vdash$

22. X ∇ ∇

 K_{21} xxq_3x \vdash

23.

 K_{22} $\times q_5 \times$ \vdash

24. ∇

 K_{23} q_5 x1 \vdash

25.

 K_{24} $q_5\nabla 11$ \vdash

26. ∇ ∇ ∇

 K_{25} q_6111

1.5.2 f(6)

1. \Downarrow ∇ ∇

 K_0 $q_011111111$ \vdash

2.

 K_1 $q_11111111$ \vdash

3. ∇

 K_2 $\times q_2 111111$ \vdash

4.

 K_3 $\times 1q_21111$ \vdash

5.

 K_4 x11 q_2 111 \vdash

6. $\nabla \parallel$ K_5 x111 q_2 11 \vdash 7. K_6 x1111 q_2 1 \vdash \mathbf{X} 1 1 1 1 $\nabla \mid \{$ 8. K_7 x11111 $q_2\nabla$ \vdash $\nabla \parallel$ 9. $\nabla \parallel$ K_8 x1111 q_3 1 \vdash X 1 1 1 10. K_9 x111 q_4 1 \vdash 11. K_{10} x11 q_4 11 \vdash X 12. K_{11} x1 q_4 111 \vdash 13. K_{12} $\times q_41111$ \vdash 14. K_{13} q_4 x1111 \vdash 15. K_{14} $\times q_1 1111$ \vdash 16. K_{15} xxq_2111 \vdash 17. K_{16} $xx1q_211$ \vdash 18. K_{17} xx11 q_2 1 \vdash



 K_{18} xx111 $q_2\nabla$ \vdash

20.



 K_{19} xx11 q_3 1 \vdash

21.



$$K_{20}$$
 $xx1q_41$ \vdash

22.

				\Downarrow				
∇	∇	Х	Х	1	1	∇	∇	∇

$$K_{21}$$
 xxq_411 \vdash

23.



$$K_{22}$$
 $\times q_4 \times 11$ \vdash

24.

				₩				_
∇	∇	Х	X	1	1	∇	∇	∇

$$K_{23}$$
 xxq_111 \vdash

25.



$$K_{24} \quad \mathbf{x}\mathbf{x}\mathbf{x}q_2\mathbf{1} \quad \vdash$$

26.



$$K_{25}$$
 $xxx1q_2\nabla$ \vdash

27.

					\Downarrow				
∇	∇	х	x	X	1	∇	∇	∇	

$$K_{26}$$
 $xxxq_31$ \vdash

28.

				\Downarrow					
∇	∇	х	х	х	∇	∇	∇	∇	

$$K_{27}$$
 xxq_4x \vdash

29.

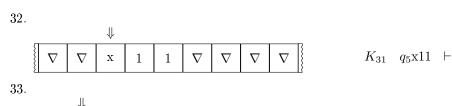
$$K_{28} \quad \mathbf{x}\mathbf{x}\mathbf{x}q_1\nabla \quad \vdash$$

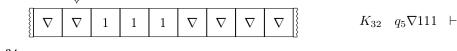
30.

$$K_{29}$$
 xxq_5x \vdash

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & \\ \hline \hline \hline & \nabla & \nabla & x & x & 1 & \nabla & \nabla & \nabla & \nabla \\ \hline \end{array}$$

$$K_{30}$$
 $\times q_5 \times 1$ \vdash







1.6 Zadanie 1.6

Zadanie 1.6 i zadanie 1.3 polegają na tym samym, różni się tylko zapis

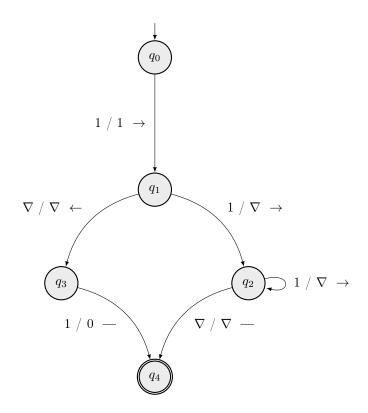
1.7 Zadanie 1.7

Zaprojektuj maszynę Turinga, które oblicza funkcję signum (znaku)

(11)
$$sgn(n) = \begin{cases} 1, & \text{jeżeli } n \ge 0, \\ 0, & \text{jeżeli } n = 0 \end{cases}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(12)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4\}, \{1\}, \{1, 0, \nabla\}, \delta, q_0, \nabla, \{q_4\})$$



1.7.1 sgn(0)

1.

	\Downarrow			
∇	1	∇	K_0	q_01

2.

		\Downarrow			
 ∇	1	∇	K_1	$1q_1\nabla$	H

3.

	\Downarrow				
∇	1	∇	K_2	q_31	H

4.

	\Downarrow			
∇	0	∇	K_3	$q_{4}0$

$1.7.2 \quad \text{sgn}(7)$

1.

	\Downarrow								
∇	1	1	1	1	1	1	1	1	∇

 $K_0 \quad q_0 1 1 1 1 1 1 1 1 1 1 \vdash$

2.

		\Downarrow							
∇	1	1	1	1	1	1	1	1	∇

 $K_1 \quad 1q_111111111 \quad \vdash$

3.



 K_2 $1\nabla q_2 1111111$ \vdash

4.

					\Downarrow					
-	∇	1	∇	∇	1	1	1	1	1	∇

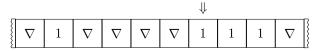
 K_3 $1\nabla\nabla q_2$ 11111 \vdash

5.

					\Downarrow				
∇	1	∇	∇	∇	1	1	1	1	∇

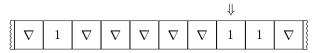
 $K_4 \quad 1\nabla\nabla\nabla q_2 1111 \quad \vdash$

6.



 $K_5 \quad 1\nabla\nabla\nabla\nabla\nabla q_2 111 \quad \vdash$

7.

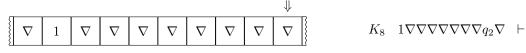


 K_6 $1\nabla\nabla\nabla\nabla\nabla q_2$ 11 \vdash

8.

								\Downarrow	
 ∇	1	∇	∇	∇	∇	∇	∇	1	∇

 K_7 $1\nabla\nabla\nabla\nabla\nabla\nabla\nabla q_21$ \vdash



10.

									\Downarrow		
∇	1	∇	K_9	$1\nabla\nabla\nabla\nabla\nabla\nabla\nabla\nabla q_4\nabla$							

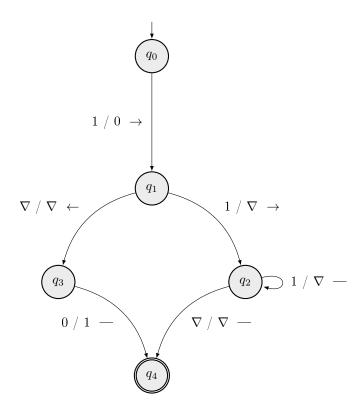
1.8 Zadanie 1.8

Zaprojektuj maszynę Turinga, które oblicza funkcję dopełnienie signum

(13)
$$\overline{sgn}(n) = \begin{cases} 0, & \text{jeżeli } n \ge 0, \\ 1, & \text{jeżeli } n = 0 \end{cases}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(14)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4\}, \{1\}, \{1, 0, \nabla\}, \delta, q_0, \nabla, \{q_4\})$$



1.8.1 $\overline{sgn}(0)$

1.



3. ∇ K_2 q_30 \vdash 4. ∇ K_3 q_41 **1.8.2** $\overline{sgn}(7)$ 1. \Downarrow K_0 $q_0111111111$ \vdash ∇ ∇ 1 1 1 2. $K_1 \quad 0q_111111111 \quad \vdash$ ∇ 3. K_2 $0\nabla q_2$ 1111111 \vdash ∇ 0 ∇ 1 1 1 1 1 4. $K_3 \quad 0\nabla\nabla q_2$ 11111 \vdash 5. $K_4 \quad 0\nabla\nabla\nabla q_2$ 1111 \vdash ∇ 0 ∇ 1 1 1 ∇ ∇ ∇ 1 6. $K_5 \quad 0\nabla\nabla\nabla\nabla\nabla q_2$ 111 \vdash 1 7. $K_6 \quad 0\nabla\nabla\nabla\nabla\nabla q_2$ 11 \vdash ∇ 0 ∇ ∇ ∇ ∇ ∇ 1 1 8. $K_7 \quad 0\nabla\nabla\nabla\nabla\nabla\nabla\nabla q_2 1 \quad \vdash$

9.

 $K_9 \quad 0\nabla\nabla\nabla\nabla\nabla\nabla\nabla\nabla q_4\nabla$

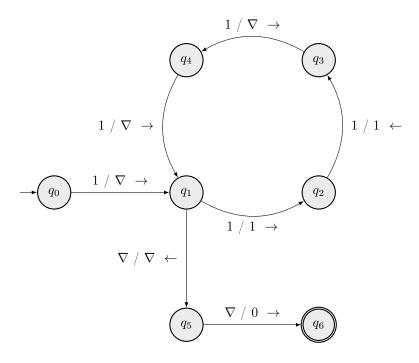
1.9 Zadanie 1.9

Zaprojektuj maszynę Turinga, które oblicza funkcję

(15)
$$f(n) = \begin{cases} 0, & \text{jeżeli } n \text{ jest parzysta,} \\ 1, & \text{jeżeli } n \text{ jest nieparzysta} \end{cases}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(16)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{1\}, \{1, 0, \nabla\}, \delta, q_0, \nabla, \{q_6\})$$



1.9.1 f(4)

1.

$$K_0$$
 $q_0111111$ \vdash

2.

$$K_1$$
 q_11111 \vdash

3.

$$K_2$$
 $1q_2111$ \vdash

4.

$$K_3$$
 q_31111 \vdash

$$K_4$$
 q_4 111 \vdash

6. K_5 q_111 \vdash 7. K_6 $1q_21$ \vdash ∇ ∇ ∇ 8. K_7 q_311 \vdash ∇ 9. ∇ $\nabla \mid \rangle$ K_8 q_41 \vdash ∇ ∇ 10. ∇ $K_9 \quad q_1 \nabla \quad \vdash$ 11. K_{10} $q_5\nabla$ \vdash $\nabla \parallel$ ∇ ∇ ∇ ∇ 12. K_{11} $0q_6\nabla$ ∇ ∇ 1.9.2 f(7) 1.

6.

 ∇ ∇

 K_0 $q_0111111111$ \vdash

2. ∇ 1 ∇ 1

 K_1 $q_111111111$ \vdash

3. ∇

 K_2 1 q_2 1111111 \vdash

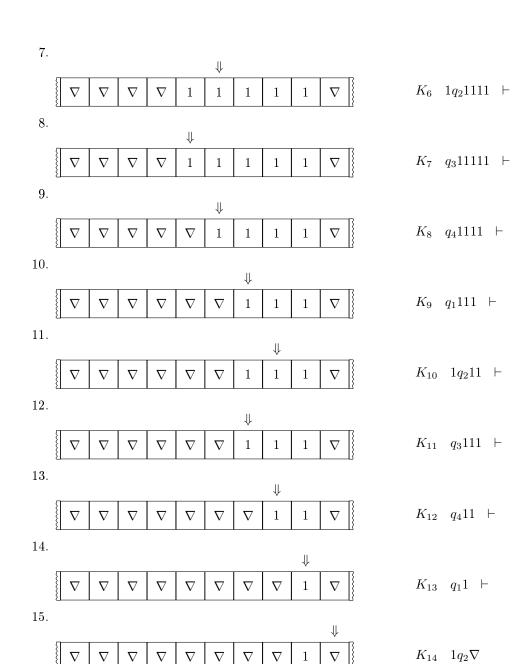
4. ∇ 1

 K_3 q_3 11111111 \vdash

5.

 K_4 q_4 1111111 \vdash

 ∇ K_5 $q_1111111$ \vdash



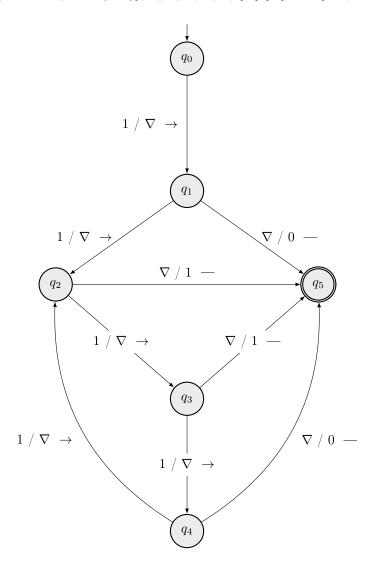
1.10 Zadanie 1.10

Zaprojektuj maszynę Turinga, które oblicza funkcję

(17)
$$f(n) = \begin{cases} 0, & \text{jeżeli } n \text{ jest podzielna przez } 3, \\ 1, & \text{jeżeli } n \text{ nie jest podzielna przez } 3 \end{cases}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(18)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{1\}, \{1, 0, \nabla\}, \delta, q_0, \nabla, \{q_5\})$$



1.10.1 f(4)

1.

· •	\Downarrow						
∇	1	1	1	1	1	∇	3

$$K_0$$
 $q_0111111$ \vdash

$$K_1$$
 q_11111 \vdash

1.10.2 f(6)

								\Downarrow			
 ∇	∇	0		K_8	q_50						

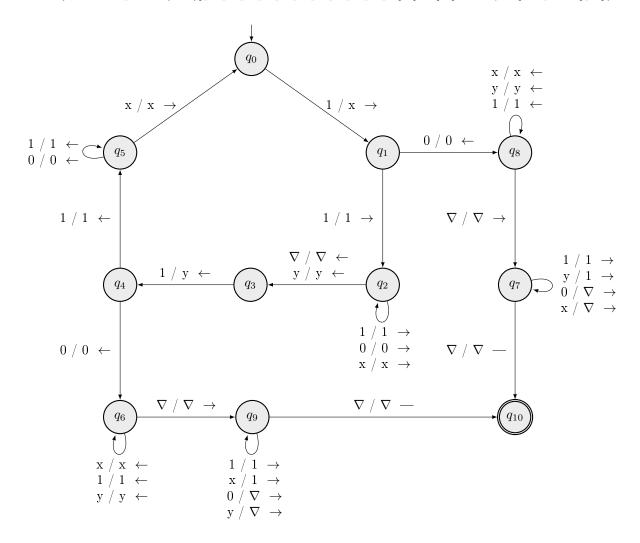
1.11 Zadanie 1.11

Zaprojektuj maszynę Turinga, które oblicza funkcję maksimum dla liczb naturalnych m i n w reprezentacji unarnej, czyli

$$(19) f(m,n) = max(m,n).$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

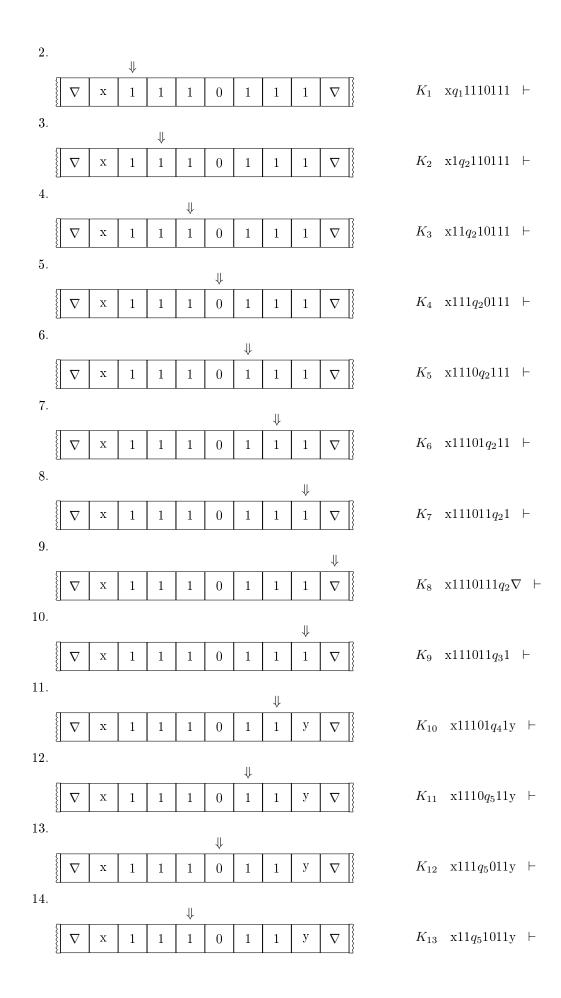
$$(20) \qquad M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}, \{1, 0\}, \{1, 0, x, y, \nabla\}, \delta, q_0, \nabla, \{q_{10}\})$$

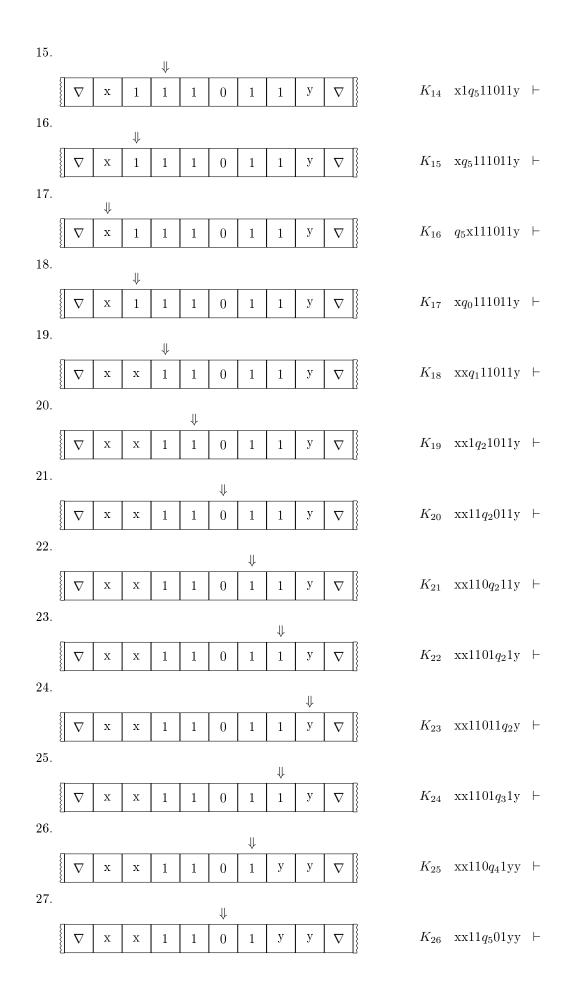


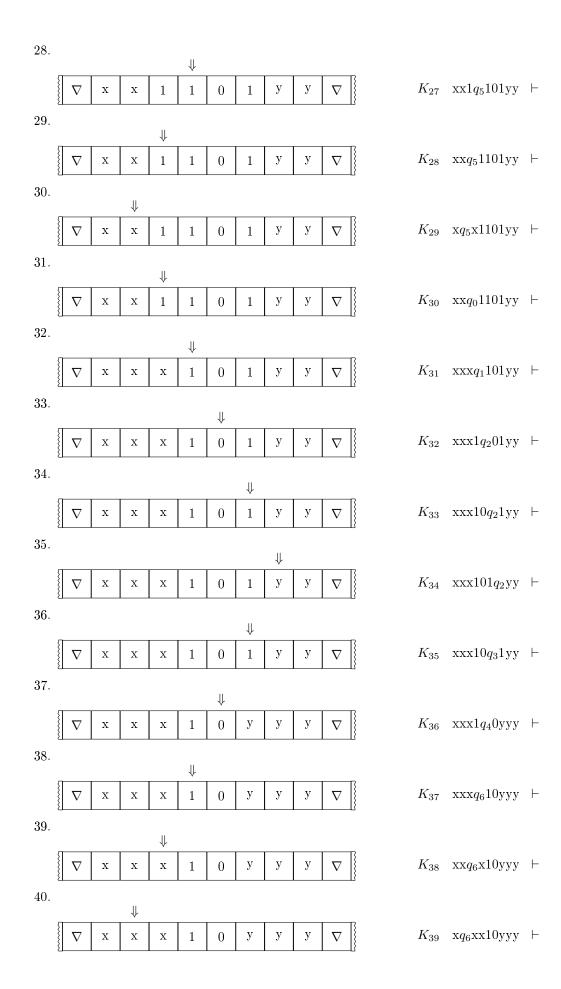
1.11.1 $\max(3, 2)$

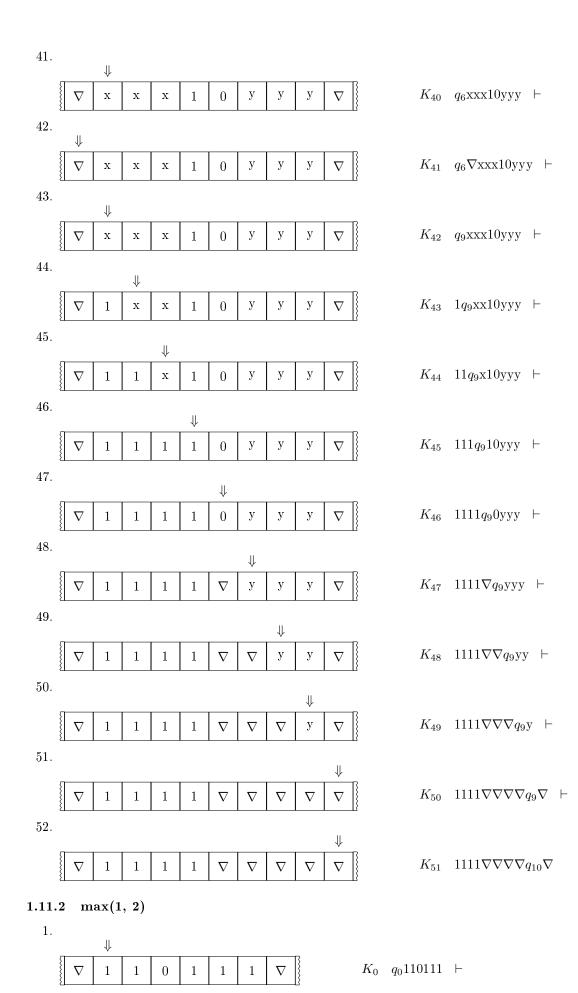
	\Downarrow									
∇	1	1	1	1	0	1	1	1	∇	

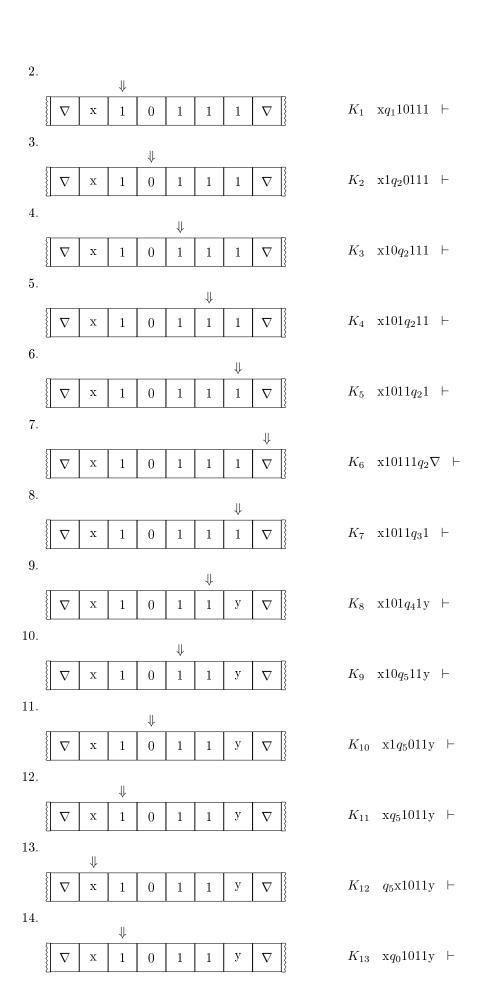
$$K_0$$
 $q_011110111$ \vdash

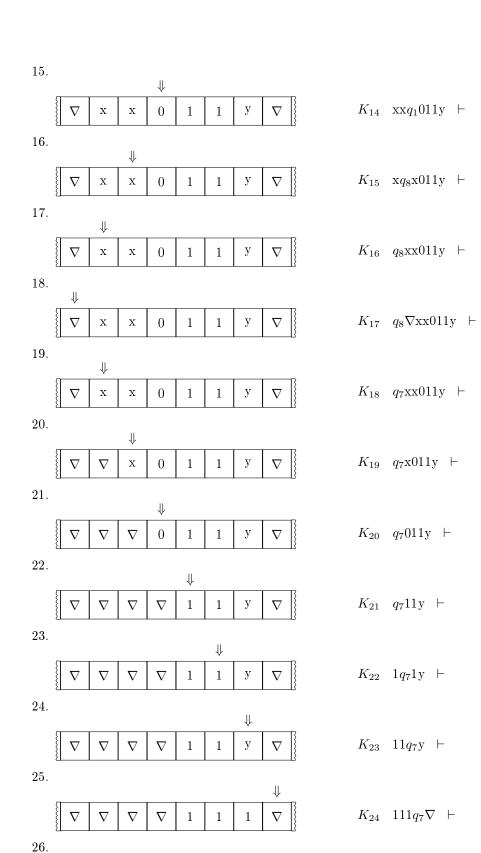












 K_{25} $111q_{10}\nabla$

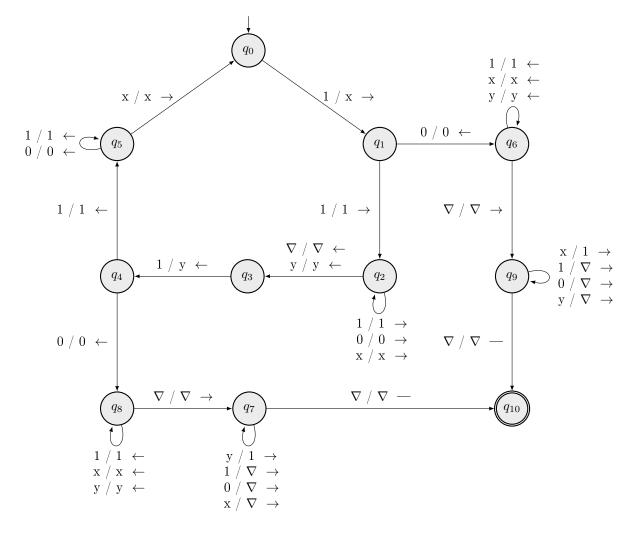
1.12 Zadanie 1.12

Zaprojektuj maszynę Turinga, które oblicza funkcję minimum dla liczb naturalnych m i n w reprezentacji unarnej, czyli

$$(21) f(m,n) = min(m,n).$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(22) M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}, \{1, 0\}, \{1, 0, x, y, \nabla\}, \delta, q_0, \nabla, \{q_{10}\})$$



$1.12.1 \quad \min(3, 1)$

 $K_0 \quad q_0 1 1 1 1 0 1 1 \quad \vdash$

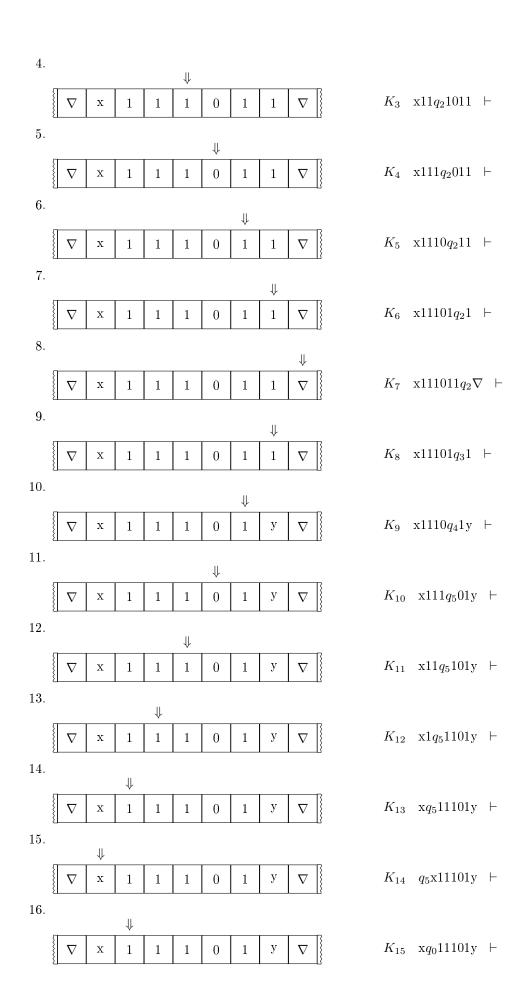
2.

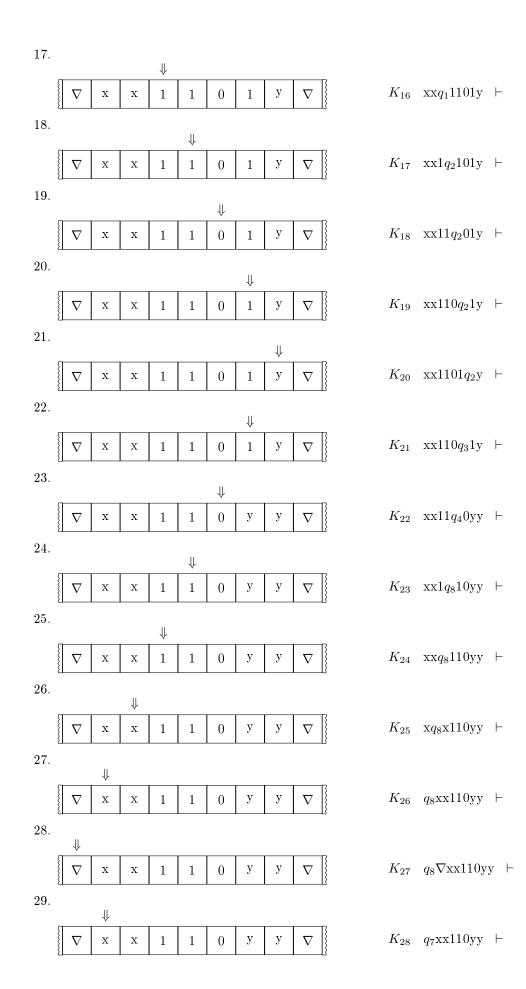
		\Downarrow						_
∇	Х	1	1	1	0	1	1	∇

$$K_1 \quad \mathbf{x} q_1 1 1 1 0 1 1 \quad \vdash$$

			\Downarrow					
∇	X	1	1	1	0	1	1	∇

$$K_2 \quad x1q_211011 \quad \vdash$$

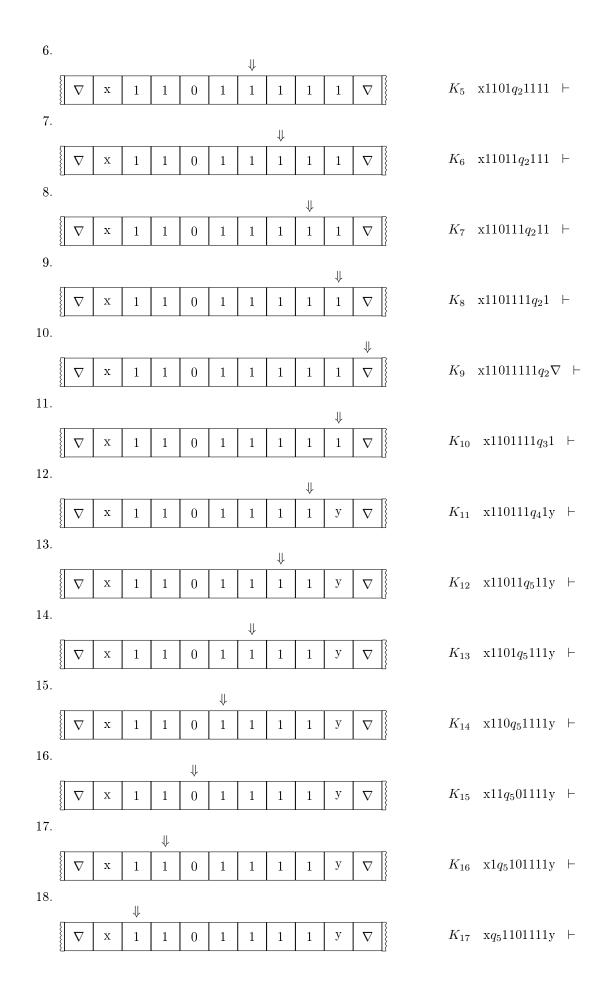


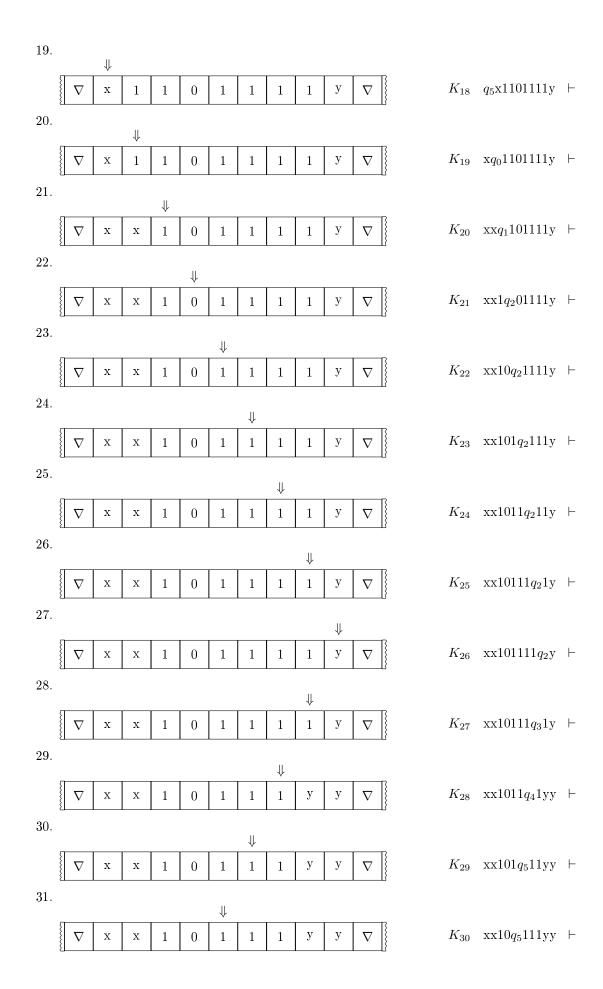


30. K_{29} q_7 x110yy \vdash ∇ x 31. K_{30} q_7110 yy \vdash ∇ ∇ 1 0 32. K_{31} q_710 yy \vdash 0 ∇ 33. K_{32} q_70yy \vdash y ∇ ∇ ∇ 0 у ∇ 34.у K_{33} $q_7 yy \vdash$ 35. K_{34} $1q_7y$ \vdash y ∇ ∇ ∇ ∇ ∇ 36. K_{35} $11q_7\nabla$ \vdash ∇ 37. K_{36} $11q_{10}\nabla$ ∇ ∇ ∇ ∇ 1 1 ∇ $1.12.2 \quad \min(2, 4)$

1. $K_0 \quad q_0 1 1 1 0 1 1 1 1 1 1 \vdash$ ∇ 1 1 1 ∇ 1 2. $K_1 \quad \mathbf{x} q_1 1 1 0 1 1 1 1 1 1 \vdash$ 3. $K_2 \quad x1q_210111111 \quad \vdash$ ∇ 1 1 4. K_3 x11 q_2 011111 \vdash 5.

 K_4 x110 q_2 111111 \vdash





32.					\Downarrow									
	∇	X	х	1	0	1	1	1	у	у	∇	K_{31}	$xx1q_50111yy$	⊢
33.							ı				<u> </u>			
	∇	X	x	1	0	1	1	1	у	у	∇	K_{32}	$xxq_510111yy$	⊢
34.	<u>{</u>						<u> </u>				<u> </u>			
	∇	X	↓	1	0	1	1	1	у	у	∇	K_{33}	$xq_5x10111yy$	⊢
35.	<u>}</u>									<u> </u>	<u> </u>			
	∇	x	x	↓ 1	0	1	1	1	у	у	∇	K_{24}	$xxq_010111yy$	F
36.	{ L	11		1	0	1	1	1	<i>,</i>		_ v _}}	54	111140101111	
	{ .	37	37	37		1	1	1	V	V		<i>K</i>	$xxxq_10111yy$	L
37.	$\left\{ \begin{array}{c} \nabla \end{array} \right\}$	X	X	X	0	1	1	1	У	У	∇	N 35	$xxxq_10111yy$	1
91.	{			#			I				<u> </u>	T.5	0444	
9.0	∇	X	X	X	0	1	1	1	У	У		K_{36}	$xxq_6x0111yy$	F
38.	ξ Ι		#					г			T			
	∇	Х	Х	X	0	1	1	1	у	У	∇	K_{37}	$xq_6xx0111yy$	F
39.										_				
	∇	х	х	Х	0	1	1	1	У	у	∇	K_{38}	q_6 xxx0111yy	\vdash
40.	\Downarrow													
	∇	X	X	X	0	1	1	1	у	у	$oxed{\nabla}$	K_{39}	$q_6 \nabla xxx0111y$	у ⊢
41.														
	∇	X	х	X	0	1	1	1	у	у	∇	K_{40}	q_9 xxx0111yy	⊢
42.			\Downarrow								_			
	∇	1	x	X	0	1	1	1	у	у	∇	K_{41}	$1q_9$ xx 0111 yy	⊢
43.				↓				ļ.						
	∇	1	1	X	0	1	1	1	у	у	∇	K_{42}	$11q_9$ x 0111 yy	\vdash
44.	·		1	l		I	I	I	I	1	<u> </u>			
	∇	1	1	1	0	1	1	1	у	у	∇	K_{43}	$111q_90111yy$	⊢
	<u>}</u>							L						

45.y K_{44} 111 ∇q_9 111yy \vdash 46. K_{45} 111 $\nabla \nabla q_9$ 11yy \vdash ∇ 1 у ∇ 1 1 1 ∇ 47. K_{46} 111 $\nabla\nabla\nabla q_9$ 1yy \vdash y 48. K_{47} 111 $\nabla\nabla\nabla\nabla\nabla q_9$ yy \vdash ∇ ∇ у 1 1 ∇ ∇ У ∇ 49. K_{48} 111 $\nabla\nabla\nabla\nabla\nabla\nabla q_9$ y \vdash 50. K_{49} 111 $\nabla\nabla\nabla\nabla\nabla\nabla\nabla q_9\nabla$ \vdash ∇ ∇ ∇ 51.

 K_{50} 111 $\nabla\nabla\nabla\nabla\nabla\nabla \nabla q_{10}\nabla$

2 Maszyny Turinga obliczające funkcje w alfabecie binarnym

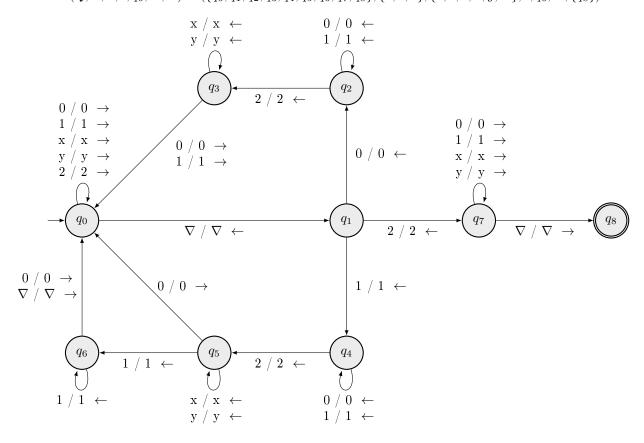
2.1 Zadanie 1.18

Zaprojektuj maszynę Turinga, która oblicza funkcję dodawania

$$(23) f(m,n) = m+n.$$

dla liczby naturalnej n w reprezentacji binarnej. Za separator przyjmij 2. Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia, w tym dodaj 3+2 (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(24) M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}, \{0, 1, 2\}, \{0, 1, 2, x, y, \nabla\}, \delta, q_0, \nabla, \{q_8\})$$



$2.1.1 \quad f(6, 4)$

1.



$$K_0 \quad q_0 1102100 \quad \vdash$$

2.

_		\Downarrow						
∇	1	1	0	2	1	0	0	∇

$$K_1 \quad 1q_0102100 \quad \vdash$$

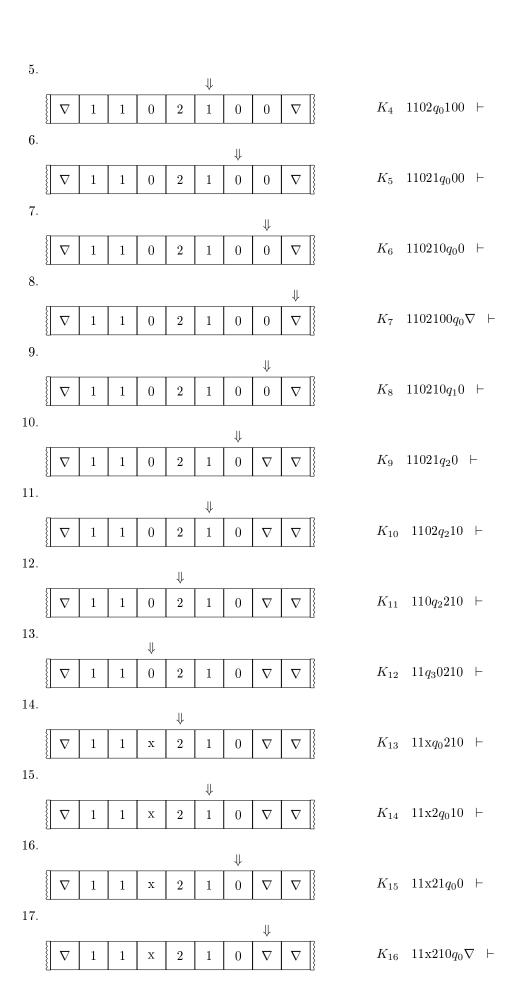
3.

_			\Downarrow					
∇	1	1	0	2	1	0	0	∇

$$K_2 \quad 11q_002100 \quad \vdash$$

_				\Downarrow				
∇	1	1	0	2	1	0	0	∇

$$K_3$$
 110 q_0 2100 \vdash



18. ∇ K_{17} $11x21q_10 \vdash$ 19. K_{18} $11x2q_21$ \vdash 1 \mathbf{X} 2 ∇ ∇ $\nabla \parallel$ 20. K_{19} $11 \times q_2 21$ \vdash 21. ∇ K_{20} 11 q_3 x21 \vdash ∇ 2 1 ∇ ∇ 22. 23. 2 24.

 K_{21} $1q_31$ x21 \vdash K_{22} 1y q_0 x21 \vdash

 K_{23} 1yx q_0 21 \vdash

25. K_{24} 1yx2 q_0 1 \vdash Х ∇

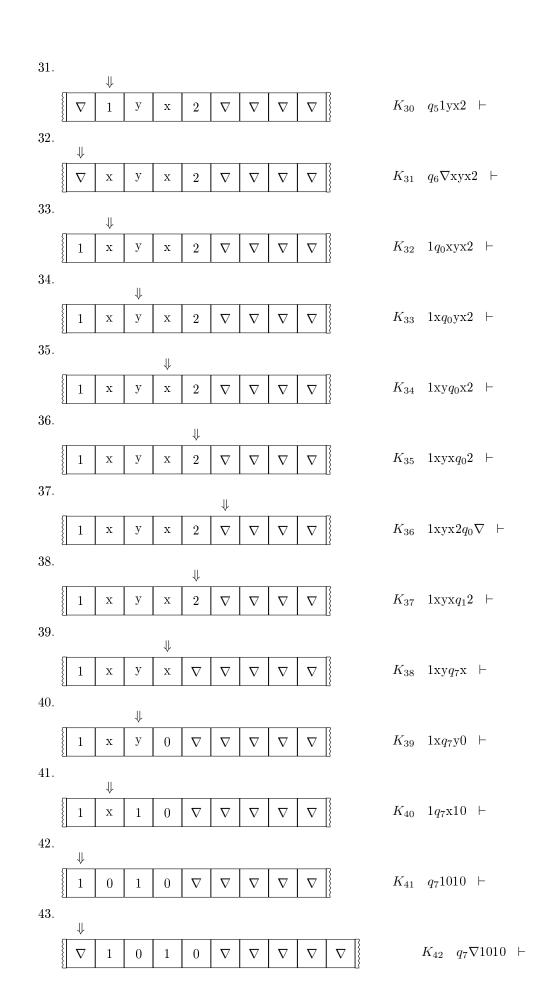
26. K_{25} 1yx21 $q_0\nabla$ \vdash

27. K_{26} 1yx2 q_1 1 \vdash

28. K_{27} 1yx q_4 2 \vdash

29. K_{28} 1y q_5 x2 \vdash

30. K_{29} $1q_5$ yx2 \vdash ∇ ∇

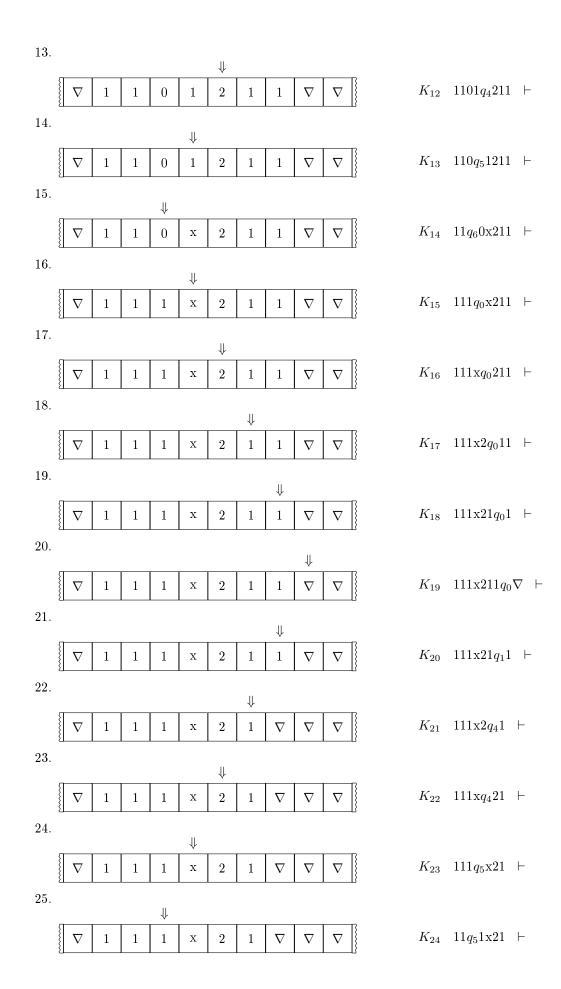


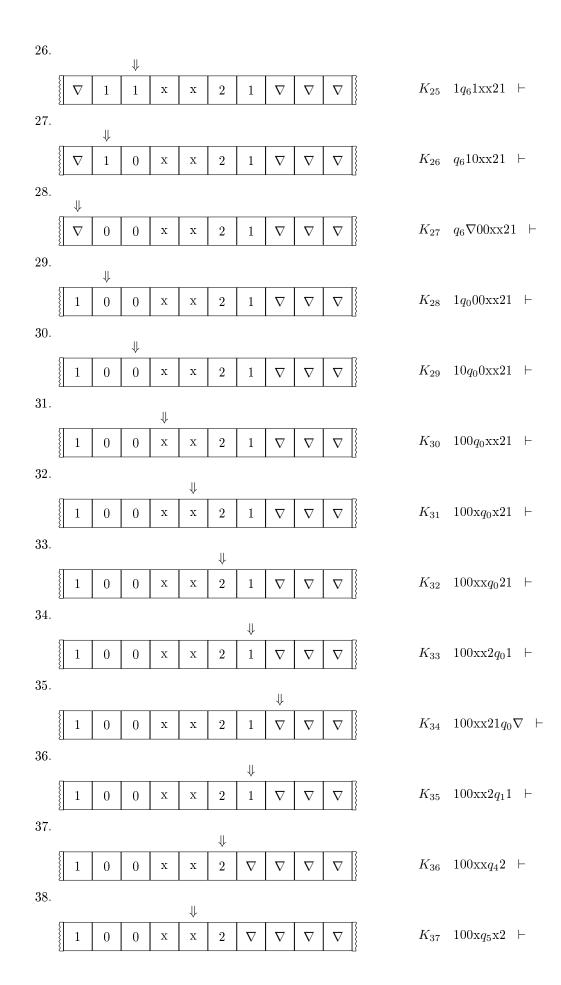
44. K_{43} q_81010 ∇ 2.1.2 f(13, 7) 1. $\nabla \parallel$ K_0 $q_011012111$ \vdash ∇ 1 2. K_1 1 q_0 1012111 \vdash ∇ ∇ 3. K_2 11 q_0 012111 \vdash ∇ 4. ∇ K_3 110 q_0 12111 \vdash 0 1 2 5. ∇ K_4 1101 q_0 2111 \vdash 2 1 6. 2 ∇ K_5 11012 q_0 111 \vdash ∇ 1 1 1 7. ∇ K_6 110121 q_0 11 \vdash 1 2 1 1 1 8. K_7 1101211 q_0 1 \vdash ∇ 2 1 ∇ 1 0 1 1 1 9. ∇ K_8 11012111 $q_0\nabla$ \vdash 2 10.

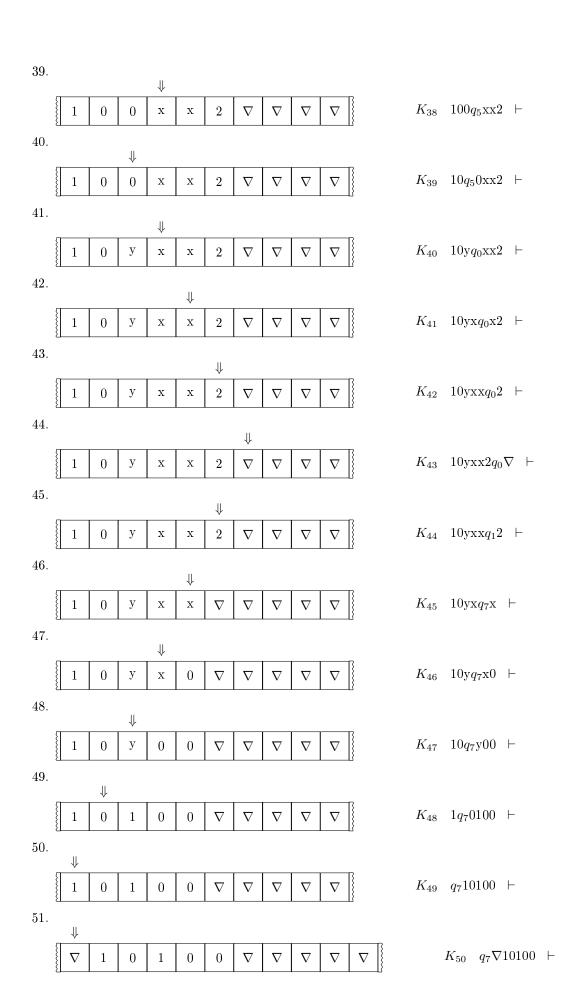
 ∇ K_9 1101211 q_1 1 \vdash 1 0 1 2 11. K_{10} 110121 q_4 1 \vdash $\nabla \parallel$

2

12. ∇ K_{11} 11012 q_4 11 \vdash 2 ∇







		\Downarrow											
-	∇	1	0	1	0	0	∇	∇	∇	∇	∇	K_{51}	q_810100

3 Maszyny Turinga akceptujące języki

3.1 Zadanie 1.19

Zaprojektuj maszynę Turinga, która kopiuje wejściowy łańcuch w dla alfabetu $\Sigma = \{a,b\}$. Rozwiązanie może nie zawierać separatora

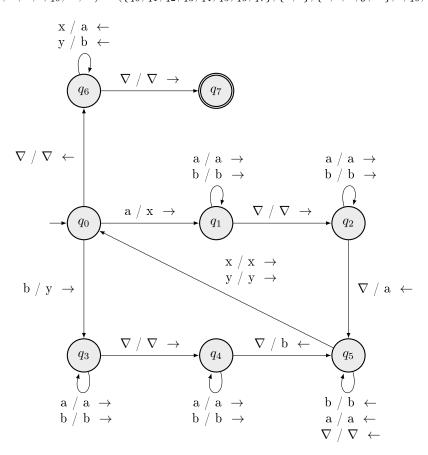
$$q_0w \stackrel{*}{\vdash} q_fww$$

lub może zawierać dowolny separator, na przykład separatorem może być blank, czyli

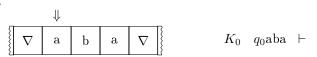
$$q_0w \stackrel{*}{\vdash} q_f w \nabla w$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(27) M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{a, b, x, y, \nabla\}, \delta, q_0, \nabla, \{q_7\})$$



3.1.1 aba



2. K_1 $\mathbf{x}q_1\mathbf{ba}$ \vdash 3. $K_2 \quad xbq_1a \quad \vdash$ ∇ x b 4. $K_3 \quad \text{xba} q_1 \nabla \quad \vdash$ 5. ∇ ∇ b 6.

 $K_4 \operatorname{xba} \nabla q_2 \nabla \vdash$

 $K_5 \quad \text{xba} q_5 \nabla \text{a} \quad \vdash$ 7.

 $K_6 \operatorname{xb} q_5 \operatorname{a} \nabla \operatorname{a} \vdash$

8. $K_7 \quad \mathbf{x}q_5\mathbf{ba}\nabla\mathbf{a} \quad \vdash$

9. K_8 q_5 xba ∇ a \vdash

10. $K_9 \quad \mathbf{x}q_0\mathbf{b}\mathbf{a}\nabla\mathbf{a} \quad \vdash$

11. K_{10} xy q_3 a ∇ a \vdash

12. K_{11} xya $q_3\nabla a$ \vdash

13. K_{12} xya ∇q_4 a \vdash

14. K_{13} xya ∇ a $q_4\nabla$ \vdash ∇

15. K_{14} xya ∇q_5 ab \vdash 16. K_{15} xya $q_5\nabla$ ab \vdash y ∇ a b 17. K_{16} xy q_5 a ∇ ab \vdash 18. K_{17} x q_5 ya ∇ ab \vdash у ∇ a ∇ b 19. K_{18} xy q_0 a ∇ ab \vdash 20. у \mathbf{X} b 21.

 $K_{19} \quad \text{xyx} q_1 \nabla \text{ab} \quad \vdash$

 K_{20} xyx ∇q_2 ab \vdash х

22. K_{21} xyx ∇aq_2b \vdash b

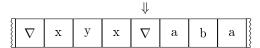
23. $K_{22} \quad \text{xyx} \nabla \text{ab} q_2 \nabla \quad \vdash$ ∇

24. K_{23} xyx ∇aq_5 ba \vdash

25. K_{24} xyx ∇q_5 aba \vdash

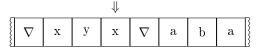
26. K_{25} xyx $q_5\nabla$ aba \vdash

27. K_{26} xy q_5 x ∇ aba \vdash



 $K_{27} \quad \text{xyx} q_0 \nabla \text{aba} \quad \vdash$

29.



 K_{28} xy q_6 x ∇ aba \vdash

30.

		\Downarrow					
∇	x	у	a	∇	a	b	a

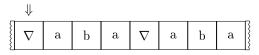
 K_{29} x q_6 ya ∇ aba \vdash

31.

	\Downarrow							
∇	х	b	a	∇	a	b	a	

 K_{30} q_6 xba ∇ aba \vdash

32.



$$K_{31}$$
 $q_6\nabla aba\nabla aba$ \vdash

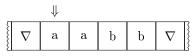
33.

_	\Downarrow						
∇	a	b	a	∇	a	b	a

 K_{32} q_7 aba ∇ aba

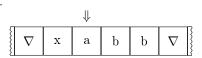
3.1.2 aabb

1.



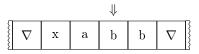
 K_0 q_0 aabb \vdash

2.



 K_1 $\times q_1$ abb \vdash

3.



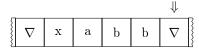
 $K_2 \quad xaq_1bb \quad \vdash$

4.

				\Downarrow	
∇	x	a	b	b	∇

 $K_3 \operatorname{xab} q_1 \mathbf{b} \vdash$

5.



 $K_4 \quad \mathrm{xabb} q_1 \nabla \quad \vdash$

6.

						\Downarrow	
∇	х	a	b	b	∇	∇	

$$K_5 \quad \mathrm{xabb} \nabla q_2 \nabla \quad \vdash$$

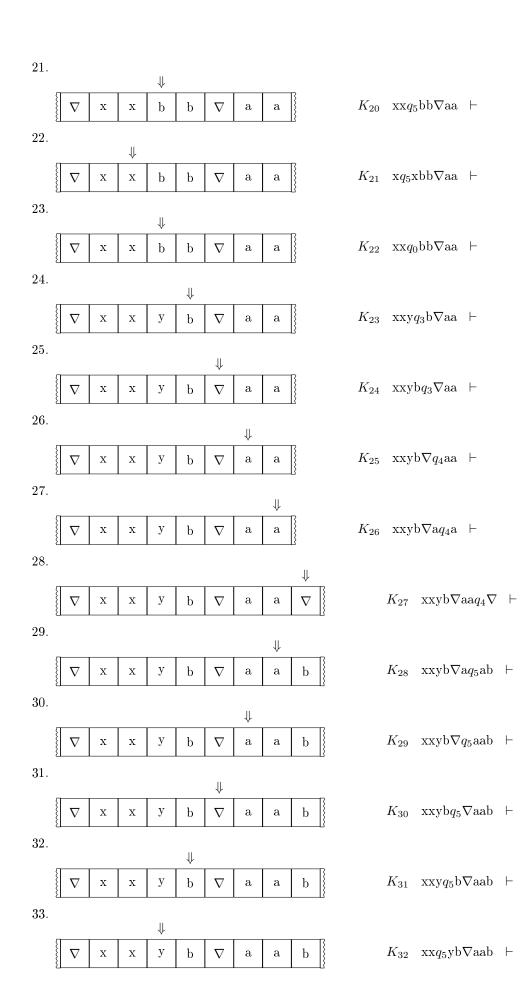
					\Downarrow	
∇	X	a	b	b	∇	a

$$K_6 \quad \text{xabb} q_5 \nabla \text{a} \quad \vdash$$

8. $K_7 \operatorname{xab} q_5 \mathrm{b} \nabla \mathrm{a} \vdash$ ∇ 9. $K_8 \quad xaq_5bb\nabla a \vdash$ x a b ∇ 10. $K_9 \quad \mathbf{x}q_5\mathbf{a}\mathbf{b}\mathbf{b}\nabla\mathbf{a} \quad \vdash$ 11. K_{10} q_5 xabb ∇ a \vdash ∇ ∇ \mathbf{a} b b 12. K_{11} $\mathbf{x}q_0\mathbf{a}\mathbf{b}\mathbf{b}\nabla\mathbf{a}$ \vdash 13. $K_{12} \quad \mathbf{x} \mathbf{x} q_1 \mathbf{b} \mathbf{b} \nabla \mathbf{a} \vdash$ x X ∇ b 14. K_{13} xxb q_1 b ∇ a \vdash X ∇ 15. $K_{14} \quad \text{xxbb} q_1 \nabla \mathbf{a} \quad \vdash$ X ∇ a 16. \Downarrow K_{15} xxbb ∇q_2 a \vdash ∇ a 17. $K_{16} \quad \text{xxbb} \nabla a q_2 \nabla \quad \vdash$ \mathbf{a} 18. K_{17} xxbb ∇q_5 aa \vdash ∇ 19. K_{18} xxbb $q_5\nabla$ aa \vdash 20.

 ∇

 $K_{19} \operatorname{xxb} q_5 \mathrm{b} \nabla \mathrm{aa} \vdash$



34. K_{33} xxy q_0 b ∇ aab \vdash ∇ b 35. K_{34} xxyy $q_3\nabla$ aab \vdash у ∇ X ∇ a a b 36. K_{35} xxyy ∇q_4 aab \vdash a 37.y y K_{36} xxyy ∇aq_4ab \vdash ∇ X ∇ a a b 38. K_{37} xxyy ∇ aa q_4 b \vdash b 39. K_{38} xxyy ∇ aab $q_4\nabla$ \vdash у ∇ a X b 40. K_{39} xxyy ∇ aa q_5 bb \vdash 41. K_{40} xxyy ∇aq_5abb \vdash ∇ X a b 42. K_{41} xxyy ∇q_5 aabb \vdash ∇ \mathbf{a} 43. K_{42} xxyy $q_5\nabla$ aabb \vdash \mathbf{a} 44. K_{43} xxy q_5 y ∇ aabb \vdash ∇ 45. K_{44} xxyy $q_0\nabla$ aabb \vdash 46. K_{45} xxy q_6 y ∇ aabb \vdash ∇ a b

47.											
			\Downarrow								
∇	X	х	у	b	∇	a	a	b	b	K_{46}	$xxq_6yb\nabla aabb \vdash$
48.											
		\Downarrow									
	x	X	b	b	∇	a	a	b	b	K_{47}	$xq_6xbb\nabla aabb \vdash$
49.											
49.	\Downarrow										
∇	х	a	b	b	∇	a	a	b	b	K_{48}	q_6 xabb ∇ aabb \vdash
F0											
50. ↓											
∇	a	a	b	b	∇	a	a	b	b	K_{49}	$q_6\nabla$ aabb ∇ aabb \vdash
51.	п										
_		,									
V	a	a	b	b	∇	a	a	b	b	K_{50}	q_7 aabb ∇ aabb

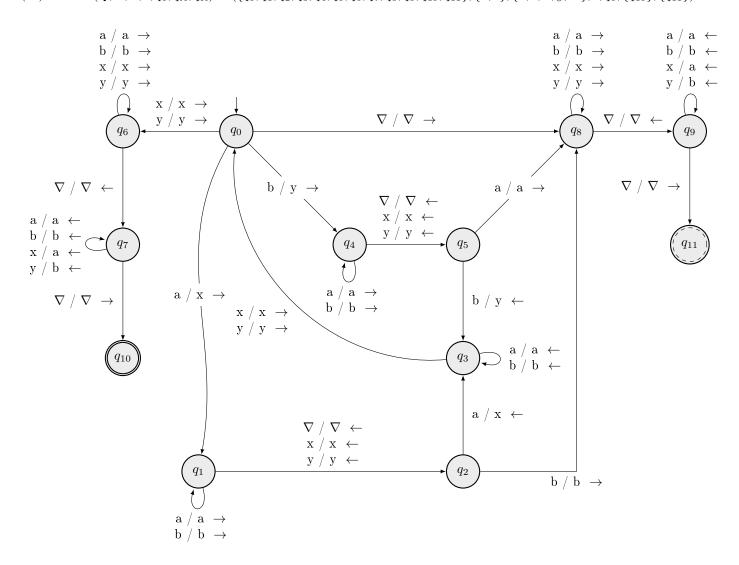
3.2 Zadanie 1.20

Zaprojektuj maszynę Turinga, która akceptuje język

$$(28) L = \{\omega\omega : \omega \in \{a, b\}^+\}$$

nad alfabetem $\Sigma = \{a, b\}$. Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(29) \quad M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_8, q_{10}, q_{11}\}, \{a, b\}, \{a, b, x, y, \nabla\}, \delta, q_0, \{q_{10}\}, \{q_{11}\})$$



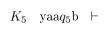
3.2.1 baab

 K_0 q_0 baab \vdash

 K_1 y q_4 aab \vdash

 K_2 ya q_4 ab \vdash

4. ∇ 5. ∇ a a 6. 7. y ∇ ∇ 8.



 K_6 ya q_3 ay \vdash

 K_3 yaa q_4 b \vdash

 K_4 yaab $q_4\nabla$ \vdash

$$K_7$$
 y q_3 aay \vdash

9.

$$K_8$$
 q_3 yaay \vdash

10.

$$K_9$$
 y q_0 aay \vdash

11.

$$K_{10}$$
 yx q_1 ay \vdash

12.

$$K_{11}$$
 yxa q_1 y \vdash

13.

$$K_{12}$$
 yx q_2 ay \vdash

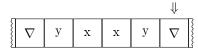
14.

$$K_{13}$$
 yq_3xxy \vdash

15.

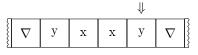
$$K_{14}$$
 yx q_0 xy \vdash

$$K_{15}$$
 $yxxq_6y$ \vdash



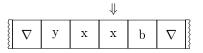
 K_{16} yxxy $q_6\nabla$ \vdash

18.



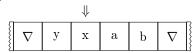
 K_{17} $yxxq_7y$ \vdash

19.



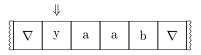
$$K_{18}$$
 yx q_7 xb \vdash

20.



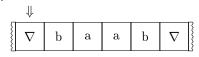
$$K_{19}$$
 y q_7 xab \vdash

21.



$$K_{20}$$
 q_7 yaab \vdash

22.



$$K_{21}$$
 $q_7 \nabla \text{baab} \vdash$

23.



$$K_{22}$$
 q_{10} baab

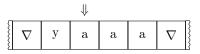
3.2.2 baaa

1.



$$K_0$$
 q_0 baaa \vdash

2.



$$K_1$$
 y q_4 aaa \vdash

3.

			\Downarrow		
∇	у	a	a	a	∇

$$K_2$$
 ya q_4 aa \vdash

4.

				\Downarrow	
∇	у	a	a	a	∇

$$K_3$$
 yaa q_4 a \vdash

5.

					\Downarrow	
∇	у	a	a	a	∇	000000

$$K_4$$
 yaaa $q_4\nabla$ \vdash

$$K_5$$
 yaa q_5 a \vdash

 ∇ y a a a ∇ K_8 ya q_9 aa \vdash 10.

 ∇

a

 K_9 y q_9 aaa \vdash

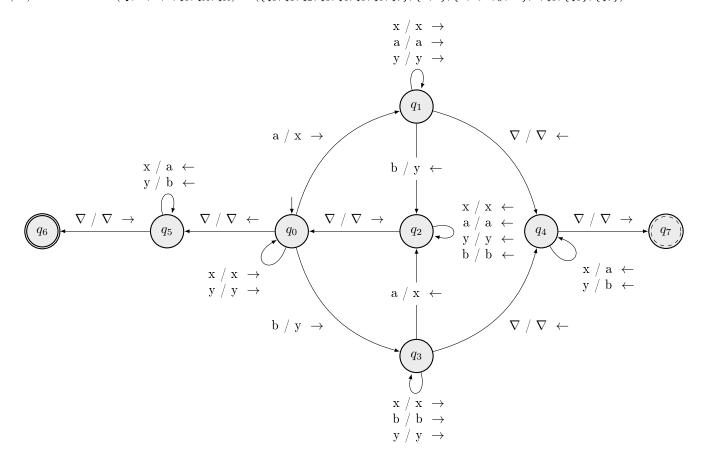
3.3 Zadanie 1.22

Zaprojektuj maszynę Turinga nad alfabetem $\Sigma = \{a, b\}$, która akceptuje język

(30)
$$L = \{\omega : \omega \text{ zawiera równą liczbę symboli a i b} \}$$

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(31) M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{a, b, x, y, \nabla\}, \delta, q_0, \{q_6\}, \{q_7\})$$



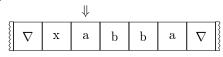
3.3.1 aabba

1.

	\Downarrow						
∇	a	a	b	b	a	∇	3

$$K_0$$
 q_0 aabba \vdash

2.



$$K_1$$
 $\times q_1$ abba \vdash

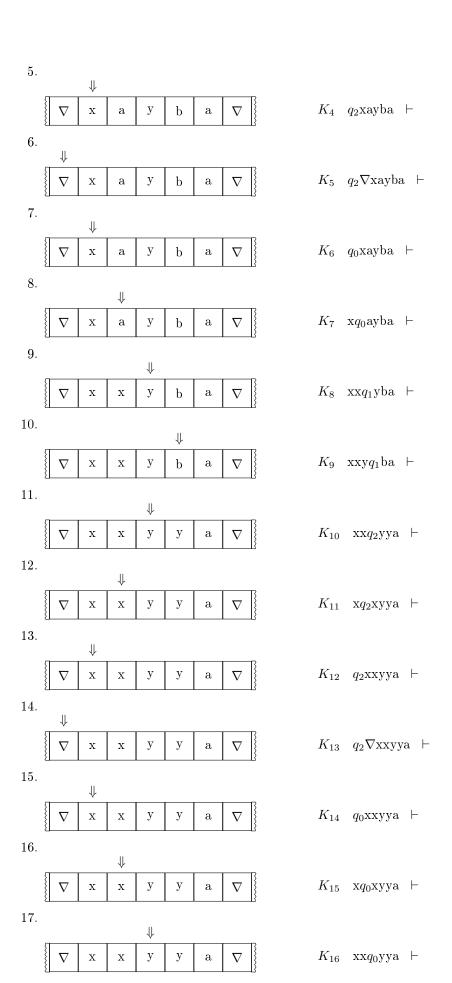
3.

			\Downarrow				
 ∇	x	a	b	b	a	∇	~~~

$$K_2 \quad xaq_1bba \quad \vdash$$

$$\begin{array}{|c|c|c|c|c|c|}\hline & & & & & \\ \hline \hline & \nabla & x & a & y & b & a & \nabla \\ \hline \end{array}$$

$$K_3$$
 $\times q_2$ ayba \vdash



18. K_{17} xxy q_0 ya \vdash 19. K_{18} xxyy q_0 a \vdash у X a ∇ 20. ∇

 K_{19} $xxyyxq_1\nabla$ \vdash 21.

 K_{20} xxyy q_4 x \vdash у ∇ 22.

 K_{21} xxy q_4 ya \vdash y ∇

23. K_{22} xx q_4 yba \vdash y b ∇

24. K_{23} $\times q_4 \times bba$ \vdash

25. K_{24} q_4 xabba \vdash

26. K_{25} $q_4\nabla$ aabba \vdash

27. K_{26} q_7 aabba a b

3.3.2 abbbaa

1. K_0 q_0 abbbaa \vdash ∇ 2. K_1 xq_1 bbbaa \vdash 3.

 K_2 q_2 xybbaa \vdash

4.											
1.	{∏	1	1					T [}		- 11
		X	У	b	b	a	a	∇	}	K_3	$q_2\nabla xybbaa \vdash$
5.		.							2		
	∇	Х	У	b	b	a	a	∇	}	K_4	q_0 xybbaa \vdash
6.			\Downarrow								
	∇	X	у	b	b	a	a	∇	}	K_5	$xq_0ybbaa \vdash$
7.			•	11		•			1		
	∇	X	у	↓ b	b	a	a	∇	}	K_6	xyq_0 bbaa \vdash
8.	<u> </u>								}		
	{ \	x	у	у	↓	a		∇	}	K_7	$xyyq_3$ baa \vdash
9.			<u> </u>	y	b	a	a	∇	}	ιιγ	хуудзыа
9.	Л	1	ı			#		1 [₹		
	∇	X	У	У	b	a	a	∇		K_8	$xyybq_3aa \vdash$
10.					\Downarrow						
	∇	X	у	у	b	х	a	∇]	K_9	$xyyq_2bxa \vdash$
11.				\Downarrow							
	∇	х	у	у	b	х	a	∇	}	K_{10}	$xyq_2ybxa \vdash$
12.	4								3		
	∇	x	<u>↓</u>	y	b	X	a	∇	}	K_{11}	$xq_2yybxa \vdash$
13.	{I	<u> </u>							}		
	1	 	1,	77	,			I . [}	IZ.	a www.brro
1.4		Х	У	У	b	Х	a	∇	}	Λ_{12}	q_2 xyybxa \vdash
14.	↓	1	ĭ						₹		
	∇	Х	у	у	b	х	a	∇	}	K_{13}	$q_2\nabla xyybxa \vdash$
15.		\Downarrow									
	∇	X	у	у	b	х	a	∇	}	K_{14}	q_0 xyybxa \vdash
16.											
	∇	x	у	у	b	x	a	∇	}	K_{15}	$xq_0yybxa \vdash$
	<u>1</u>		I			L			}		

17. $\nabla \parallel$ K_{16} xy q_0 ybxa \vdash 18. K_{17} xyy q_0 bxa \vdash у X ∇ 19. K_{18} xyyy q_3 xa \vdash X 20. K_{19} xyyyx q_3 a \vdash у y y ∇ X 21. K_{20} xyyy q_2 xx \vdash ∇ 22. K_{21} xyy q_2 yxx \vdash у X 23. K_{22} xyq_2yyxx \vdash 24. K_{23} $\times q_2 yyyxx$ \vdash 25. K_{24} q_2 xyyyxx \vdash 26. K_{25} $q_2 \nabla xyyyxx \vdash$ 27. K_{26} q_0 xyyyxx \vdash 28. K_{27} $\times q_0 yyyxx$ \vdash 29.

 K_{28} xyq_0yyxx \vdash



 K_{30} xyyy q_0 xx \vdash

y

y

X

 ∇

 K_{31} xyyyx q_0 x \vdash

 K_{32} xyyyxx $q_0\nabla$ \vdash

 K_{33} xyyyx q_5 x \vdash

 K_{34} xyyy q_5 xa \vdash

 K_{35} xyy q_5 yaa \vdash

 K_{36} xy q_5 ybaa \vdash

 K_{37} x q_5 ybbaa \vdash

 K_{38} q_5 xbbbaa \vdash

 K_{39} $q_5\nabla$ abbbaa \vdash

 K_{40} q_6 abbbaa

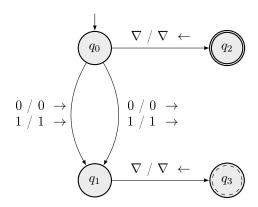
3.4 Zadanie 1.23

Zaprojektuj maszynę Turinga, która akceptuje język

(32)
$$L = \{\omega : |\omega| \text{ jest parzysta}\}\$$

nad alfabetem $\Sigma = \{0, 1\}$. Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

(33)
$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R) = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, \nabla\}, \delta, q_0, \{q_2\}, \{q_3\})$$



3.4.1 100111

1.

	\Downarrow						
∇	1	0	0	1	1	1	∇

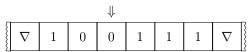
$$K_0 \quad q_0 100111 \quad \vdash$$

2.

			\Downarrow						
لسسم	∇	1	0	0	1	1	1	∇	7

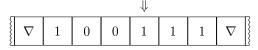
$$K_1 \quad 1q_100111 \quad \vdash$$

3.



$$K_2 \quad 10q_00111 \quad \vdash$$

4.



$$K_3 \quad 100q_1111 \quad \vdash$$

5.

					\Downarrow		
∇	1	0	0	1	1	1	∇

$$K_4 \quad 1001q_011 \quad \vdash$$

6.

							\Downarrow	
*	∇	1	0	0	1	1	1	∇

$$K_5 \quad 10011q_11 \quad \vdash$$

								\Downarrow	
~~~	$\nabla$	1	0	0	1	1	1	$\nabla$	

$$K_6 \quad 100111q_0 \nabla \quad \vdash$$

						$\Downarrow$				
 $\nabla$	1	0	0	1	1	1	$\nabla$	}	$K_7$	$10011q_21$

## 3.4.2 11011

1.

	$\Downarrow$					
$\nabla$	1	1	0	1	1	$\nabla$

 $K_0$   $q_011011$   $\vdash$ 

2.

		$\Downarrow$				
$\nabla$	1	1	0	1	1	$\nabla$

 $K_1$   $1q_11011$   $\vdash$ 

3.

•			$\Downarrow$			
$\nabla$	1	1	0	1	1	$\nabla$

 $K_2$  11 $q_0$ 011  $\vdash$ 

4.

				$\Downarrow$		
$\nabla$	1	1	0	1	1	$\nabla$

 $K_3$  110 $q_1$ 11  $\vdash$ 

5.

					$\Downarrow$	
$\nabla$	1	1	0	1	1	$\nabla$

 $K_4$  1101 $q_0$ 1  $\vdash$ 

6.

							$\Downarrow$	
-	$\nabla$	1	1	0	1	1	$\nabla$	}  }

 $K_5$  11011 $q_1\nabla$   $\vdash$ 

7.

					₩	
$\nabla$	1	1	0	1	1	$\nabla$

 $K_6 \quad 1101q_31$ 

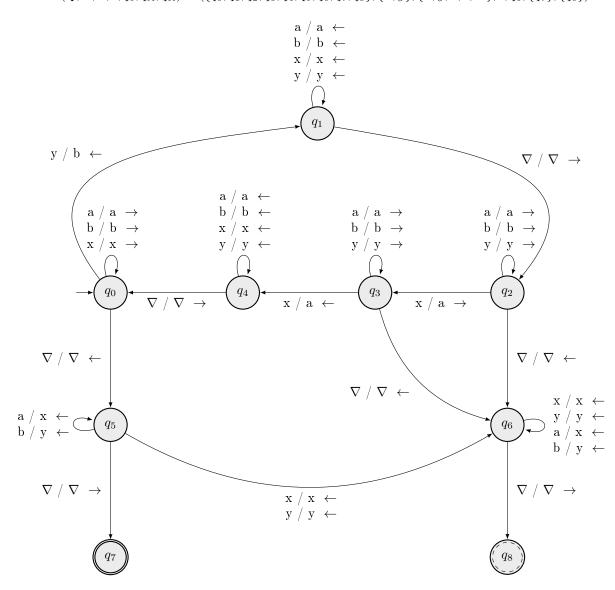
### 3.5 Zadanie 1.24

Zaprojektuj maszynę Turinga nad alfabetem  $\Sigma = \{x, y\}$ , która akceptuje język

(34) Maszyna Turinga akceptująca język  $L = \{\omega : |\omega| \text{ ma dwa razy wiecej symboli } x \text{ niz } y\}$ 

Narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

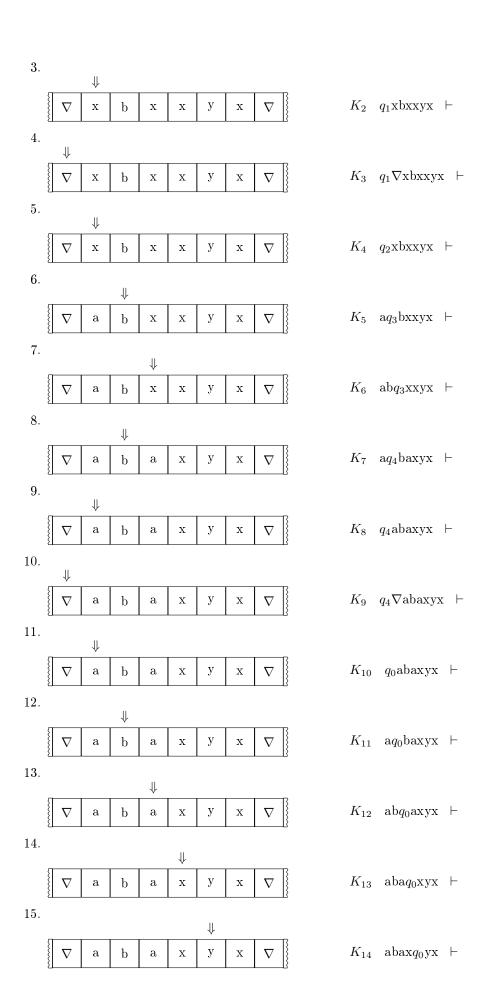
$$(35) \hspace{1cm} M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}, \{x, y\}, \{x, y, a, b, \nabla\}, \delta, q_0, \{q_7\}, \{q_8\})$$

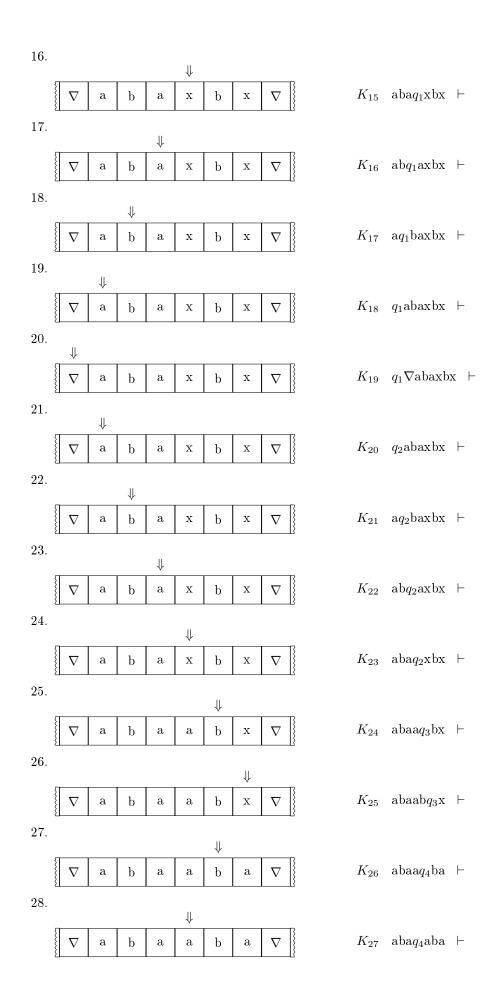


### 3.5.1 xyxxyx

$$K_0$$
  $q_0$ xyxxyx  $\vdash$ 

$$K_1$$
  $\times q_0 y \times x y \times \vdash$ 





29.				<b></b>								
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{28}$	$abq_4aaba$	$\vdash$
30.			<b></b>									
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{29}$	$aq_4$ baaba	$\vdash$
31.	<u> </u>						I	<u> </u>	<b>?</b>			
	$\nabla$	↓ a	b	a	a	b	a	$\nabla$	}	$K_{30}$	$q_4$ abaaba	⊢
32.	{ <b>]</b> •							[}	}	30	11	
<u> </u>	₩							T _ 18	}	17	<del></del>	
2.2	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{31}$	$q_4\nabla$ abaab	a ⊢
33.		<b></b>					·		,			
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{32}$	$q_0$ abaaba	$\vdash$
34.			$\Downarrow$									
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{33}$	$aq_0$ baaba	$\vdash$
35.				<b>.</b>								
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{34}$	$\mathrm{ab}q_0\mathrm{aaba}$	$\vdash$
36.	<u> </u>					l	<u> </u>	1 13	}			
	$\nabla$	a	b	a	<b>↓</b> a	b	a	$\nabla$		$K_{35}$	aba $q_0$ aba	⊢
37.			L D			_ D		$\nabla$		1135	abaqqaba	'
91.	<u>η</u>					<b>#</b>	ı	T B	}			
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{36}$	abaa $q_0$ ba	F
38.	_						<b></b>					
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{37}$	abaab $q_0$ a	$\vdash$
39.								<b></b>				
	$\nabla$	a	b	a	a	b	a	$\nabla$		$K_{38}$	abaaba $q_0$	7 -
40.						•						
	$\nabla$	a	b	a	a	b	↓ a	$\nabla$		$K_{39}$	abaab $q_5$ a	$\vdash$
41.	<u> </u>						<u> </u>	1 3	?			
		9	l h	9	9	\\ \  \  \  \  \  \  \  \  \  \  \  \  \		$\nabla$		$K_{40}$	${ m abaa}q_5{ m bx}$	⊢
	$\mid \nabla \mid$	a	b	a	a	b	x			<b>41</b> 40	avaaq5 DX	1

42.

\[ \nabla \quad \text{a b a a y x } \nabla \]

43.

\[ \nabla \quad \text{a b a x y x } \nabla \]

44.

 $K_{43}$  a $q_5$ bxxyx  $\vdash$ 

 $K_{44}$   $q_5$ ayxxyx  $\vdash$ 

 $K_{41}$  aba $q_5$ ayx  $\vdash$ 

 $K_{42}$  ab $q_5$ axyx  $\vdash$ 

y

 $\nabla$ 

 $K_{46}$   $q_7$ xyxxyx

 $\mathbf{3.5.2} \quad \mathbf{xxyyyy}$ 

 $K_0$   $q_0$ xxyyyy  $\vdash$ 

 $K_1$   $\times q_0 \times y \times y \times \vdash$ 

 $K_2 \quad xxq_0yyyy \quad \vdash$ 

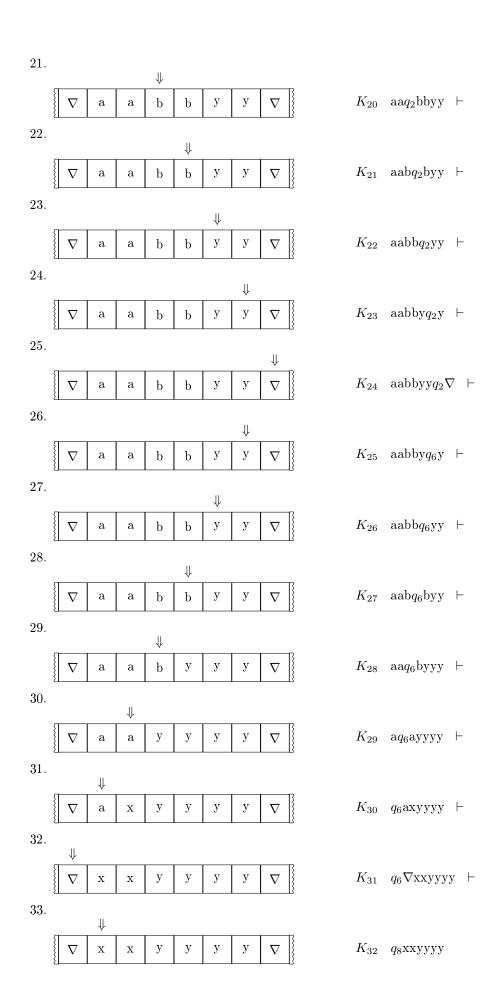
 $K_3$   $\times q_1 \times \text{byyy}$   $\vdash$ 

 $K_4$   $q_1$ xxbyyy  $\vdash$ 

 $K_5 \quad q_1 \nabla x x b y y y \vdash$ 

 $K_6$   $q_2$ xxbyyy  $\vdash$ 

8.											
	$\nabla$	a	<b>↓</b>	b	у	у	у	$\nabla$	$K_7$	$aq_3x$ byyy	⊢
9.	{ ·			.5				<u> </u>	•	10 000	
	<u> </u>	<del> </del>		,			T	T _ R	TZ.	1.	
	$\nabla$	a	a	b	У	У	У	$oxed{\nabla}$	$\kappa_8$	$q_4$ aabyyy	Η
10.	<b>#</b>						,				
	$\nabla$	a	a	b	у	у	У		$K_9$	$q_4\nabla$ aabyyy	·  -
11.		$\Downarrow$									
	$\nabla$	a	a	b	у	у	у	$\nabla$	$K_{10}$	$q_0$ aabyyy	$\vdash$
12.	ч							<u> </u>			
	$\nabla$	a	↓ a	b	у	у	у	$\nabla$	$K_{11}$	$aq_0abyyy$	⊢
13.	{  <b>v</b>			О		<u> </u>		_ <b>'</b> _}}	11	10 333	
10.	{I			<b></b>		ı		T B		,	
	$\nabla$	a	a	b	У	У	У		$K_{12}$	$aaq_0$ byyy	⊢
14.	_				$\Downarrow$			_			
	$\nabla$	a	a	b	у	у	у		$K_{13}$	$\mathrm{aab}q_0$ yyy	$\vdash$
15.				$\Downarrow$							
	$\nabla$	a	a	b	b	у	у	$\nabla$	$K_{14}$	$aaq_1bbyy$	$\vdash$
16.	υ	l				l	I	<u> </u>			
	$\nabla$	a	↓ a	b	b	у	у	$\nabla$	$K_{15}$	$aq_1abbyy$	⊢
17.	{  <b>v</b>			Б				<b>v</b>  }	15	aqrassyy	
<b></b>	Я	<b>#</b>				1	1	ТВ			
		a	a	b	b	У	У		$K_{16}$	$q_1$ aabbyy	⊢
18.											
	$\nabla$	a	a	b	b	у	у	$\nabla$	$K_{17}$	$q_1\nabla$ aabby	y F
19.		<b></b>									
	$\nabla$	a	a	b	b	у	у	$\nabla$	$K_{18}$	$q_2$ aabbyy	⊢
20.	Ъ	I			I	I	I	1 13			
	{ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	e e	<b>↓</b>	h	h	у	у	$\nabla$	<i>K</i>	$\mathrm{a}q_2\mathrm{abbyy}$	⊢
	$\nabla$	a	a	b	b	^y	^y		11 19	ayzanny y	1



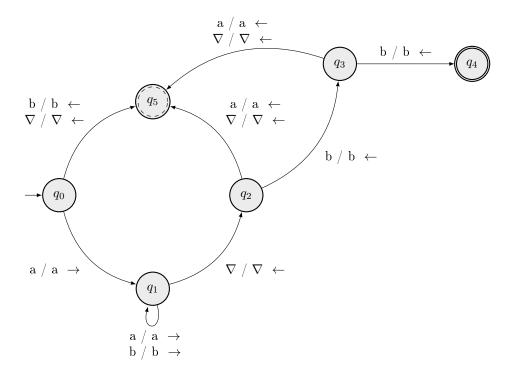
# 4 Maszyny Turinga i wyrażenia regularne

## 4.1 Zadanie 1.46

Wypisz cztery przykładowe łańcuchy opisywane przez wyrażenie  $\mathbf{a}(\mathbf{a}+\mathbf{b})^*\mathbf{b}\mathbf{b}$ . Czy można skonstruować (deterministyczną) maszynę Turinga, która akceptuje język

$$(36) L = L(a(a+b)^*bb)$$

$$(37) M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R) = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \{a, b, \nabla\}, \delta, q_0, \{q_4\}, \{q_5\})$$

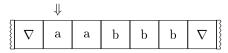


### 4.1.1 Przykładowe łańcuchy

- 1. abb
- 2. aabb
- 3. abbb
- 4. aabbb

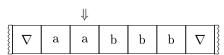
#### 4.1.2 aabbb

1.



$$K_0$$
  $q_0$ aabbb  $\vdash$ 

2.



$$K_1$$
 a $q_1$ abbb  $\vdash$ 

				$\downarrow$			
20000	$\nabla$	a	a	b	b	b	$\nabla$

$$K_2$$
 aa $q_1$ bbb  $\vdash$ 

4. $K_3$  aab $q_1$ bb  $\vdash$ 5.  $K_4$  aabb $q_1$ b  $\vdash$ b a b  $\nabla$ 6.  $K_5$  aabbb $q_1\nabla$   $\vdash$  $\nabla$ 7.  $K_6$  aabb $q_2$ b  $\vdash$ b b b  $\nabla \mid$ 8.  $K_7$  aab $q_3$ bb  $\vdash$  $\nabla$ 9.

 $K_8$  aa $q_4$ bbb b b  $\nabla$ 

## 4.1.3 aaaba

1.  $K_0$   $q_0$ aaaba  $\vdash$  $\nabla \parallel$ 2.  $K_1$  a $q_1$ aaba  $\vdash$  $\nabla$ 

3.  $K_2$  aa $q_1$ aba  $\vdash$  $\nabla \parallel$ 

4.  $K_3$  aaa $q_1$ ba  $\vdash$  $\nabla$ 

5.  $K_4$  aaab $q_1$ a  $\vdash$  $\nabla$ 

6.  $K_5$  aaaba $q_1\nabla$   $\vdash$ 

7.  $\nabla$  $K_6$  aaab $q_2$ a  $\vdash$ 

				$\Downarrow$				
$\nabla$	a	a	a	b	a	$\nabla$	$K_7$	aaa $q_5$ ba

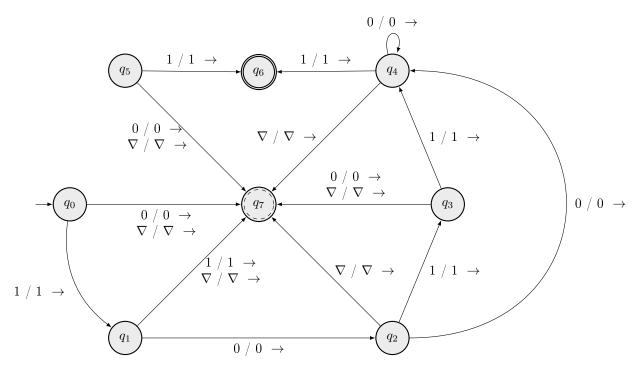
#### 4.2 Zadanie 1.47

Wypisz cztery przykładowe łańcuchy opisywane przez wyrażenie  $\mathbf{10} + (\mathbf{0} + \mathbf{11})\mathbf{0}^*\mathbf{1}$ . Czy można skonstruować (deterministyczną) maszynę Turinga, która akceptuje język

$$(38) L = L(10 + (0+11)0^*1)$$

Jeżeli można, to narysuj diagram przejść i dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśmy i zapisz konfiguracje).

$$(39) \hspace{3.1em} M=(Q,\Sigma,\Gamma,\delta,q_0,q_A,q_R)=(\{q_0,q_1,q_2,q_3,q_4,q_5,q_6,q_7\},\{1,0\},\{1,0,\nabla\},\delta,q_0,\{q_6\},\{q_7\})$$

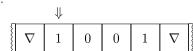


#### 4.2.1 Przykładowe łańcuchy

- 1. 1001
- 2. 10111
- 3. 100001
- 4. 101101

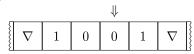
#### 4.2.2 1001

1.



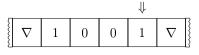
$$K_0 \quad q_0 1001 \quad \vdash$$

$$K_1$$
  $1q_1001$   $\vdash$ 



 $K_2$  10 $q_2$ 01  $\vdash$ 

4.



 $K_3 \quad 100q_41 \quad \vdash$ 

5.

					$\Downarrow$
$\nabla$	1	0	0	1	$\nabla$

 $K_4$  1001 $q_6\nabla$ 

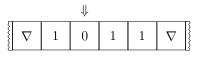
## 4.2.3 1011

1.



 $K_0$   $q_01011$   $\vdash$ 

2.



 $K_1$   $1q_1011$   $\vdash$ 

3.

			$\Downarrow$		
$\nabla$	1	0	1	1	$\nabla$

 $K_2 \ 10q_211 \ \vdash$ 

4.

				₩	
$\nabla$	1	0	1	1	$\nabla$

 $K_3$  101 $q_3$ 1  $\vdash$ 

5.

					$\Downarrow$	
$\nabla$	1	0	1	1	$\nabla$	3

 $K_4$  1011 $q_4\nabla$   $\vdash$ 

6.

						$\Downarrow$	
 $\nabla$	1	0	1	1	$\nabla$	$\nabla$	~~~~

 $K_5$   $1011\nabla q_7\nabla$ 

# 5 Zadania z wielotaśmowej maszyny Turinga

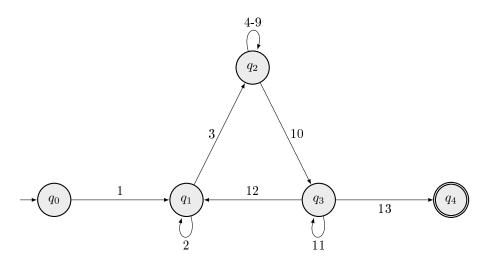
## 5.1 Zadanie 1.51

Używając unarnej reprezentacji liczb, zaprojektuj wielotaśmową maszynę Turinga, która oblicza funkcję

$$(40) f(n) = \lceil \log_2 n \rceil.$$

Podaj tabelę przejść i narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśm i zapisz konfiguracje).

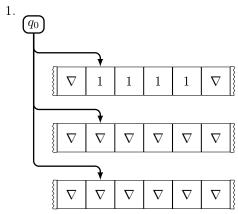
(41) 
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4\}, \{1\}, \{1, \nabla\}, \delta, q_0, \nabla, \{q_4\})$$



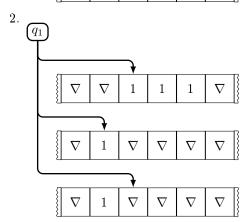
Nr	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
1	$q_0$	$[1,\  abla,\  abla]$	$[\nabla,\ 1,\ 1]$	$q_1$	$[\rightarrow, -\!\!\!-, \rightarrow]$
2	$q_1$	$[1,\ 1,\ \nabla]$	$[1,\ 1,\ 1]$	$q_1$	$[, \rightarrow, \rightarrow]$
3	$q_1$	$[1,\  abla,\  abla]$	$[1,\ 1,\  abla]$	$q_2$	$[\leftarrow, \leftarrow, \leftarrow]$
4	$q_2$	[1,1,1]	$[1,\ 1,\ 1]$	$q_2$	$[\leftarrow, \leftarrow, \leftarrow]$
5	$q_2$	$[\nabla,\ 1,\ 1]$	$[\nabla,1,1]$	$q_2$	$[,\leftarrow,\leftarrow]$
6	$q_2$	$[1,\ \nabla,\ 1]$	$[1,\ \nabla,\ 1]$	$q_2$	$[\leftarrow,, \leftarrow]$
7	$q_2$	$[1,\ 1,\  abla]$	$[1,\ 1,\  abla]$	$q_2$	$[\leftarrow, \leftarrow,]$
8	$q_2$	$[\nabla,\ \nabla,\ 1]$	$[\nabla,\ \nabla,\ 1]$	$q_2$	[─-, ─-, ←]
9	$q_2$	$[1, \ \nabla, \ \nabla]$	$[1, \nabla, \nabla]$	$q_2$	[←, —, —]
10	$q_2$	$[\nabla,\ \nabla,\ \nabla]$	$[ abla, \  abla, \  abla]$	$q_3$	$[\rightarrow,\rightarrow,\rightarrow]$
11	$q_3$	[1,1,1]	$[1,\ 1,\ 1]$	$q_3$	[ o,  o,  o]
12	$q_3$	$[1,\ 1,\ \nabla]$	$[1,\ 1,\ \nabla]$	$q_1$	[,,]
13	$q_3$	$[\nabla,\ 1,\ 1]$	$[ abla,\ 1,\ 1]$	$q_4$	[,,]

Wynik na środkowej taśmie

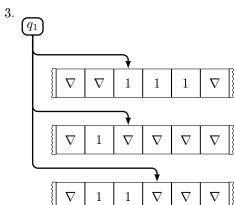
## 5.1.1 f(3)



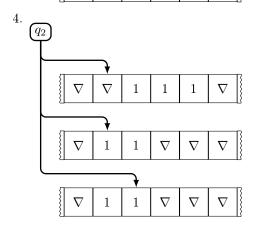
$$K_0=(q_0,\uparrow 1111,\uparrow \nabla,\uparrow \nabla)$$



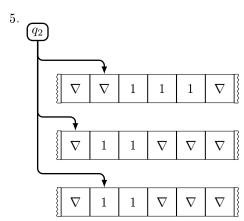
$$K_1=(q_1,\uparrow 111,\uparrow 1,1\uparrow 
abla)$$



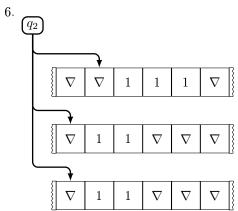
$$K_2=(q_1,\uparrow 111,\,1\uparrow 
abla,\,11\uparrow 
abla)$$



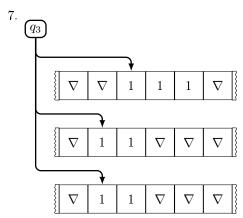
$$K_3=(q_2,\uparrow 
abla 111,\uparrow 11,1\uparrow 1)$$



$$K_4=(q_2,\uparrow
abla 111,\uparrow
abla 11,\uparrow
abla 11)$$

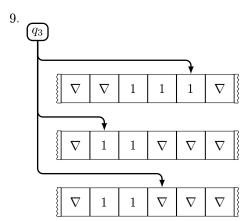


$$K_5=(q_2,\uparrow 
abla 111,\uparrow 
abla 11,\uparrow 
abla 11)$$

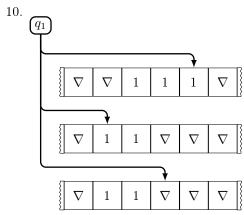


$$K_6=(q_3,\uparrow 111,\uparrow 11,\uparrow 11)$$

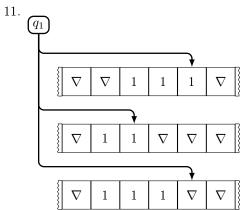
$$K_7=(q_3,\,1{\uparrow}11,\,{\uparrow}11,\,1{\uparrow}1)$$



$$K_8=(q_3,\,11{\uparrow}1,\,{\uparrow}11,\,11{\uparrow}
abla)$$

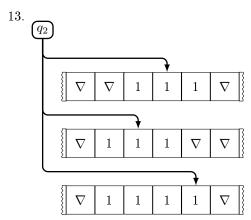


$$K_9=(q_1,\,11{\uparrow}1,\,{\uparrow}11,\,11{\uparrow}
abla)$$

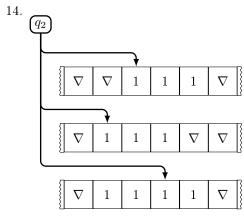


$$K_{10}=(q_1,\,11\uparrow 1,\,1\uparrow 1,\,111\uparrow \nabla)$$

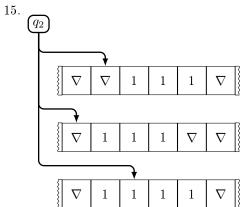
$$K_{11}=(q_1,\ 11{\uparrow}1,\ 11{\uparrow}
abla,\ 1111{\uparrow}
abla)$$



$$K_{12}=(q_2,\,1{\uparrow}11,\,1{\uparrow}11,\,111{\uparrow}1)$$

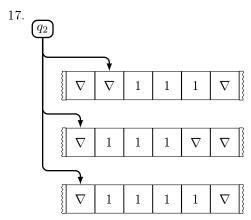


$$K_{13}=(q_2,\uparrow 111,\uparrow 111,11\uparrow 11)$$

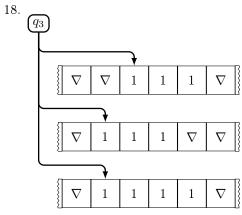


$$K_{14}=(q_2,\uparrow 
abla 111,\uparrow 
abla 111,1 \uparrow 1111)$$

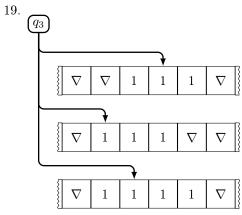
$$K_{15}=(q_2,\uparrow 
abla 111,\uparrow 
abla 1111)$$



$$K_{16}=(q_2,\uparrow 
abla 111,\uparrow 
abla 111,\uparrow 
abla 1111)$$

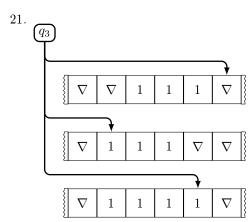


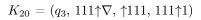
$$K_{17}=(q_3,\uparrow 111,\uparrow 111,\uparrow 1111)$$

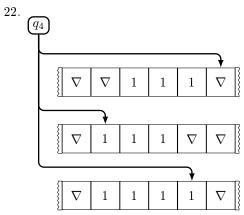


$$K_{18}=(q_3,\,1{\uparrow}11,\,{\uparrow}111,\,1{\uparrow}111)$$

$$K_{19}=(q_3,\,11\uparrow 1,\,\uparrow 111,\,11\uparrow 11)$$

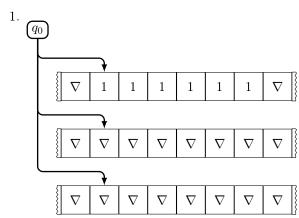




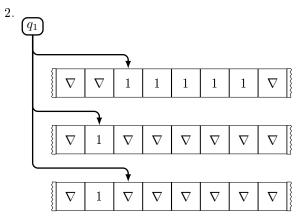


$$K_{21}=(q_4,\,111\uparrow
abla,\,\uparrow111,\,111\uparrow1)$$

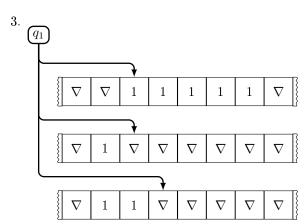
## 5.1.2 f(5)



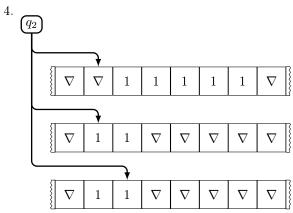
$$K_0=(q_0,\uparrow 111111,\uparrow 
abla,\uparrow 
abla)$$



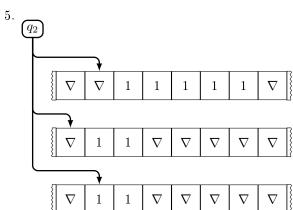
$$K_1=(q_1,\uparrow 11111,\uparrow 1,1\uparrow \nabla)$$



$$K_2=(q_1,\uparrow 11111,1\uparrow 
abla,\,11\uparrow 
abla)$$

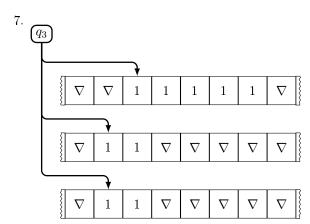


$$K_3=(q_2,\uparrow\nabla 11111,\uparrow 11,1\uparrow 1)$$

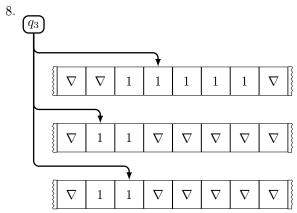


$$K_4=(q_2,\uparrow 
abla 11111,\uparrow 
abla 11,\uparrow 11)$$

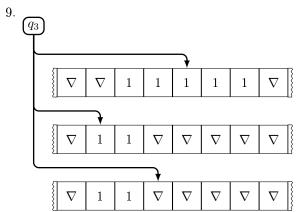
$$K_5=(q_2,\uparrow \nabla 11111,\uparrow \nabla 11,\uparrow \nabla 11)$$



$$K_6=(q_3,\uparrow 11111,\uparrow 11,\uparrow 11)$$

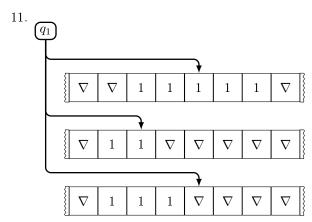


$$K_7=(q_3,\,1\uparrow1111,\,\uparrow11,\,1\uparrow1)$$

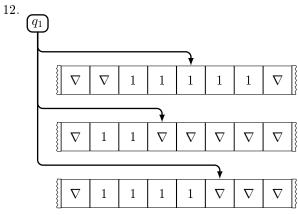


$$K_8=(q_3,\,11{\uparrow}111,\,{\uparrow}11,\,11{\uparrow}
abla)$$

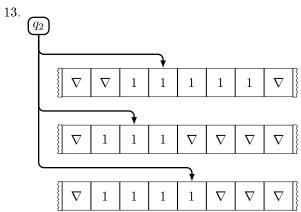
$$K_9=(q_1,\,11{\uparrow}111,\,{\uparrow}11,\,11{\uparrow}
abla)$$



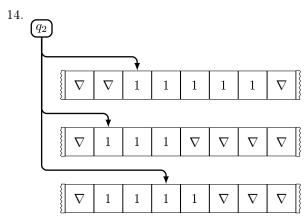
 $K_{10}=(q_1,\,11{\uparrow}111,\,1{\uparrow}1,\,111{\uparrow}
abla)$ 



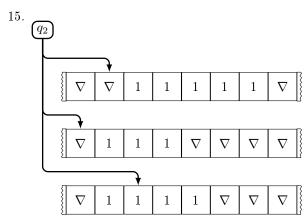
 $K_{11}=(q_1,\,11{\uparrow}111,\,11{\uparrow}
abla,\,1111{\uparrow}
abla)$ 



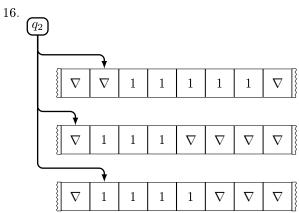
 $K_{12}=(q_2,\,1\!\!\uparrow\!\!1111,\,1\!\!\uparrow\!\!11,\,111\!\!\uparrow\!\!1)$ 



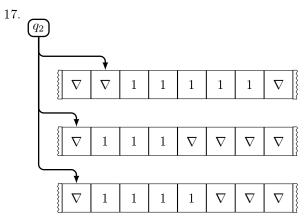
 $K_{13}=(q_2,\uparrow 11111,\uparrow 111,\,11\uparrow 11)$ 



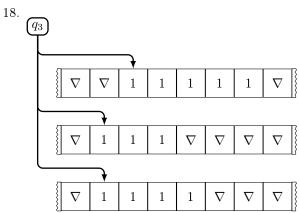
$$K_{14}=(q_2,\uparrow 
abla 11111,\uparrow 
abla 1111)$$



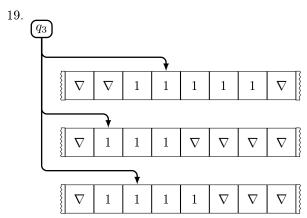
$$K_{15}=(q_2,\uparrow 
abla 11111,\uparrow 
abla 1111)$$



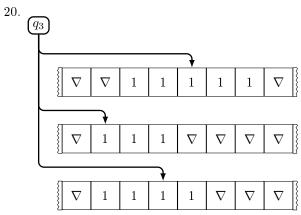
$$K_{16}=(q_2,\uparrow 
abla 11111,\uparrow 
abla 1111)$$



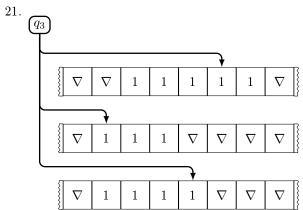
$$K_{17}=(q_3,\uparrow 11111,\uparrow 111,\uparrow 1111)$$



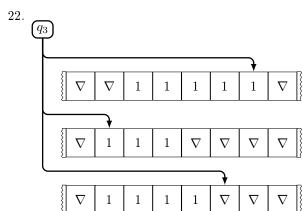
 $K_{18}=(q_3,\,1\!\!\uparrow\!\!1111,\,\uparrow\!\!111,\,1\!\!\uparrow\!\!111)$ 



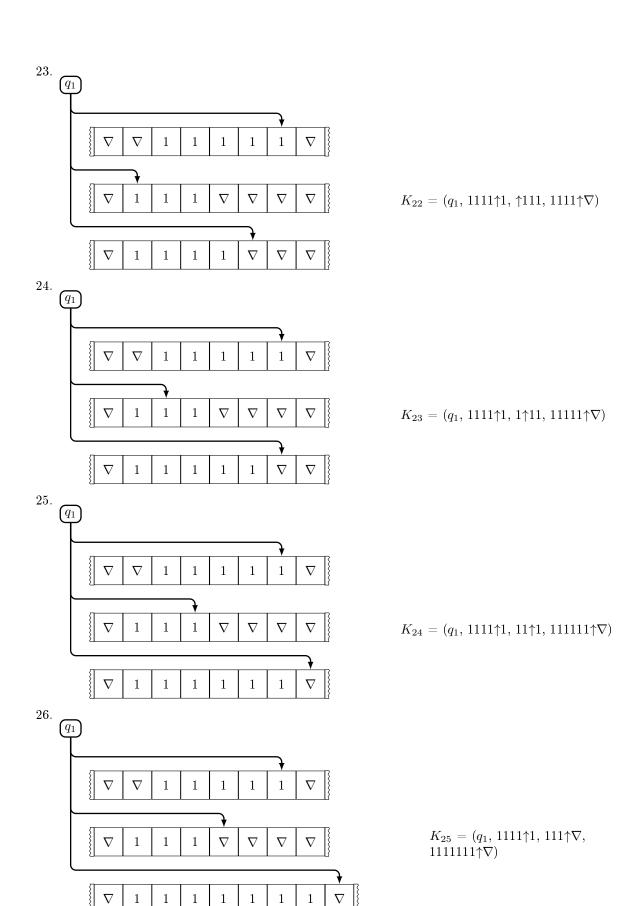
 $K_{19}=(q_3,\,11{\uparrow}111,\,{\uparrow}111,\,11{\uparrow}11)$ 

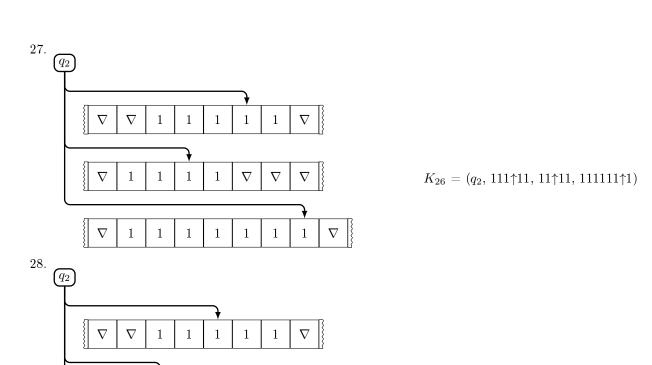


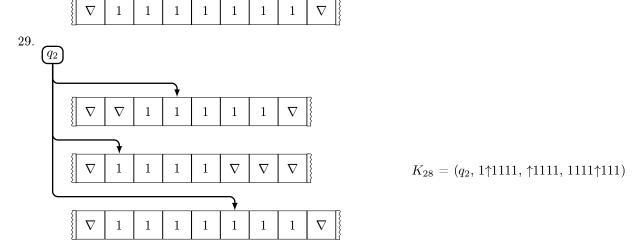
 $K_{20}=(q_3,\,111{\uparrow}11,\,{\uparrow}111,\,111{\uparrow}1)$ 



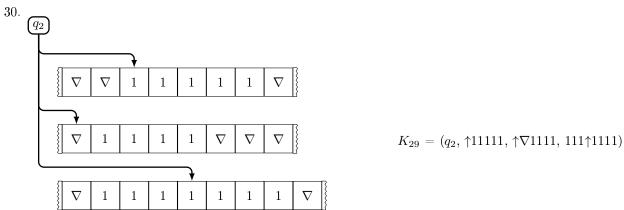
 $K_{21}=(q_3,\,1111\uparrow 1,\,\uparrow 111,\,1111\uparrow \nabla)$ 

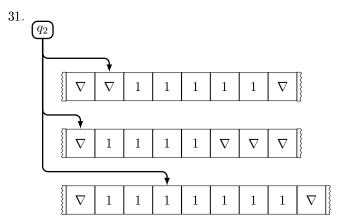




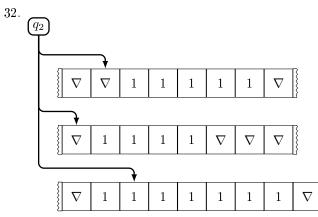


 $K_{27}=(q_2,\,11{\uparrow}111,\,1{\uparrow}111,\,11111{\uparrow}11)$ 

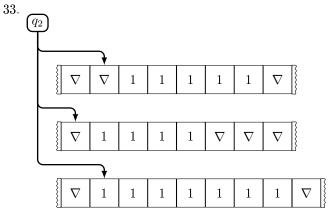




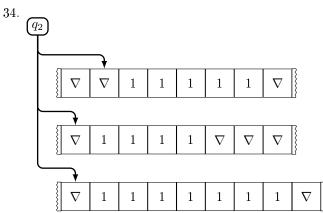
 $K_{30}=(q_2,\uparrow\nabla 11111,\uparrow\nabla 1111,\\11\uparrow11111)$ 



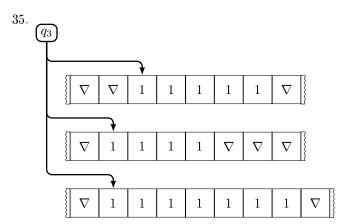
 $\begin{array}{l} K_{31}=(q_2,\uparrow\nabla 11111,\uparrow\nabla 1111,\\ 1\uparrow 111111) \end{array}$ 



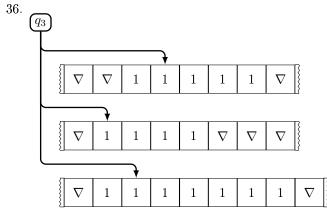
 $K_{32} = (q_2, \uparrow \nabla 11111, \uparrow \nabla 1111, \uparrow 1111111)$ 



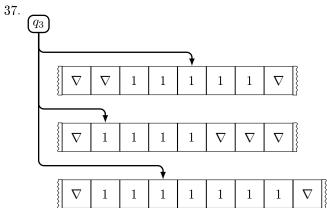
 $K_{33} = (q_2, \uparrow \nabla 11111, \uparrow \nabla 1111, \uparrow \nabla 11111, \uparrow \nabla 111111)$ 



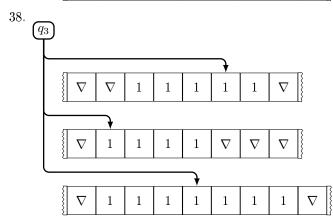
 $K_{34}=(q_3,\uparrow 11111,\uparrow 1111,\uparrow 1111111)$ 



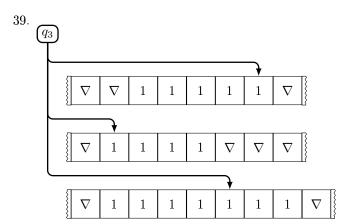
 $K_{35}=(q_3,\,1\!\!\uparrow\!\!1111,\,\uparrow\!\!1111,\,1\!\!\uparrow\!\!111111)$ 



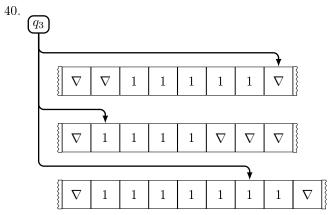
 $K_{36}=(q_3,\,11{\uparrow}111,\,{\uparrow}1111,\,11{\uparrow}11111)$ 



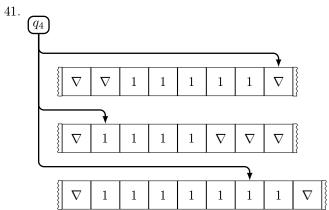
 $K_{37}=(q_3,\,111{\uparrow}11,\,{\uparrow}1111,\,111{\uparrow}1111)$ 



 $K_{38}=(q_3,\,1111\uparrow 1,\,\uparrow 1111,\,1111\uparrow 111)$ 



 $K_{39}=(q_3,\,11111\uparrow\nabla,\,\uparrow1111,\,11111\uparrow11)$ 



 $K_{40}=(q_4,\,111111\uparrow
abla,\,\uparrow 1111,\,11111\uparrow 11)$ 

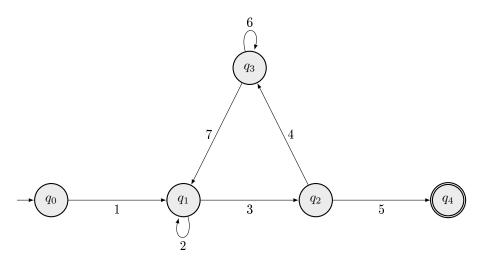
## 5.2 Zadanie 1.53

Używając unarnej reprezentacji liczb, zaprojektuj wielotaśmową maszynę Turinga, która oblicza funkcję

$$(42) f(n) = n^2$$

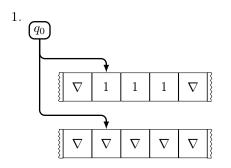
Podaj tabelę przejść i narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśm i zapisz konfiguracje).

(43) 
$$M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4\}, \{1\}, \{1, x, \nabla\}, \delta, q_0, \nabla, \{q_4\})$$

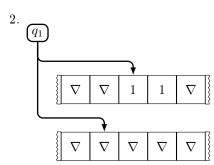


$\mathbf{Nr}$	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
1	$q_0$	$[1, \nabla]$	[B, B]	$q_1$	[ o,]
2	$q_1$	$[1, \nabla]$	$[\mathbf{x},\ 1]$	$q_1$	[ o,   o]
3	$q_1$	[ abla,   abla]	[B, B]	$q_2$	[←, —]
4	$q_2$	$[\mathrm{x},\;  abla]$	$[\mathbf{B},\ \mathbf{B}]$	$q_3$	[←, —]
5	$q_2$	[ abla,   abla]	[B, 1]	$q_4$	[,]
6	$q_3$	$[\mathrm{x},\;  abla]$	[1, 1]	$q_3$	$[\leftarrow, \rightarrow]$
7	$q_3$	[ abla,   abla]	[B, B]	$q_1$	[ o,]

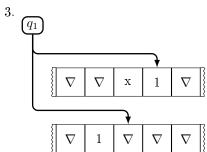
#### $5.2.1 \quad f(2)$



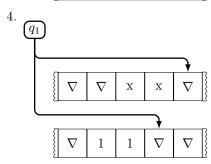
$$K_0=(q_0,\uparrow 111,\uparrow \nabla)$$



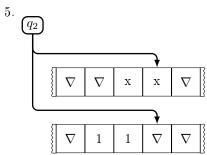
$$K_1=(q_1,\uparrow 11,\uparrow \nabla)$$



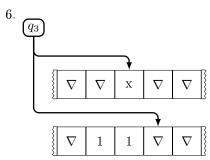
$$K_2 = (q_1, x \uparrow 1, 1 \uparrow \nabla)$$



$$K_3 = (q_1, xx\uparrow\nabla, 11\uparrow\nabla)$$



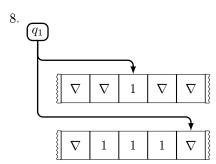
$$K_4 = (q_2, x \uparrow x, 11 \uparrow \nabla)$$



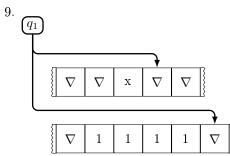
$$K_5 = (q_3, \uparrow \mathbf{x}, 11 \uparrow \nabla)$$

7. 
$$q_3$$
 $\nabla$ 
 $1$ 
 $1$ 
 $1$ 
 $1$ 

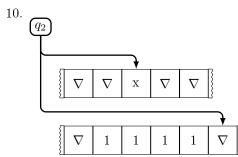
$$K_6=(q_3,\uparrow\nabla 1,\,111\uparrow\nabla)$$



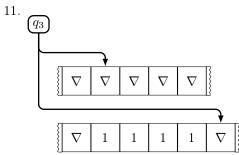
$$K_7=(q_1,\uparrow 1,\, 111\uparrow \nabla)$$



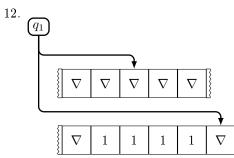
$$K_8 = (q_1, x \uparrow \nabla, 1111 \uparrow \nabla)$$



$$K_9 = (q_2, \uparrow \mathbf{x}, 1111 \uparrow \nabla)$$

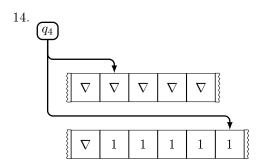


$$K_{10}=(q_3,\uparrow\nabla,1111\uparrow\nabla)$$



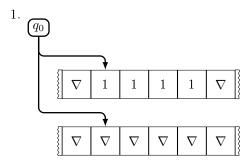
$$K_{11}=(q_1,\uparrow 
abla,1111\uparrow 
abla)$$

$$K_{12}=(q_2,\uparrow 
abla,\,1111 \uparrow 
abla)$$

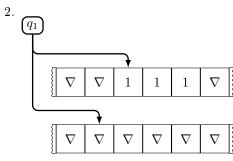


$$K_{13}=(q_4,\uparrow 
abla,1111\uparrow 1)$$

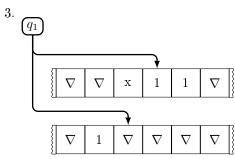
## 5.2.2 f(3)



$$K_0=(q_0,\uparrow 1111,\uparrow \nabla)$$

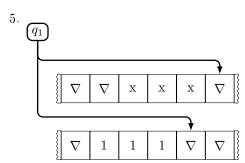


$$K_1=(q_1,\uparrow 111,\uparrow \nabla)$$

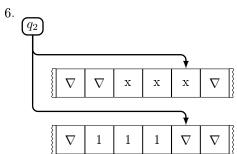


$$K_2 = (q_1, \, \mathrm{x} {\uparrow} 11, \, 1 {\uparrow} \nabla)$$

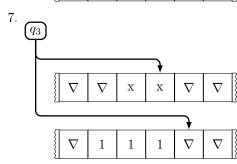
$$K_3=(q_1,\,\mathrm{xx}\!\uparrow\!1,\,11\!\uparrow\!\nabla)$$



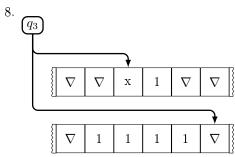
$$K_4 = (q_1, \, \text{xxx} \uparrow \nabla, \, 111 \uparrow \nabla)$$



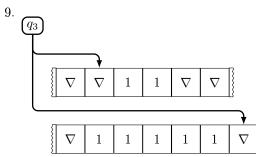
$$K_5 = (q_2, \, \mathbf{xx} \uparrow \mathbf{x}, \, 111 \uparrow \nabla)$$



$$K_6=(q_3,\,\mathrm{x}\!\uparrow\!\mathrm{x},\,111\!\uparrow\!
abla)$$

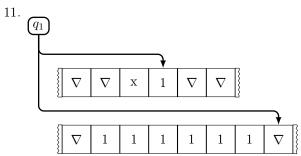


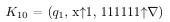
$$K_7 = (q_3, \uparrow \mathbf{x}1, 1111 \uparrow \nabla)$$

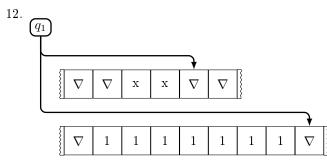


$$K_8=(q_3,\uparrow 
abla 11,11111 \uparrow 
abla)$$

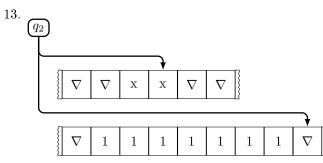
$$K_9=(q_1,\uparrow 11,\, 111111\uparrow \nabla)$$



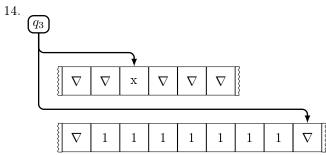




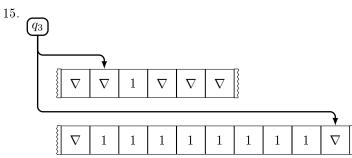
$$K_{11}=(q_1,\,\mathrm{xx}\!\!\uparrow\!\!
abla,\,11111111\!\!\uparrow\!\!
abla)$$



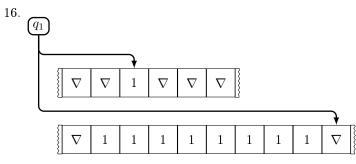
$$K_{12} = (q_2, x \uparrow x, 11111111 \uparrow \nabla)$$



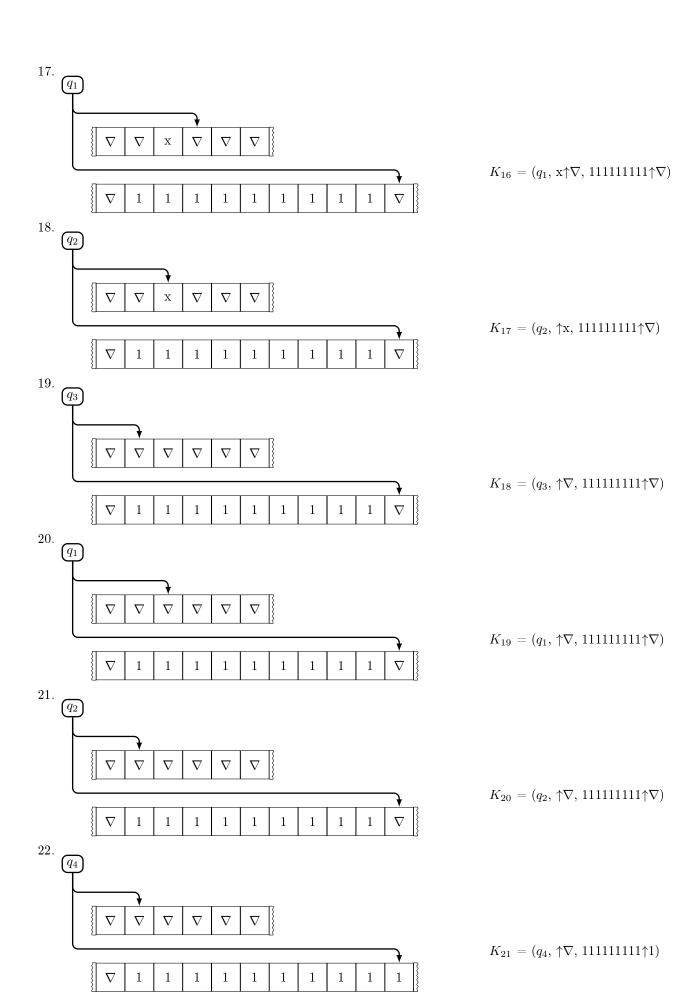
$$K_{13} = (q_3, \uparrow \mathbf{x}, 11111111\uparrow \nabla)$$



$$K_{14}=(q_3,\uparrow\nabla 1,\,11111111\uparrow\nabla)$$



$$K_{15}=(q_1,\uparrow 1,111111111\uparrow \nabla)$$



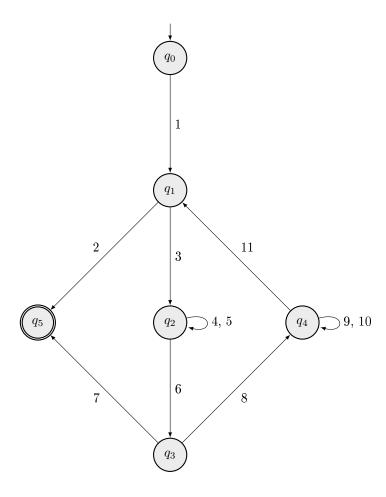
## 5.3 Zadanie 1.54

Zaprojektuj wielotaśmową maszynę Turinga, które oblicza funkcję mnożenia f dla liczb naturalnych m i n w reprezentacji unarnej

$$(44) f(m,n) = m \cdot n$$

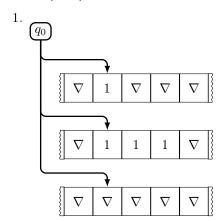
Podaj tabelę przejść i narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśm i zapisz konfiguracje).

$$(45) \hspace{3.1em} M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{1\}, \{1, \nabla\}, \delta, q_0, \nabla, \{q_5\})$$

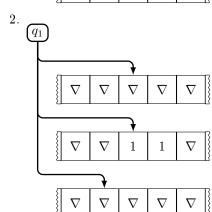


$\mathbf{Nr}$	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
1	$q_0$	$[1,\ 1,\  abla]$	$[\nabla,\ \nabla,\ \nabla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!\!-]$
2	$q_1$	$[\nabla,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_5$	[,,]
3	$q_1$	$[1,\ 1,\  abla]$	$[\nabla,\ 1,\ 1]$	$q_2$	$[, \rightarrow, \rightarrow]$
4	$q_2$	$[1,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_2$	$[, \rightarrow, \rightarrow]$
5	$q_2$	$[\nabla,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_2$	$[, \rightarrow, \rightarrow]$
6	$q_2$	$[\nabla,\ \nabla,\ \nabla]$	$[\nabla,\ \nabla,\ \nabla]$	$q_3$	$[\rightarrow,\leftarrow,-\!\!-\!]$
7	$q_3$	$[\nabla,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_5$	[,,]
8	$q_3$	$[1,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_4$	$[,\leftarrow,\rightarrow]$
9	$q_4$	$[1,\ 1,\  abla]$	$[\nabla,\ 1,\ 1]$	$q_4$	$[,\leftarrow,\rightarrow]$
10	$q_4$	$[\nabla,\ 1,\ \nabla]$	$[\nabla,\ 1,\ 1]$	$q_4$	$[,\leftarrow,\rightarrow]$
11	$q_4$	$[\nabla,\ \nabla,\ \nabla]$	$[ abla, \  abla, \  abla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!-\!]$

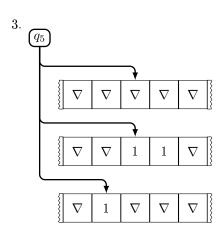
# $5.3.1 \quad f(0, 2)$



$$K_0=(q_0,\uparrow 1,\uparrow 111,\uparrow 
abla)$$

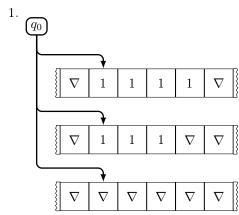


$$K_1=(q_1,\uparrow
abla,\uparrow
abla,\uparrow
abla)$$

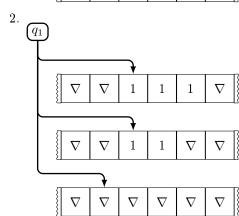


$$K_2=(q_5,\uparrow
abla,\uparrow11,\uparrow1)$$

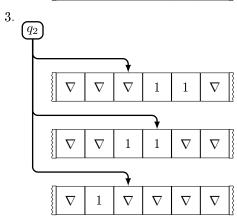
## $5.3.2 \quad f(3, 2)$



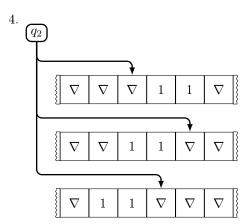
$$K_0=(q_0,\uparrow 1111,\uparrow 111,\uparrow \nabla)$$



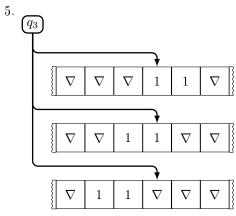
$$K_1=(q_1,\uparrow 111,\uparrow 11,\uparrow \nabla)$$



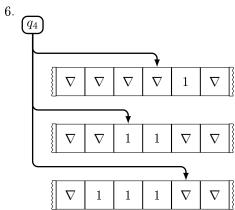
$$K_2=(q_2,\uparrow 
abla 11,\, 1\uparrow 1,\, 1\uparrow 
abla)$$



$$K_3=(q_2,\uparrow 
abla 11,\, 11 {\uparrow} 
abla,\, 11 {\uparrow} 
abla)$$

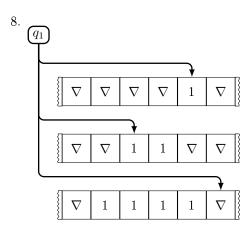


$$K_4=(q_3,\uparrow 11,\,1\uparrow 1,\,11\uparrow \nabla)$$

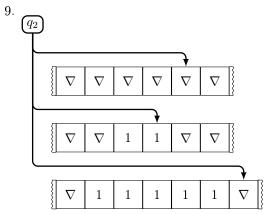


$$K_5=(q_4,\uparrow 
abla 1,\uparrow 11,111 \uparrow 
abla)$$

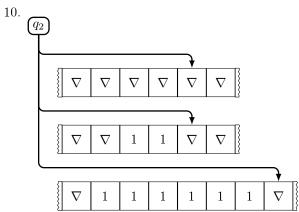
$$K_6=(q_4,\uparrow \nabla 1,\uparrow \nabla 11,1111\uparrow \nabla)$$



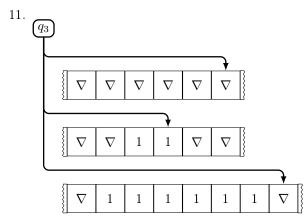
$$K_7=(q_1,\uparrow 1,\uparrow 11,\, 1111\uparrow 
abla)$$



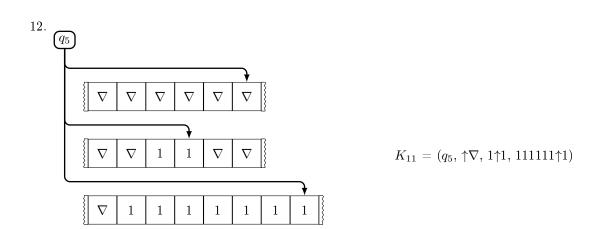
$$K_8=(q_2,\uparrow 
abla,\,1\uparrow 1,\,11111\uparrow 
abla)$$



$$K_9=(q_2,\uparrow 
abla,111\uparrow 
abla,111111\uparrow 
abla)$$



$$K_{10}=(q_3,\uparrow \nabla,1 \uparrow 1,111111 \uparrow \nabla)$$



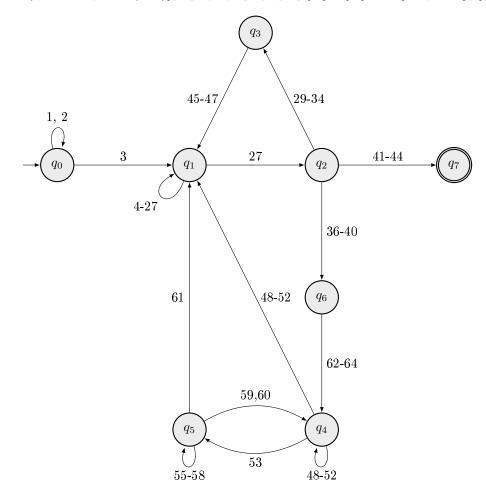
#### 5.4 Zadanie 1.57

Używając binarnej reprezentacji liczb, zaprojektuj wielotaśmową maszynę Turinga, która oblicza funkcję

$$(46) f(n) = n^2$$

Podaj tabelę przejść i narysuj diagram przejść. Dla zaprojektowanej maszyny wykonaj dwa obliczenia (wykonaj rysunki taśm i zapisz konfiguracje).

$$(47) M = (Q, \Sigma, \Gamma, \delta, q_0, \nabla, F) = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{1, 0\}, \{1, 0, \nabla\}, \delta, q_0, \nabla, \{q_7\})$$



Nr	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
1	$q_0$	$[0,\ \nabla,\ \nabla]$	$[0,0,\nabla]$	$q_0$	$[\rightarrow,\rightarrow,\rightarrow]$
2	$q_0$	$[1,\  abla,\  abla]$	$[1,\ 1,\  abla]$	$q_0$	$[\rightarrow,\rightarrow,\rightarrow]$
3	$q_0$	$[\nabla,\;\nabla,\;\nabla]$	$[\nabla,\ \nabla,\ \nabla]$	$q_2$	$[\leftarrow,\leftarrow,\leftarrow]$

Tabela 1: Tabela przejścia dla stanu  $q_0$ 

Nr	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
4	$q_1$	[0,0,0]	$[0,\ 0,\ 0]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
5	$q_1$	[0,0,1]	$[0,\ 0,\ 1]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
6	$q_1$	[0,1,0]	$[0,\ 1,\ 0]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
7	$q_1$	[0,1,1]	$[0,\ 1,\ 1]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
8	$q_1$	[1,  0,  0]	$[1, \ 0, \ 0]$	$q_1$	$[\to,\to,\to]$
9	$q_1$	$[1,\ 0,\ 1]$	$[1,\ 0,\ 1]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
10	$q_1$	[1, 1, 0]	$[1,\ 1,\ 0]$	$q_1$	$[\to,\to,\to]$
11	$q_1$	[1,1,1]	$[1,\ 1,\ 1]$	$q_1$	$[\rightarrow,\rightarrow,\rightarrow]$
12	$q_1$	$[\nabla,\ 0,\ 0]$	$[\nabla,\ 0,\ 0]$	$q_1$	$[, \rightarrow, \rightarrow]$
13	$q_1$	$[\nabla,\ 0,\ 1]$	$[\nabla,\ 0,\ 1]$	$q_1$	$[, \rightarrow, \rightarrow]$
14	$q_1$	$[\nabla,\ 1,\ 0]$	$[\nabla,\ 1,\ 0]$	$q_1$	$[, \rightarrow, \rightarrow]$
15	$q_1$	$[\nabla,\ 1,\ 1]$	$[\nabla,\ 1,\ 1]$	$q_1$	$[, \rightarrow, \rightarrow]$
16	$q_1$	$[0,\;\nabla,\;0]$	$[0,\ \nabla,\ 0]$	$q_1$	$[\rightarrow,-\!\!-\!,\rightarrow]$
17	$q_1$	$[0,\;\nabla,\;1]$	$[0,\ \nabla,\ 1]$	$q_1$	$[\rightarrow,-\!\!-\!,\rightarrow]$
18	$q_1$	$[1,\;  abla,\; 0]$	$[1,\;\nabla,\;0]$	$q_1$	$[\rightarrow,-\!\!-\!,\rightarrow]$
19	$q_1$	$[1,\;  abla,\; 1]$	$[1,\ \nabla,\ 1]$	$q_1$	$[\rightarrow,-\!\!-\!,\rightarrow]$
20	$q_1$	$[0,0,\nabla]$	$[0,\ 0,\ \nabla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!-\!]$
21	$q_1$	$[0,1,\nabla]$	$[0,\ 1,\ \nabla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!-]$
22	$q_1$	$[1,0,\nabla]$	$[1,\ 0,\ \nabla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!-]$
23	$q_1$	$[1,\ 1,\  abla]$	$[1,\ 1,\ \nabla]$	$q_1$	$[\rightarrow,\rightarrow,-\!\!-]$
24	$q_1$	[ abla,   abla,  0]	$[ abla, \  abla, \ 0]$	$q_1$	$[,,\rightarrow]$
25	$q_1$	$[\nabla,\ \nabla,\ 1]$	$[\nabla,\ \nabla,\ 1]$	$q_1$	$[,,\to]$
26	$q_1$	$[0,\ \nabla,\ \nabla]$	$[0,\ \nabla,\ \nabla]$	$q_1$	$[\rightarrow,,]$
27	$q_1$	$[1,\;  abla,\;  abla]$	$[1,\;\nabla,\;\nabla]$	$q_1$	$[ o,  -\!\!\!-\!\!\!-,  -\!\!\!\!-]$
28	$q_1$	$[ abla,\  abla,\  abla]$	$[\nabla,\ \nabla,\ \nabla]$	$q_2$	$[\leftarrow, \leftarrow, \leftarrow]$

Tabela 2: Tabela przejścia dla bieżącego stanu  $q_1$ 

$\mathbf{Nr}$	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
29	$q_2$	[1,0, abla]	$[1,\  abla,\  abla]$	$q_3$	[ o,  o,  o]
30	$q_2$	$[0,0,\nabla]$	$[0,\ \nabla,\ \nabla]$	$q_3$	$[\rightarrow,-\!\!-\!,-\!\!-\!]$
31	$q_2$	[1,  0,  1]	$[1,\  abla,\ 1]$	$q_3$	$[\rightarrow,,]$
32	$q_2$	[1,  0,  0]	$[1,\ \nabla,\ 0]$	$q_3$	$[\rightarrow,-\!\!-\!,-\!\!-\!]$
33	$q_2$	[0,0,1]	$[0,\ \nabla,\ 1]$	$q_3$	$[\rightarrow,,]$
34	$q_2$	[0,0,0]	$[0,\ \nabla,\ 0]$	$q_3$	$[\rightarrow,,]$
35	$q_2$	[1,1, abla]	$[1,\;  abla,\;  abla]$	$q_6$	$[\rightarrow,,]$
36	$q_2$	$[0,1,\nabla]$	$[0,\ \nabla,\ \nabla]$	$q_6$	$[\rightarrow,,]$
37	$q_2$	[1, 1, 1]	$[1,\  abla,\ 1]$	$q_6$	$[\rightarrow,,]$
38	$q_2$	[1, 1, 0]	$[1,\;\nabla,\;0]$	$q_6$	$[\rightarrow,,]$
39	$q_2$	[0,1,1]	$[0,\ \nabla,\ 1]$	$q_6$	$[\rightarrow,,]$
40	$q_2$	[0,1,0]	$[0,\ \nabla,\ 0]$	$q_6$	$[\rightarrow,,]$
41	$q_2$	$[0,\;  abla,\; 0]$	$[0,\ \nabla,\ 0]$	$q_7$	[,,]
42	$q_2$	$[0,\;\nabla,\;1]$	$[0,\ \nabla,\ 1]$	$q_7$	[,,]
43	$q_2$	$[1, \  abla, \ 0]$	$[1,\ \nabla,\ 0]$	$q_7$	[,,]
44	$q_2$	$[1,\  abla,\ 1]$	$[1,\ \nabla,\ 1]$	$q_7$	[,,]

Tabela 3: Tabela przejścia dla bieżącego stanu  $q_2\,$ 

Nr	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
45	$q_3$	$[\nabla,\ \nabla,\ \nabla]$	$[\nabla,\;\nabla,\;0]$	$q_1$	[←, <del></del> , <del></del> ]
46	$q_3$	$[\nabla,\nabla,1]$	$[0,\ \nabla,\ 1]$	$q_1$	$[\rightarrow,-\!\!-\!,-\!\!-\!]$
47	$q_3$	$[\nabla,\ \nabla,\ 0]$	$[0,\ \nabla,\ 0]$	$q_1$	$[\rightarrow, -\!\!\!-\!\!\!-, -\!\!\!-\!\!\!]$

Tabela 4: Tabela przejścia dla bieżącego stanu  $q_3$ 

Nr	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
48	$q_4$	$[1,\;  abla,\;  abla]$	$[1,\  abla,\ 1]$	$q_4$	[←, ─, ←]
49	$q_4$	$[1,\;\nabla,\;0]$	$[1,\  abla,\ 1]$	$q_4$	$[\leftarrow,, \leftarrow]$
50	$q_4$	$[0,\;\nabla,\;1]$	$[0,\ \nabla,\ 1]$	$q_4$	[←, ─, ←]
51	$q_4$	$[0,\ \nabla,\ \nabla]$	$[0,\;\nabla,\;0]$	$q_4$	$[\leftarrow,, \leftarrow]$
52	$q_4$	$[0,\;\nabla,\;0]$	$[0,\ \nabla,\ 0]$	$q_4$	[←, ─, ←]
53	$q_4$	$[1,\;\nabla,\;1]$	$[1,\; oldsymbol{ abla},\; 0]$	$q_5$	$[\leftarrow,, \leftarrow]$
54	$q_4$	$[\nabla,\ \nabla,\ \nabla]$	$[\nabla,\;\nabla,\;\nabla]$	$q_1$	$[\rightarrow, -\!\!\!-, \rightarrow]$

Tabela 5: Tabela przejścia dla bieżącego stanu  $q_4$ 

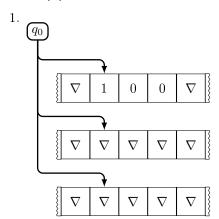
$\mathbf{Nr}$	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
55	$q_5$	$[1,\;\nabla,\;1]$	$[1,\  abla,\ 1]$	$q_5$	$[\leftarrow,, \leftarrow]$
56	$q_5$	$[0,\;\nabla,\;1]$	$[0,\ \nabla,\ 0]$	$q_5$	$[\leftarrow,, \leftarrow]$
57	$q_5$	$[1,\;\nabla,\;0]$	$[1,\;\nabla,\;0]$	$q_5$	$[\leftarrow,, \leftarrow]$
58	$q_5$	$[1,\;\nabla,\;\nabla]$	$[1,\;\nabla,\;0]$	$q_5$	$[\leftarrow,, \leftarrow]$
59	$q_5$	$[0,\;\nabla,\;0]$	$[0,\ \nabla,\ 1]$	$q_4$	$[\leftarrow,, \leftarrow]$
60	$q_5$	$[0,\ \nabla,\ \nabla]$	$[0,\ \nabla,\ 1]$	$q_4$	$[\leftarrow,, \leftarrow]$
61	$q_5$	$[\nabla,\ \nabla,\ \nabla]$	$[ abla, \  abla, \ 1]$	$q_1$	$[\rightarrow,,\rightarrow]$

Tabela 6: Tabela przejścia dla bieżącego stanu  $q_{5}\,$ 

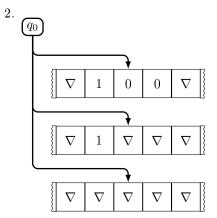
$\mathbf{Nr}$	Stan bieżący	Czytany symbol	Nowy symbol	Nowy stan	Kierunki głowic
62	$q_6$	$[\nabla,\;\nabla,\;\nabla]$	$[\nabla,\ \nabla,\ \nabla]$	$q_4$	[←, <del></del> , <del></del> ]
63	$q_6$	$[\nabla,\ \nabla,\ 1]$	$[0,\ \nabla,\ 1]$	$q_4$	[,]
64	$q_6$	$[\nabla,\ \nabla,\ 0]$	$[0,\ \nabla,\ 0]$	$q_4$	[,]

Tabela 7: Tabela przejścia dla bieżącego stanu  $q_6$ 

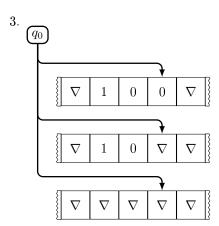
### $5.4.1 \quad f(4)$



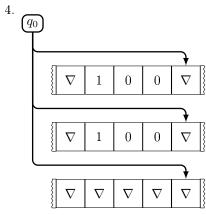
$$K_0=(q_0,\uparrow 100,\uparrow 
abla,\uparrow 
abla)$$



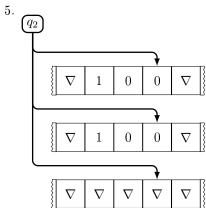
$$K_1=(q_0,\,1{\uparrow}00,\,1{\uparrow}
abla,\,{\uparrow}
abla$$



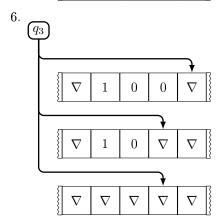
$$K_2=(q_0,\,10{\uparrow}0,\,10{\uparrow}
abla,\,{\uparrow}
abla)$$



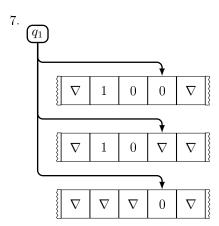
$$K_3 = (q_0, 100 \uparrow \nabla, 100 \uparrow \nabla, \uparrow \nabla)$$



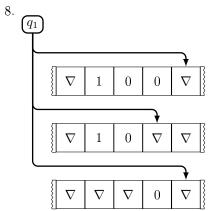
$$K_4=(q_2,\,10{\uparrow}0,\,10{\uparrow}0,\,{\uparrow}\nabla)$$



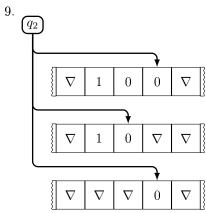
$$K_5=(q_3,\,100{\uparrow}
abla,\,10{\uparrow}
abla,\,\uparrow
abla)$$



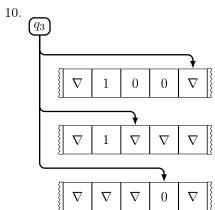
$$K_6 = (q_1, 10 \uparrow 0, 10 \uparrow \nabla, \uparrow 0)$$



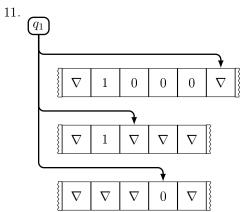
$$K_7=(q_1,\,100{\uparrow}
abla,\,10{\uparrow}
abla,\,0{\uparrow}
abla,\,0{\uparrow}
abla)$$



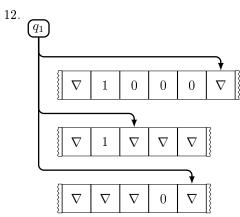
$$K_8=(q_2,\,10{\uparrow}0,\,1{\uparrow}0,\,{\uparrow}0)$$



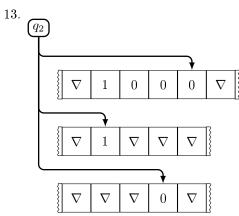
$$K_9 = (q_3, 100 \uparrow \nabla, 1 \uparrow \nabla, \uparrow 0)$$





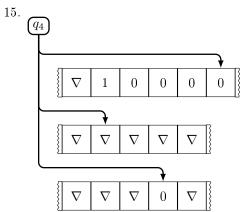


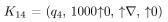
$$K_{11} = (q_1, 1000 \uparrow \nabla, 1 \uparrow \nabla, 0 \uparrow \nabla)$$

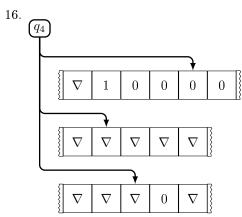


$$K_{12}=(q_2,\,100\uparrow 0,\,\uparrow 1,\,\uparrow 0)$$

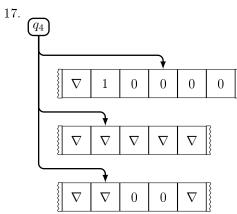
$$K_{13} = (q_6, 1000 \uparrow \nabla, \uparrow \nabla, \uparrow 0)$$





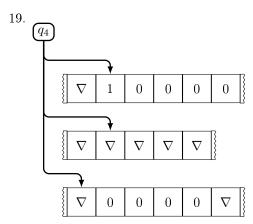


$$K_{15} = (q_4, 100\uparrow 00, \uparrow \nabla, \uparrow \nabla 0)$$

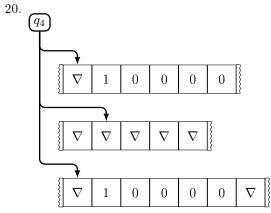


$$K_{16} = (q_4, 10 \uparrow 000, \uparrow \nabla, \uparrow \nabla 00)$$

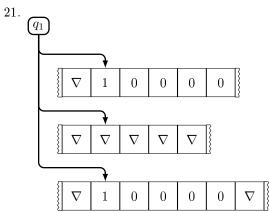
$$K_{17}=(q_4,\,1\uparrow 0000,\,\uparrow \nabla,\,\uparrow \nabla 000)$$



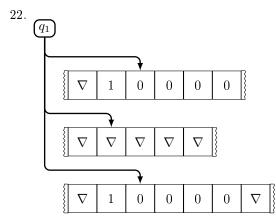
 $K_{18}=(q_4,\uparrow 10000,\uparrow \nabla,\uparrow \nabla 0000)$ 



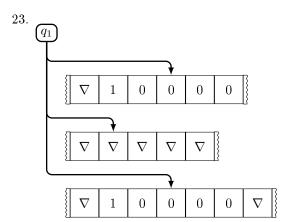
 $K_{19} = (q_4, \uparrow \nabla 10000, \uparrow \nabla, \uparrow \nabla 10000)$ 



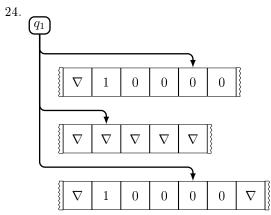
 $K_{20}=(q_1,\uparrow 10000,\uparrow 
abla,\uparrow 10000)$ 



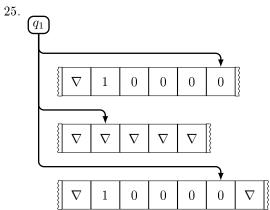
 $K_{21}=(q_1,\,1\uparrow 0000,\,\uparrow \nabla,\,1\uparrow 0000)$ 



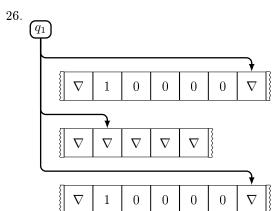




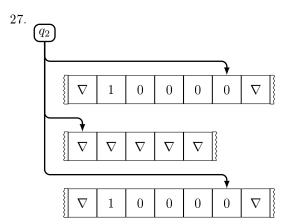
 $K_{23}=(q_1,\,100{\uparrow}00,\,{\uparrow}
abla,\,100{\uparrow}00)$ 



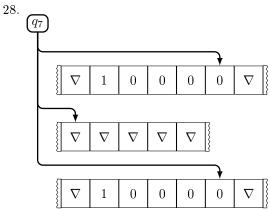
 $K_{24}=(q_1,\,1000{\uparrow}0,\,{\uparrow}
abla,\,1000{\uparrow}0)$ 



 $K_{25}=(q_1,\,10000\uparrow
abla,\,\uparrow
abla,\,10000\uparrow
abla)$ 

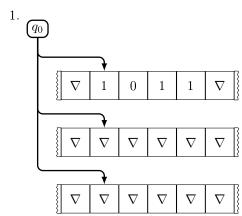


$$K_{26}=(q_2,\,1000{\uparrow}0,\,{\uparrow}
abla,\,1000{\uparrow}0)$$



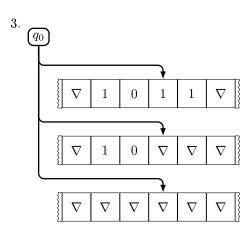
$$K_{27}=(q_7,\,1000{\uparrow}0,\,{\uparrow}
abla,\,1000{\uparrow}0)$$

## 5.4.2 f(11)

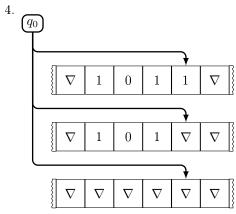


$$K_0=(q_0,\uparrow 1011,\uparrow \nabla,\uparrow \nabla)$$

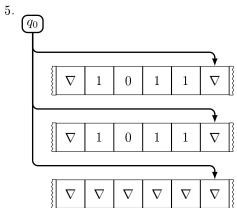
$$K_1=(q_0,\,1{\uparrow}011,\,1{\uparrow}
abla,\,{\uparrow}
abla)$$



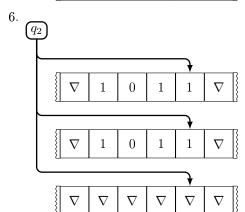
$$K_2=(q_0,\,10{\uparrow}11,\,10{\uparrow}
abla,\,{\uparrow}
abla)$$



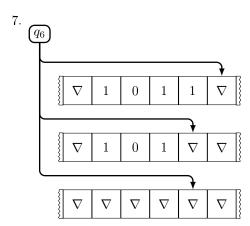
$$K_3 = (q_0, 101\uparrow 1, 101\uparrow \nabla, \uparrow \nabla)$$



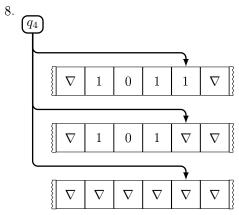
$$K_4 = (q_0, 1011 \uparrow \nabla, 1011 \uparrow \nabla, \uparrow \nabla)$$



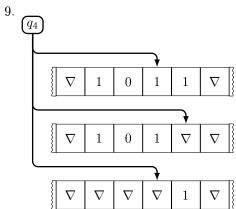
$$K_5=(q_2,\,101{\uparrow}1,\,101{\uparrow}1,\,{\uparrow}
abla)$$



$$K_6=(q_6,\,1011{\uparrow}
abla,\,101{\uparrow}
abla,\,{\uparrow}
abla$$

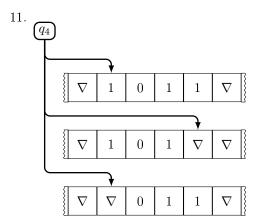


$$K_7 = (q_4, 101 \uparrow 1, 101 \uparrow \nabla, \uparrow \nabla)$$

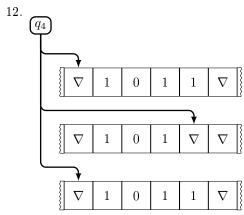


$$K_8=(q_4,\,10{\uparrow}11,\,101{\uparrow}
abla,\,{\uparrow}
abla1)$$

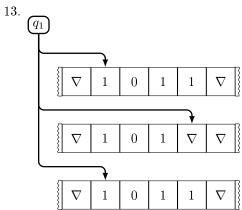
$$K_9=(q_4,\,1\uparrow 011,\,101\uparrow \nabla,\,\uparrow \nabla 11)$$



$$K_{10}=(q_4,\uparrow 1011,\, 101\uparrow \nabla,\, \uparrow \nabla 011)$$

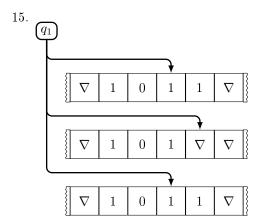


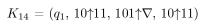
$$K_{11} = (q_4, \uparrow \nabla 1011, 101 \uparrow \nabla, \uparrow \nabla 1011)$$

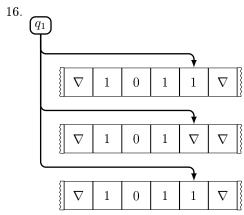


$$K_{12}=(q_1,\uparrow 1011,\, 101\uparrow 
abla,\, \uparrow 1011)$$

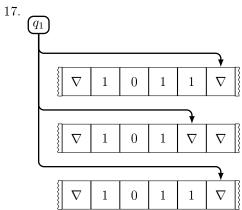
$$K_{13}=(q_1,\,1\uparrow 011,\,101\uparrow \nabla,\,1\uparrow 011)$$



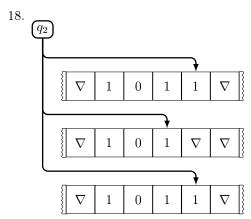




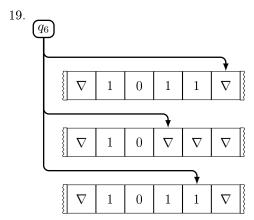
$$K_{15}=(q_1,\,101{\uparrow}1,\,101{\uparrow}\nabla,\,101{\uparrow}1)$$



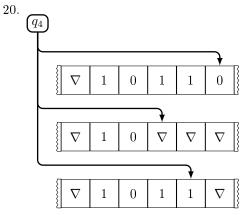
$$K_{16}=(q_1,\,1011{\uparrow}
abla,\,101{\uparrow}
abla,\,1011{\uparrow}
abla)$$



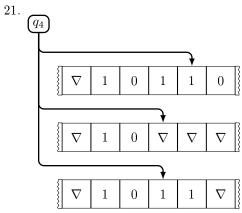
$$K_{17}=(q_2,\,101{\uparrow}1,\,10{\uparrow}1,\,101{\uparrow}1)$$



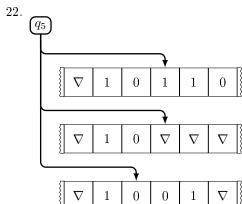
 $K_{18}=(q_6,\,1011\uparrow
abla,\,10\uparrow
abla,\,101\uparrow
abla)$ 



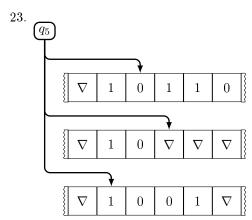
 $K_{19}=(q_4,\,1011\uparrow 0,\,10\uparrow \nabla,\,101\uparrow 1)$ 



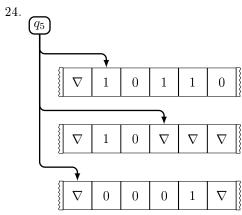
 $K_{20} = (q_4, 101 \uparrow 10, 10 \uparrow \nabla, 10 \uparrow 11)$ 



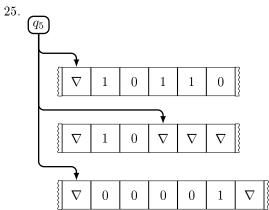
 $K_{21}=(q_5,\,10{\uparrow}110,\,10{\uparrow}
abla,\,1{\uparrow}001)$ 



$$K_{22}=(q_5,\,1{\uparrow}0110,\,10{\uparrow}
abla,\,{\uparrow}1001)$$

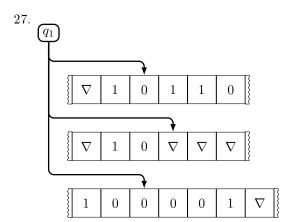


$$K_{23}=(q_5,\uparrow 10110,\,10\uparrow \nabla,\uparrow \nabla 0001)$$

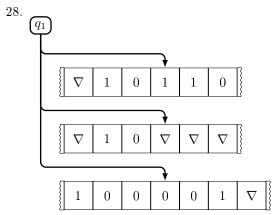


$$K_{24}=(q_5,\uparrow 
abla 10110,\, 10 \uparrow 
abla,\, \uparrow 
abla 00001)$$

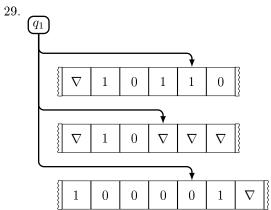
$$K_{25}=(q_1,\uparrow 10110,\,10\uparrow \nabla,\,1\uparrow 00001)$$



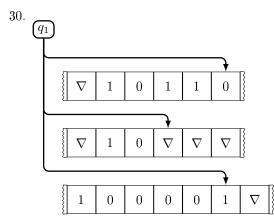
 $K_{26}=(q_1,\,1{\uparrow}0110,\,10{\uparrow}
abla,\,10{\uparrow}0001)$ 



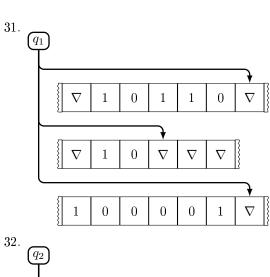
 $K_{27}=(q_1,\,10{\uparrow}110,\,10{\uparrow}
abla,\,100{\uparrow}001)$ 



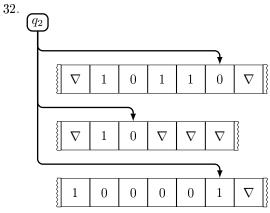
 $K_{28}=(q_1,\,101{\uparrow}10,\,10{\uparrow}
abla,\,1000{\uparrow}01)$ 



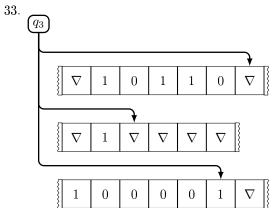
 $K_{29}=(q_1,\,1011{\uparrow}0,\,10{\uparrow}
abla,\,10000{\uparrow}1)$ 



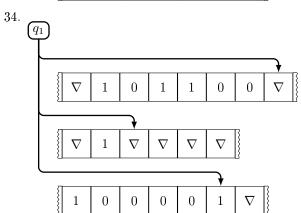
 $\begin{array}{l} K_{30} = (q_1,\, 10110 {\uparrow} \nabla,\, 10 {\uparrow} \nabla, \\ 100001 {\uparrow} \nabla) \end{array}$ 



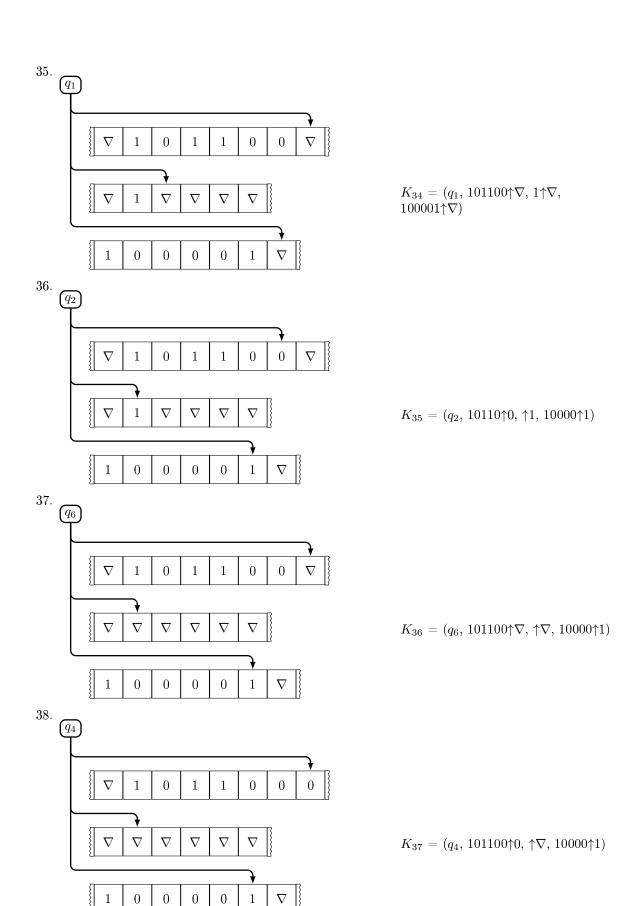
 $K_{31}=(q_2,\,1011{\uparrow}0,\,1{\uparrow}0,\,10000{\uparrow}1)$ 

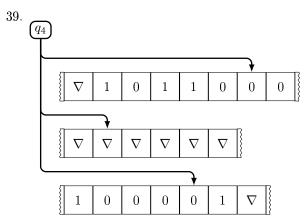


 $K_{32}=(q_3,\,10110{\uparrow}
abla,\,1{\uparrow}
abla,\,10000{\uparrow}1)$ 

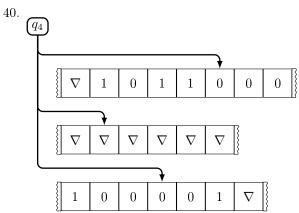


 $K_{33}=(q_1,\,101100\uparrow\nabla,\,1\uparrow\nabla,\,10000\uparrow1)$ 

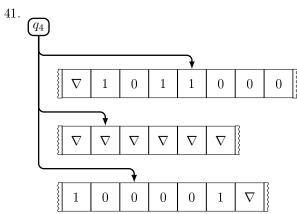




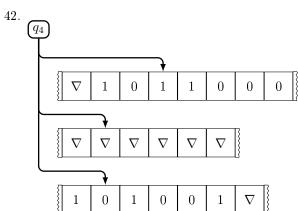
 $K_{38}=(q_4,\,10110{\uparrow}00,\,{\uparrow}
abla,\,1000{\uparrow}01)$ 



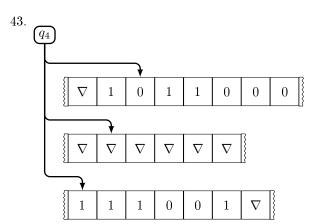
 $K_{39}=(q_4,\,1011\uparrow000,\,\uparrow\nabla,\,100\uparrow001)$ 



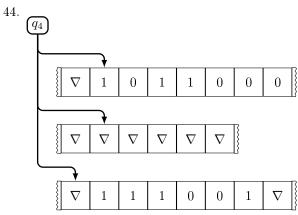
 $K_{40} = (q_4, 101 \uparrow 1000, \uparrow \nabla, 10 \uparrow 0001)$ 



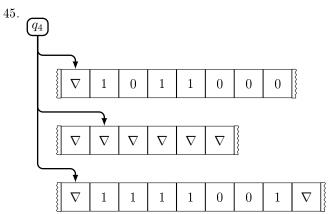
 $K_{41}=(q_4,\,10{\uparrow}11000,\,{\uparrow}
abla,\,1{\uparrow}01001)$ 





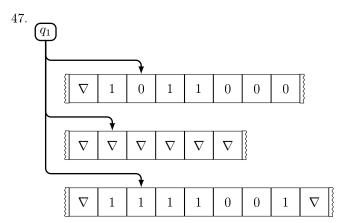


$$K_{43}=(q_4,\uparrow 1011000,\uparrow \nabla,\uparrow \nabla 111001)$$

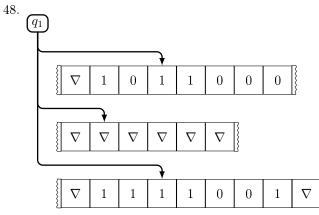


$$K_{44} = (q_4, \uparrow \nabla 1011000, \uparrow \nabla, \uparrow \nabla 1111001)$$

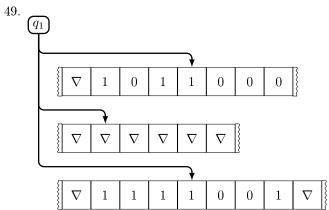
$$K_{45} = (q_1, \uparrow 1011000, \uparrow \nabla, \uparrow 1111001)$$



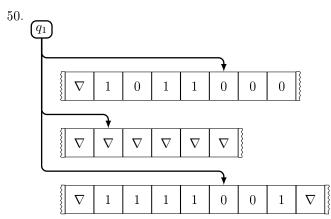
 $K_{46}=(q_1,\,1{\uparrow}011000,\,{\uparrow}
abla,\,1{\uparrow}111001)$ 



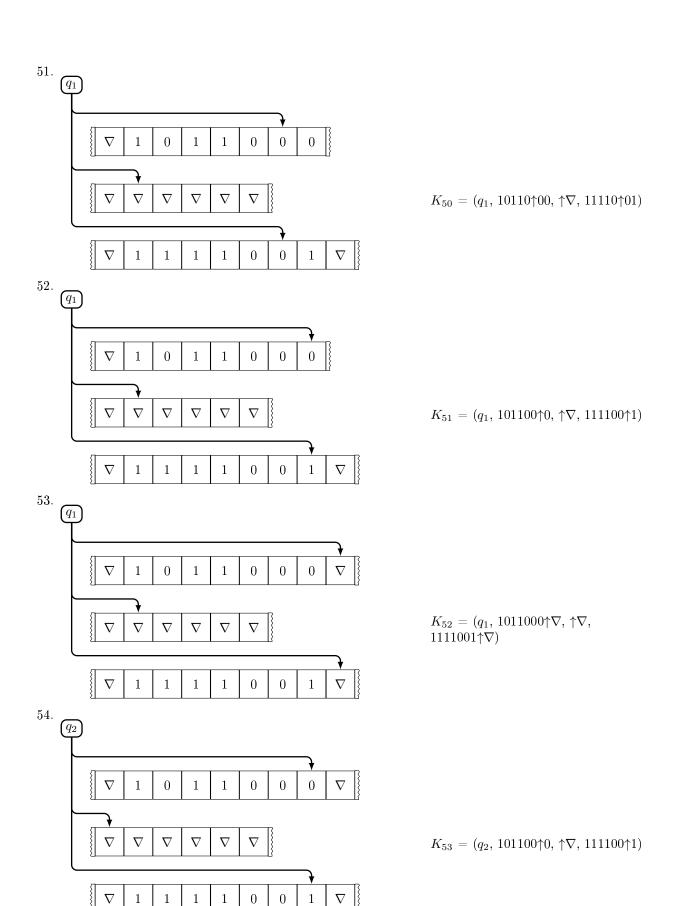
 $K_{47}=(q_1,\,10{\uparrow}11000,\,{\uparrow}\nabla,\,11{\uparrow}11001)$ 

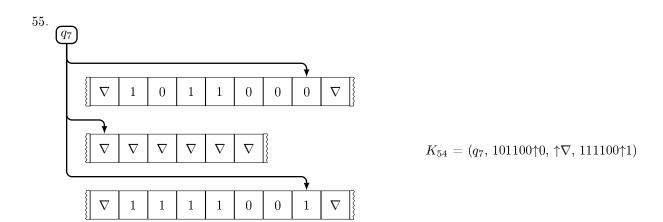


 $K_{48}=(q_1,\,101{\uparrow}1000,\,{\uparrow}
abla,\,111{\uparrow}1001)$ 



 $K_{49}=(q_1,\,1011\uparrow 000,\,\uparrow \nabla,\,1111\uparrow 001)$ 

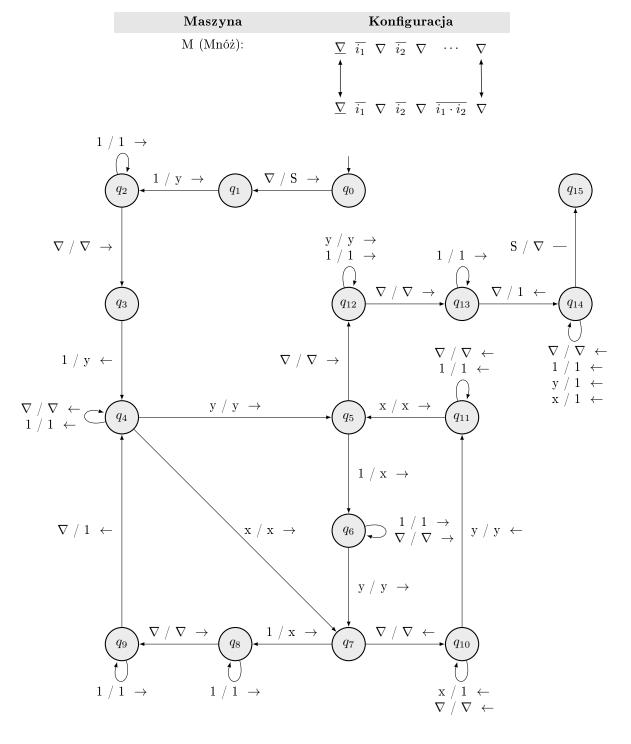




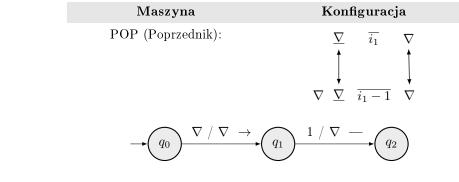
## 5.5 Zadanie 1.70

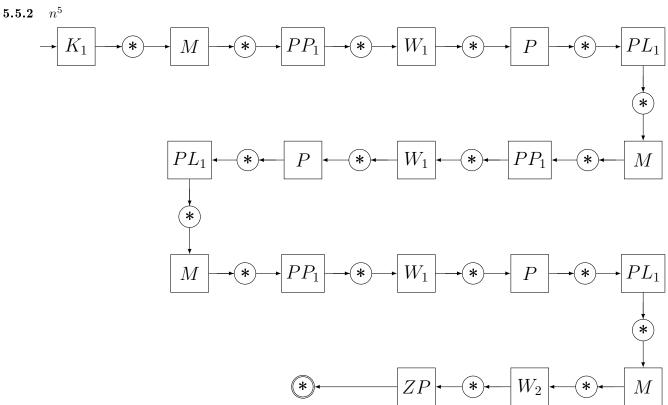
## 5.5.1 Makra

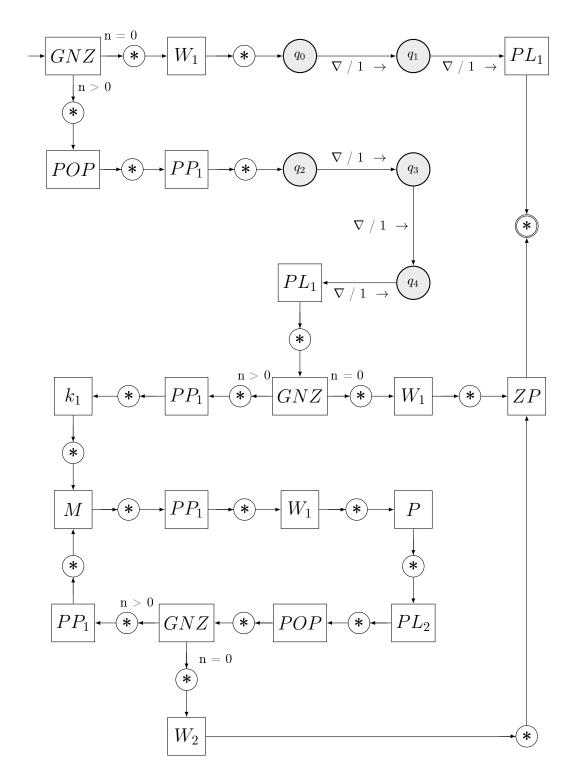
Makro M (Mnóż)



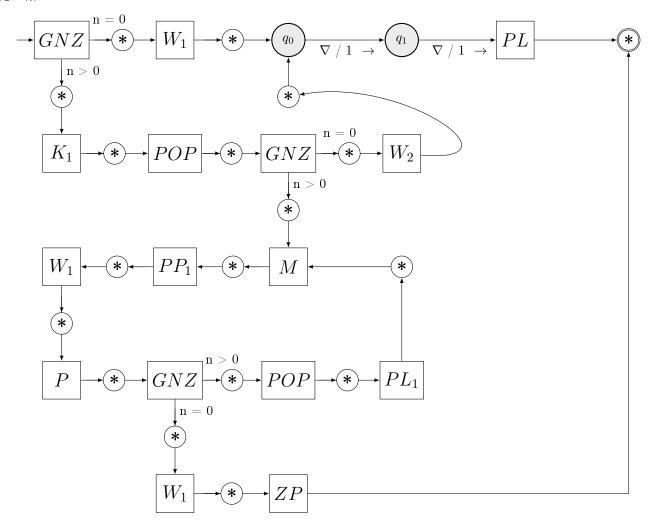
Makro POP (Poprzednik)







## **5.5.4** n!



5.5.5  $n^{n!}$ 

