Logic for Computer Scientists 2019/20. Mid-term exam.

1. (10 points) A *system* is a triple (P, R, S) where P is a finite set and $R, S \subseteq P \times P$ are two binary relations on P. For every $n \in \mathbb{N}$, an *n-solution* is a function $c : \{0, \dots, n\} \times \{0, \dots, n\} \to P$ s.t., for every $0 \leq i < n$ and $0 \leq j \leq n$, we have

$$R(c(i,j), c(i+1,j)) \quad \text{and} \quad S(c(j,i), c(j,i+1)). \tag{1}$$

   A *solution* is a function $c : \mathbb{N} \times \mathbb{N} \to P$ s.t., for every $i, j \in \mathbb{N}$, (1) holds.

   (a) Prove that if a system has an n-solution for every n, then it has a solution.

   (b) Would this hold without the assumption of finiteness of P? Motivate your answer.

2. (10 points) Find a relational signature Σ and a sentence of first-order logic φ over Σ s.t., for every $n \in \mathbb{N}$, φ has precisely n^n non-isomorphic models of cardinality n.

3. (10 points) A 3-CNF formula of propositional logic $\varphi \equiv \varphi_1 \wedge \cdots \wedge \varphi_m$ has *overlapping degree* k if there are k propositional variables $\{p_1, \dots, p_k\}$ s.t. every clause φ_i contains at least one literal from $\{p_1, \neg p_1, \dots, p_k, \neg p_k\}$. Show that the SAT problem for 3-CNF formulas of overlapping degree $O(\log n)$ and size $O(n)$ admits an algorithm running in deterministic time $O(p(n))$, where p is a polynomial.

Instructions: Write your solutions to each problem in English/Polish. Submit your solution by committing it in readable PDF or image format to Github by **12pm sharp**. (The deadline is strict since Github will automatically collect the solution by this time.) This can be achieved by scanning a clear picture of physical paper sheets containing your solution. A PDF obtained by a document processing system such as Word/LaTeX is accepted too, but be warned that it may take extra time.

You are allowed to use notes during the exam. You are not allowed to communicate with each other during the exam; we rely on your maturity to abide by this rule.

IMPORTANT: Write your full name and matriculation number (UW email) in readable handwriting on the first page of the submitted solution. Insert the same information in the file `name.txt`. Submitted solutions not satisfying this requirement will not be accepted.