FILIP PLATA fp 371335

PROBLEM 1

a) We will use COMPACTNESS theorem. Let us define

$A_{i,j,p}$ as ~~[illegible]~~ sentence with indended meaning $c(i,j) = p$, $p \in P$

We have sentences:

a1) $A_{i,j,p_1} \vee \dots \vee A_{i,j,p_k}$ for $i,j \in N$ $(p_1, \dots, p_k \in P)$

b1) $\neg(A_{i,j,p_a} \wedge A_{i,j,p_b})$ for $i,j \in N$, $p_a \neq p_b$

c1) ~~[illegible]~~ $R(p_a, p_b) \wedge A_{i,j,p_a} \wedge A_{i+1,j,p_b}$

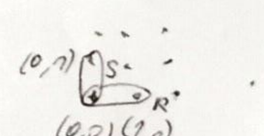
d1) $S(p_a, p_b) \wedge A_{i,j,p_a} \wedge A_{i,j+1,p_b}$

We ask if all those sentences, Δ, are satisfiable. We will prove every finite subset is satisfiable and use COMPACTNESS.

Let Δ_0 be any finite subset of Δ. Let use choose maximum index i and j, and call it N (first and second index of As).

We know we have solution c_N, this means we ~~[illegible]~~ can set to true every $A_{i,j,c_N(i,j)}$. We satisfy every sentence of Δ for indices smaller than N, and thus Δ_0 also.

By Compactness, Δ is satisfiable, and from valuation we immediately obtain a function.

b) We can think of problem as of a latice: 

We need some structure that „cannot go on forever". Well ordered sets come to mind.

Let us take $\{p_1, \dots\}$ set of consecutive prime numbers, as R and S take the „>" comparison, Then we can build structures of the kind: . We easily obtain c_n for any n, but global c is impossible, because it would imply infinite descending chain of natural numbers.

~~[illegible]~~ →

p_k		
p_{k+1}	p_k	
p_{k+2}	p_{k+1}	p_k

pdd