Firm Cyclicality and Financial Frictions*

WORK IN PROGRESS

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Abstract

In this paper we use data from the universe of Danish firms to investigate what drives the cyclicality of firms by firm age and firm size over the business cycle and study the nuanced effects of interaction between size and age using a quantitative heterogeneous firm model. Our main finding is that it is crucial to look at cyclicality by *joint* age-size bin to see the whole picture over the business cycle. Young firms are more cyclical than old firms, but cyclicality by size is more complicated. Among the youngest firms, small firms are more cyclical than old firms, but for older firms this relationship flips, and large firms are actually more cyclical than small firms. We then investigate the role of financial frictions in explaining these results, both by looking at averages over the firm lifecycle, and the cyclicality of financial variables by group. We argue that financial frictions are likely to explain the excess cyclicality of "young and small" firms, but not the excess cyclicality of "large and old" firms. We finally turn to understanding the implications of our results for quantitative heterogeneous firm models with financial frictions. We find that a standard model fails to match the cyclicality of firms by joint age-size bin, and we propose twists (including heterogeneous returns to scale) which bring it closer to the data. These twists affect the power of various government policies, by changing how they affect firms of different ages and sizes.

Keywords: firm age, firm size, cyclicality, financial frictions

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1 Introduction

Which firms drive changes in employment and turnover over the business cycle? Is it younger, growing firms, or older established firms? Is it small firms, or large firms? More fundamentally, why do certain firms respond more to the business cycle than others? The answers to these questions are important for many reasons, particularly that they might help us understand the fundamental shocks and channels driving the business cycle itself.

In this paper we use firm-level balance sheet data from the universe of Danish firms to make three contributions. Firstly, we document the sensitivity to the business cycle of firms by joint age-size bin, and find novel patterns in the data. In particular, while young firms are more cyclical than old, whether large firms are more cyclical than small depends on whether one looks within younger or older firms. Secondly, we use financial data to provide evidence of a role for financial frictions, especially for younger firms. Thirdly, we build a quantitative heterogeneous firm model, and show that standard calibrations fail to replicate our data. We propose extensions to the model that can, and find that these extensions affect the power of various policies, by altering how these policies transmit through the firm age and size distribution.

A long literature dating back to Gertler and Gilchrist (1994) emphasizes that firm size may act as a proxy for financial frictions, and thus argues that small firms being more cyclical than large firms could be interpreted as evidence in favor of financial frictions. More recently, it has been shown that firm age is a more important predictor of both the average level and cyclicality of firm growth than firm size (see Fort et al. (2013); Haltiwanger et al. (2013), for evidence from the US). Small firms are only more cyclical to the extent that they tend to be young, while older small firms display no excess cyclicality. Thus, the relationship between age, size, and cyclicality, including any underlying role of financial frictions, is complicated, and all elements must be studied at once in order to create a full picture.

With this in mind, what distinguishes our paper from previous work is i) the coverage and detail of our data, and ii) that we exploit this coverage to investigate not just patterns by firm age or size separately, but by joint age-size group. We have rich data for all firms in the Danish economy, and thus can study the behavior of younger and smaller firms, who are typically unlisted and hence not available in public databases of listed firms. Additionally, our data is detailed, including measures of employment, turnover, firm age, and a broad range of financial data from firms' balance sheets. Accordingly, we can directly investigate the role of financial frictions across the whole firm age/size distribution using balance sheet data, which are typically hard to find for young, unlisted firms.

Our first contribution is to analyze the lifecycle averages of firms by (joint) age and size, and to analyze the cyclicalities of these various firm groups. We split firms into four age bins covering entrant firms (age 0-3 years) up to mature firms (age 20+). We split into four firms size groups, defining size by employment, focusing on the 0-30th, 30-60th, 60-90th, and 90+ percentile groups. All our analyses are carried out on joint age-size bin, meaning that we distinguish between, for example, "old and small" versus "old and large" firms. Looking first at the lifecycle average of basic firm outcomes reveals the importance of both the age and size dimensions. The size distribution is wide, with the smallest (largest) bins having average employment of 1 (30) employees respectively. However, even conditioning on firm size, firm age is a strong predictor of the *growth rate* of a firm, with young firms often growing by over 10% per year, while older firms grow very little on average.

Moving on to firm cyclicality, our main result is to show that the cyclicality of firms depends not just on their age or size but, crucially, on the *joint* relationship between age and size. In particular, we measure cyclicality by regressing firm-level growth rates on aggregate GDP growth, with interactions for joint age-size bin. We first document that young firms are more cyclical than old firms, even conditioning on firm size. Since young firms are likely to be financially constrained, this fact could be due to financial frictions, a topic to which we turn later in our analysis. Secondly, we find that the relationship between firm size and cyclicality is more complicated, and actually depends on the firm age group one studies. Among young firms (age 0-3) we find that small firms are more cyclical than large firms. However, among old firms (age 20+) we find the opposite, that large firms are more cyclical than small. For intermediate age groups, firms of all sizes have similar cyclicalities. This finding highlights the importance of studying firm age and size together. Attempting to infer the role of a cyclical channel which should operate via firm age is not possible by observing data on firm size, despite firm age and size being correlated. At the business cycle level, the relationship between age and size is confounded by forces which move firms of different sizes differently depending on their age.

The excess sensitivity of younger firms regardless of size is particularly interesting, because it suggestive of a role for financial frictions. As shown by Haltiwanger et al. (2013), small firms can be divided into small young firms, and small old firms, both with very different behaviors. Small and old firms can be thought of as businesses who have a small scale of operation, and have reached this size and are not drastically growing or creating jobs. Small and young firms, on the other hand, include firms who are growing very fast, having just recently started, and are creating many jobs on the path to potentially becoming much larger firms in the future. It is this second group of young and small firms who are particularly likely to be affected by financial frictions. Raising finance may be hard for any small firm, because accessing bond or equity markets is typically something that only larger firms can do. However, only small and young firms are likely to be up against their borrowing constraints, since they are in the process of growing, and hence this difficulty in accessing finance will have a material impact on their real outcomes such as hiring or turnover. Small and old firms may find it hard to access finance, but this will be less of a concern for them since they are not trying to grow, and so may have had time to accumulate financial buffers in order to protect themselves from financial shocks.

Our final empirical results investigate the lifecycle and cyclicality of firm-level financial variables directly, in order to shed light on the role of finance in our cyclicality results. Our ability to do so follows from our dataset, which contains detailed financial data across the whole age and size distribution. We look at measures of total firm debt, as well as their leverage, measured as the debt to asset ratio. Starting with the firm lifecycle, we find that leverage is higher at young firms, even conditioning for firm size. This suggests that young firms rely on debt more extensively, and hence are more likely to be financially constrained. Moreover, young firms also have the highest growth rates of debt and leverage, suggesting that they are actively trying to increase their debt, and hence more likely to be affected if the access to debt is restricted in a recession. Older firms, on the other hand, all have shrinking leverage ratios, showing that they are reducing their reliance on debt (presumably either by accumulating retaining earnings or switching to equity financing). Finally, we find that, after controlling for age, firms of all sizes have very similar leverage ratios, as well as similar growth rates of their leverage. This suggests that financial constraints are unlikely to bind more at larger firms than small firms. Our results on the cyclicality of leverage ratios are, on the other hand, somewhat less consistent, perhaps due to valuation effects in the measurement of assets over the business cycle. However, overall the results paint a suggestive picture of the role of finance in driving cyclicality over the business cycle. We observe that young firms are more cyclical than old, which is plausibly linked to financial frictions since young firms have higher leverage. On the other hand, we observe that large firms are more cyclical than small (among older firms) which is unlikely to be driven by finance, since these firms all have similar leverage levels.

Our final contribution is to investigate the implications of our results for quantitative heterogeneous firm models. Our strategy is to first set up a relatively "standard" model, and confront it with our new facts on the cyclicality of firms by joint age-size bin. We then discuss the modifications needed to the model in order to match our data, and the policy implications of these modifications.

Our "standard" calibration builds on the seminal work of Khan and Thomas (2013), who set up a heterogeneous firm model with financial frictions, which they use for business cycle analysis. Firms are born poor, and hence financially constrained, and grow out of these financial constraints as they age and accumulate capital. We extend the model to allow permanent productivity differences across firms, in order to replicate the size distribution of firms in our Danish dataset. Our calibrated model matches the age and size distributions of firms well, making it a natural laboratory to study the cyclicality of firms by age and size over the cycle.

Our first set of quantitative results investigate the response of this standard model to a range of aggregate business cycle shocks. We study two financial shocks (a tightening of collateral constraints and a rise in discount rates) and one real shock (a decline in aggregate TFP). While the effects of these shocks differ, none of them are able to generate the full joint age-size cyclicality patterns we find in our data. For example, a tightening of the collateral constraint hurts young firms more than old firms, in line with the data. However, it also hurts small firms more than large firms, which is contrary to the data. This follows from the correlation between age and size in the model: young firms tend to be smaller, since financial constraints limit their size. By contrast a discount rate shock or TFP shock affect old firms more than young (and hence also large firms more than small) since financially unconstrained firms are more able to adjust in response to the shock.

Further, no combination of these shocks can replicate our patterns, since the responses by age and size are so tightly linked. Hence, our data reveal that tweaks are needed to the basic framework. We wish to stress that this is not meant to suggest that there are deep failings of the basic model, which already delivers a lot, and on which our analysis indeed naturally builds upon. Instead, our results point to areas where the model can be extended, in light of new data and the specific context of the period we study. Nonetheless, these changes have interesting implications for the standard model.

Having established where the model deviates from our data, we then extend the model along two dimensions in order to allow it to replicate our facts. Firstly, we introduce heterogeneous returns to scale across firms. As we show, heterogeneous returns to scale are a natural candidate for explaining why smaller firms are less cyclical than large, since returns to scale simultaneously drive firm size and their responsiveness to shocks. By adding this feature to the model, it can simultaneously generate that young firms are more cyclical than old, while large firms are more cyclical than small. Secondly, we introduce differences in the initial net worth that firms enter with, depending on their size. Specifically, the calibration suggests that large entrants start less financially constrained than small entrants. According to our data, this follows from the fact that, among large firms, young firms are no more cyclical than old firms. Hence there should be no excess cyclicality of the young—which is driven by financial frictions in our model—among large firms. With these two twists, we show that our new calibration is able to replicate the cyclicality of firms by joint age-size bin in our data.

We finally use our calibrated model to assess the effects of two kinds of policies aimed to stimulate the economy. We are particularly interested in how these policies transmit through the joint firm age-size distribution, and how this differs between our model and the standard calibration. The first policy we call an "incentive" type policy, which consists of a labor subsidy. This type of policy acts to change the incentives faced by firms to hire and produce. The second policy we call a "balance sheet" policy, which consists of giving debt forgiveness to firms in order to boost their net worth. We find that the incentive type policy is stronger in our calibration, while the balance sheet policy is weaker. This follows directly from the way that firms of different ages and sizes respond to the policies. The incentive policy becomes stronger, because large firms become more responsive to the policy and small firms become less responsive, and large firms dominate aggregate employment. The balance sheet policy becomes weaker because we identify large young firm as being less financially constrained than small young firms, and hence financial policies act more strongly on a group of firms less important for aggregate employment.

Related Literature. Our paper fits into the broad literature investigating the importance of firm size and firm age in determining firm cyclicality over the business cycle. An early contribution is Gertler and Gilchrist (1994), who investigate the cyclicality of small versus large firms and find that small firms are more sensitive to periods of credit market tightening than large firms. Khan and Thomas (2013) show that small firms contracted more than large firms during the financial crisis.

On the other hand, Moscarini and Postel-Vinay (2012) find that larger firms (in terms of number of employees) are more cyclical, when aggregate conditions are measured using the (HP-filtered) level of the unemployment rate. Similarly, Mian and Sufi (2014) show that larger establishments contracted more in areas with larger declines in house prices. Fort et al. (2013) discuss the conflicting results by firm size, and add age to this analysis. They find that young firms are more cyclical than old firms, and that this difference is much more important than the differential between small and large firms. While they do not have direct financial data at the firm level, they use state-level house price data to argue that financial frictions may drive this result.

This paper is most related to papers which investigate differences in cyclicality across firms using direct financial data at the firm level, paying particular attention to financial frictions. Sharpe (1994) uses Compustat data to document that high leverage firms are more cyclical than low leverage firms. Giroud and Mueller (2017) combine Compustat data with establishment-level employment data to show that the decline in house prices during the Great Recession, as investigated by Mian and Sufi (2014), was transmitted to declines in employment through high leverage firms. Conversely, Ottonello and Winberry (2018) use Compustat data find that firms with low default risk, including those with low debt burdens, are the most responsive to monetary shocks. Relative to their paper, our sample includes non-listed firms, and thus younger firms who may behave differently to financial frictions than older, listed firms. Jeenas (2019) investigates the role of liquidity and leverage in driving heterogeneous investment dynamics, and finds that leverage ceases to be important once liquidity is controlled for.

Within this literature are papers which directly investigate the interaction between financial constraints and firm size or age. Chodorow-Reich (2014) matches firms to banks and finds that firms borrowing from financially distressed banks contracted more during the Great Recession, and that this effect is largest at smaller firms. Crouzet and Mehrotra (2020) utilise new confidential US Census data to investigate cyclicality by firm size. They find that only the 1% of largest firms by balance sheet are more cyclical, and that cyclicality appears to be largely unrelated to measures of financial frictions. Relative to their paper, we are also able to focus on firm age and have data on employment. We find that financial variables appear to be important when investigating the age dimension.

Cloyne et al. (2018) use data for the US and UK to show that younger, non-dividend paying firms exhibit the largest and most significant changes in investment following monetary policy shocks. Due to data availability, they measure age as time since incorporation, rather than foundation, whereas we are able to measure age since foundation, and focus on overall cyclicality rather than the response to identified monetary policy shocks. Dinlersoz et al. (2018a) merge balance sheet data from Compustat and Orbis into the US Longitudinal Business Database (LBD). Similarly to our analysis, they are able to analyse both private and public firms and can measure firms employment and age since foundation. Since their balance sheet data are not from registry sources, they do not have complete coverage and under-represent low revenue firms in their sample. They argue that small private firms are plausibly financially constrained both before and after the financial crisis, while larger private firms may have only become constrained during the crisis, and large public firms appear to never be financially constrained.

Early contributions developing the theory of financial frictions include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999). More recently, Jermann and Quadrini (2012) develop a theory of financial shocks over the business cycle, and Khan and Thomas (2013) build a heterogeneous firm model with financial shocks. Finally, we connect to a literature investigating the cyclicality of firm financing. Using aggregate data, Jermann and Quadrini (2012) investigate the cyclicality debt and equity issuance. Covas and Haan (2011) show that the cyclicality of financing is different across firms of different sizes, with the procyclicality of equity issuance decreasing monotonically with firm size. Crouzet (2017) studies the choice of bank and bond financing in a calibrated heterogeneous firm model calibrated using the US' Quarterly Financial Report.

Poeschl (2018) studies the business cycle dynamics of maturity structure, and finds that the aggregate share of long-term debt in total debt is pro-cyclical, and more so for small firms. Nikolov et al. (2018) use firm-level data to identify the sources of financial constraints for different groups of firms, and find results which favour trade-off models for larger Compustat firms, limited commitment models for smaller firms, and moral hazard models for private firms.

The rest of this paper is organised as follows. In Section 2 we discuss the data and construction of our key variables. In Section 3 we present our empirical results on the lifecycle and cyclicality of firms by age and size. Finally, in Section 4 we present our quantitative model findings.

2 Data

In this section, we briefly describe the firm-level micro data that we use in our empirical analysis. We begin by discussing the datasets we use, and then the construction of our key variables of interest.

2.1 Datasets: FIRE and FIRM

Our data sources are confidential administrative datasets, covering the universe of Danish companies. The data is provided by Statistics Denmark (DST). In order to analyse firm outcomes and financial balance sheet data together, we merge two datasets ("data registers"): the FIRE dataset ("Regnskabsstatistikken"), which broadly contains data on accounting variables, is merged with the FIRM dataset ("Firmastatistik"), containing data regarding "economic, employment and accounting information at company level. Both datasets are yearly.

It is worth noting that the quality of this data is generally believed very high, as Statistics Denmark is a government agency, and most of the variables we use is originally collected by SKAT,[1] the national

[1] Sales, assets, liabilities, investment and information about employment based on payroll.

tax authority. Additionally, DST also runs independent checks on the datasets. Individual firms are identified by unique number that is generated at the time of registration, regardless if the given firm is actually active or not. The merging of the datasets is done using this identifier, and thus provides exact matches.

Subject to some minimal threshold on economic activity,[2] all firms are legally obliged to report data to SKAT or DST, which are then collected in these databased. We drop all observations that we deem as inactive by our definition, that is firms that provide no information about employment, sales, value added, or profits.

We also drop all firms that never in their life employ more than one worker.[3] Finally, we also drop firms listed as non-profits as well as entities controlled by government at any level. In our baseline exercises, we include only firms that do not exit in the current or the next year. We thus do not separately investigate the role of firm entry or exit in driving cyclicality.

The dataset from DST has near universal coverage of all firms in the Danish economy over this period. Our final dataset is a unbalanced panel of approximately 2 million firm-year observations and covers the time periods 2001-2019, with approximately 100,000 firms per year (in the early 2000's the number of firms is around 80,000 but it grows to 120,000 at the end of the sample). Crucially, our dataset therefore contains firms which are both publicly listed on the stock exchange and privately owned, and which span the entire distribution of firm age and size. Combined with the fact that we have financial data for all firms, this makes our dataset uniquely suited to studying the role of financial frictions across the whole firm life cycle, especially at younger firms. This is also very different to the datasets like Compustat, more similar to ORBIS.[4] Dinlersoz et al. (2018b) build a dataset that matches LBD with ORBIS and COMPUSTAT and covers 2005-2012. One advantage of our dataset is thus longer timeseries which is less centered around the run up the financial crising and its aftermath. Crouzet and Mehrotra (2020) use the micro data from the US Census Bureau's Quarterly Financial Report. Their dataset is at quarterly frequency and covers longer time period, but the total number of observation is lower, suggesting that it ours still covers smaller firms.

Finally, for our cyclicality analysis we will compare firm-level growth rates to an indicator of aggregate economic conditions. As our aggregate indicator, we follow Crouzet and Mehrotra (2020) (CM hereafter) and use the growth rate of real GDP. Aggregate GDP data are publicly available and collected from DST National accounts.

2.2 Key variables

Since or focus is on firm cyclicality and financial frictions, we will mostly use firm production and balance sheet variables from our merged dataset. On the production side, we use data on turnover, employment, and investment. On the financial side, we use data on total assets, total liabilities, holdings of cash or cash equivalents, and the stock of long term debt. We also use data on a firm's sector of operation.

We define the firm size by its employment. For robustness, we also follow Crouzet and Mehrotra (2020), and measure firm size by the value of its assets (we report the alternative results in the appendix A). The firm size measure is thus not static within a firm, and varies as the firm grows or shrinks as it ages and is hit by shocks. For our empirical work, we put firms into bins based on certain quantile

[2] In most situation, firms that report employment that corresponds to less than 0.5 full time worker are considered inactive by DST, but still present in our data.

[3] We do this to eliminate sole proprietorship firms and also firms that exist due to tax optimisation purposes.

[4] However, recently there has been some doubts whtehr ORBIS dataset is trully representative, see Bajgar et al. (2020).

thresholds of size across the population of firms active that year. We introduce four size bins: 0-30th, 30-60th, 60-90th and 90+. In Figure **??** we plot the asset thresholds defining these size bins, and how they evolve over time. The thresholds are relatively stable over time, with a minor expansion of the largest firms at the end of the sample.[5] since the financial crisis. The firm size distribution is heavily skewed: the top size three bins (containing only 10% of the largest firms) represent over 70% of aggregate employment.

We measure firm age from the moment the firm is registered.[6] This is measured as the first time the firm's unique identifier appears in our database. This is thus the true age since foundation of the firm, which distinguishes us from other datasets which can only measure age since, for example, the firm was publicly listed on stock markets. As with our size measure, in our empirical work we do not work with age directly, but put firms into age bins. We use four age bins: 0-3, 4-8, 9-19, and 20+ years old.

2.3 Estimation framework

To study the intricate interplay between firm size and age allow interaction between size and age bins, so being large might have different implications for small and large firms. First, we construct averages of various variables of interest by size and age and then we directly study the firm cyclicality in analogous setting. Formally, we run two types of regressions:

$$x_{i,t} = \sum_j \sum_k \alpha_{j,k} \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} + \sum_l \gamma_l \mathbb{1}_{i \in S(l)}, \tag{1}$$

$$\hat{g}_{x_{i,t}} = \sum_j \sum_k (\alpha_{j,k} + \beta_{j,k} y_t) \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} + \sum_l (\gamma_l + \delta_l y_t) \mathbb{1}_{i \in S(l)}, \tag{2}$$

where $\hat{g}_{x_{i,t}}$ denotes the firm-level normalised growth-rate of either turnover or employment at firm i. The indices j, k, and l index firm size bins, firm age bins, and firm sectors respectively.[7] $\mathbb{1}_{i \in I_t^j}$ is an indicator variable for firm i being in size group j at time t, and similarly for age and sector.

The regression equation (1) is used to gain insight about basic distribution of variables of interest and we do so by reporting the coefficients α. While we plot the results with age bins on the x-axis, the results should not be interepted as life-cycle profiles as the dataset is not a balanced panel due to firm exit. Equation (2) is used to study firm cyclicality. For each size bin, α_{jk} captures the marginal effect on the average growth rate of firms of being in that size bin. We are more interested in the β_{jk} parameters, which capture how the firm-level growth rates, $\hat{g}_{x_{i,t}}$, are differently related to the aggregate growth rate, $y_{i,t}$. The interpretation of β_j is that a 1pp increase in aggregate growth is on average associated with a "β_{jk}"pp increase in firm-level growth for firms in size group j, on top of any additional effects captured by age or sector. Thus, the β_{jk} capture the cyclicalities of each firm size$\times$age group. Similarly, δ_l coefficients control the cyclicalities of the different age groups and sectors.

To analyse firm-level cyclicality, we regress firm level growth rates, $\hat{g}_{x_{i,t}}$, on aggregate growth, y_t, using dummy variables to separately estimate the cyclicality of different groups of firms. We include industry-level controls to strip out the potentially differing average cyclicality of different industries. Thus, the effects when comparing coefficients from different, for example, age groups should be inter-

[5]For assets, the pattern is similar, see figure **??**.

[6]Given that it takes a litte time to start a new firm, we believe there is not a large need to formally register the firm long before the firm becomes economically active.

[7]We use 10 sector industrial classification.

preted as within-industry effects.

We measure firm-level outcomes using the normalised growth rates suggested by Haltiwanger et al. (2013). Let i index firms and t index years. For any firm-level variable $x_{i,t}$, we measure growth from $t-1$ to t as

$$\hat{g}_{x_{i,t}} \equiv \frac{x_{i,t} - x_{i,t-1}}{\frac{1}{2}(x_{i,t} + x_{i,t-1})}.$$

As discussed by Haltiwanger et al. (2013), this growth rate, which uses the average of the current and past value as the denominator, rather than just the past value, is more robust and typically has better properties in firm-level data. For our cyclical measure, we use the standard growth rate of aggregate GDP, which we denote as $y_t \equiv \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$.

This specification is an extension of CM's regression (their equation (1)) to include firm age categories interacted with the size bins. The equation is essentially a regression of firm-level growth rates on a constant term and the aggregate growth rates, with interaction terms allowing for group-specific means and loadings on the aggregate growth rate. Dinlersoz et al. (2018a) also estimate the effect of age and size. To capture the effect, they employ quadratic terms in age or (log of) size. The contribution of the present paper is to measure the effect of size *as it changes with age* (or vice versa). Technically the difference is in inclusion of the interaction term and using size and age bins rather than having a parametric specification for the effect. They also study the effect of the Great recession, which we do not do.

3 How does firm age and size determine firm outcomes?

3.1 Levels

To give an overview of the distribution of various variables in the data, people usually include a table that show how different variables change as firms' age or size changes. These moments are provided in table 1.

Table 1: Basic description along age and time dimension

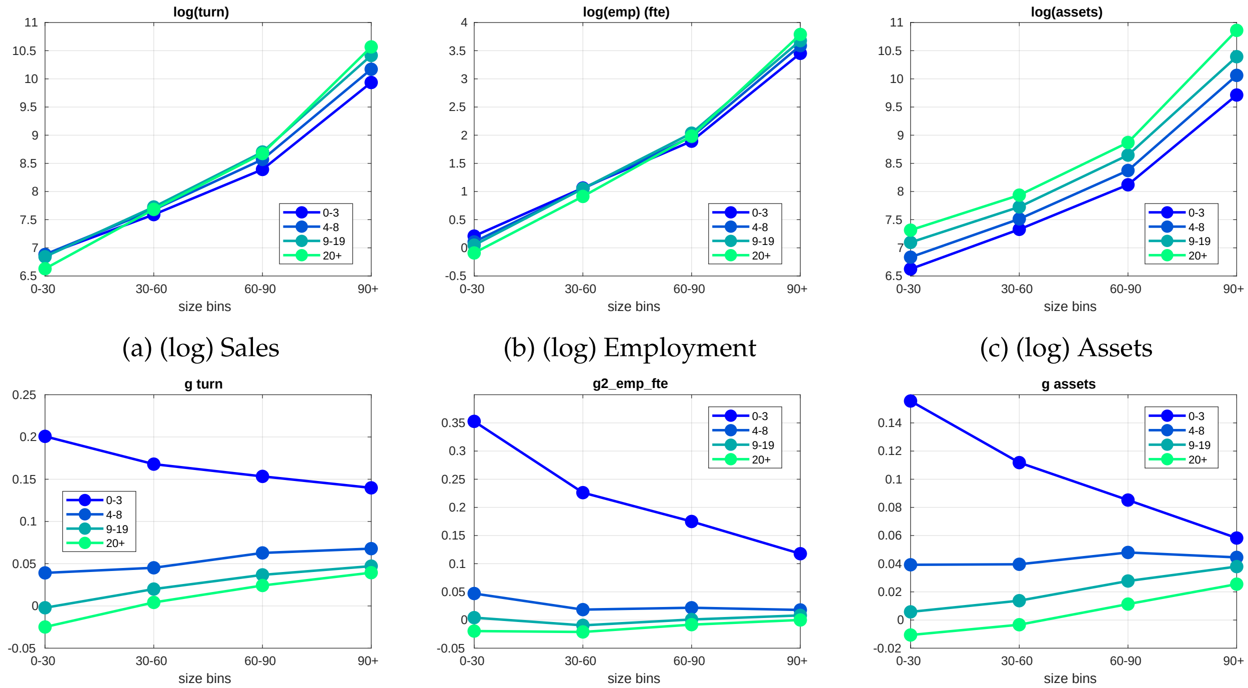
	Age groups				Size groups			
	0-3	4-8	9-19	20+	0-30	30-60	60-90	90+
Employment	7.6	12.2	17.5	26.1	1.6	4.1	11.2	115.9
Sales	17453	28044	45902	93202	4224	9344	25006	320682
Assets	18776	32856	56861	128062	10119	20448	24032	360372
Debt	10594	17611	30200	69014	5236	10806	12759	192740
Equity	4622	11136	21028	49048	4191	7606	9889	143875
DA (w)	0.85	0.79	0.69	0.62	0.75	0.75	0.73	0.68
CA (w)	0.17	0.16	0.15	0.12	0.18	0.15	0.13	0.10

Note: Sales, assets, debt and equity in thausands of DKK.

However, in the present paper we argue that the interaction between size and age is important, so we need the interaction between size and age. However, we would need a 3D table for that. So instead, we show series of plots where we group firms by age groups and plot how various characteristics change along the size dimension (after taking away the sector contributions).

Figure 1 shows the distributions of sales, employment and assets. Our definition of size is based on employment (unlike CM, where the size is defined by assets), so by construction the distribution gradient over age is rather flat for employment and also to certain extent sales. In contrast, the assets are clearly increasing with age, to the extent that entrants (the first age bin) in the second size bins have roughly the same assets as firms in the first size bins that have been around for 20+ years (the last age bin). If we interpret assets as firms' capital, production function optimisation would suggest that factors should grow proportionally. One possible explanation is via selection, that is that the firms that survive to 20+ years on average use more capital centered production technology.

Figure 1: Average levels with full interaction of size and age (regression (1)).



(a) (log) Sales (b) (log) Employment (c) (log) Assets

(d) Growth rate of sales (e) Growth rate of employment (f) Growth rate of assets

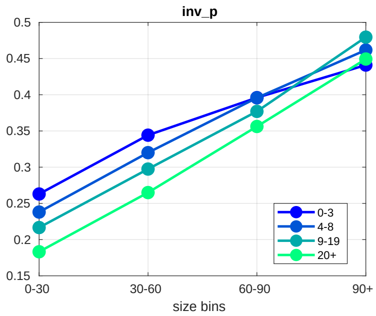
Note: Average level is computed as coefficient α_{lk} *from regression* (1)*. Different lines correspond to firms of different size bins whereas firm age bin is on x axis.*

The panels in the second row of figure 1 shows the growth rates of the same variables by size $\times$ age groups. Not suprisingly, the growth rate for all variables is the highest for the entrants, suggesting that on average firms start below their optimal size. The entrants that start in the smallest bin also grow the fastest, both the inputs (employment and assets) and also output-sales, suggesting that the dispersion of the starting size is actually than the disperison in long run optimal size. Figure 1 thus shows that Gibrat's law, i.e. that the growth rates should be independent of size, fails. Separating the different age groups also allows us to notice that the directionality of the failure is different for entrants, for whom it fails the most entrants and firms of all other ages.

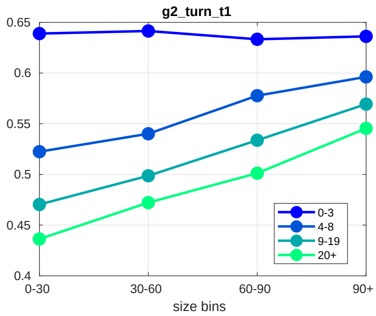
Moving beyond entrants, we find that older firms on average grow slower, but the age gradient is much lower once firms move beyond the first age group, i.e. before the age of 4. While firms of all sizes seem to grow at somilar rates in terms of employment, the pattern reverses for sales and assets; the larger firms tend to grow more on average once we move beyond the first two age bins.

To get more insight about the differences in growth of firms of different sizes and ages, we look

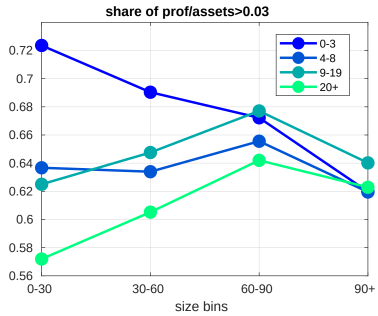
Figure 2: Description of ditribution of investment, growth of sales and profitability, full interaction of size and age (regression (1)).



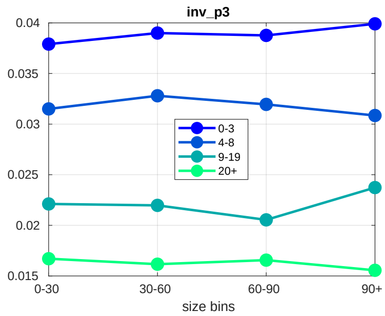
(a) Share with positive investment



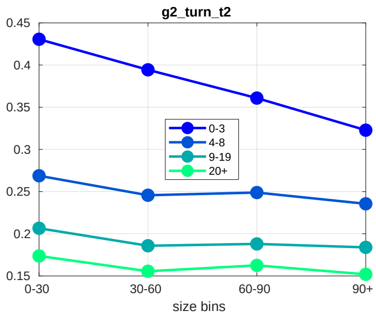
(b) Share with above median growth in sales



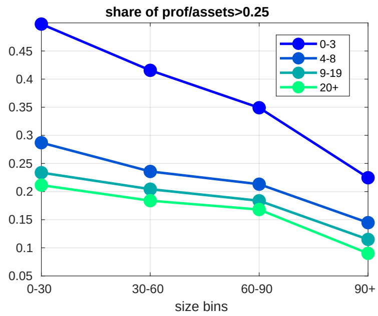
(c) Share with midly positive profits



(d) Share with high investment



(e) Share with high sales growth



(f) Share with high profits

Note: high investment defined as investment/assets>0.25. Mild profits are defined as profits/assets>0.03, high profits are defined as profits/assets>0.25. High sales growth is defined as sales growth larger than 75th percentile of sales growth distribution.

beyond the averages. In figure 2 plot the share of firms (of the given size x age bin) that perform better than some threshold in either investment, profits and growth rate of sales.

There is a difference with respect to investment. Whereas the rate at which firms engage in large projects (invesment/assets$>$ 0.25, panel (d)) does only depend on age and it falls with age, the rate at which firms engage in small or any projects, investemnt$>$ 0, is increasing in size much more than it is decreasing in age. This suggest that investment is lumpy which makes it difficult for small firms to engage in small projects (among small firms, it is still the younger than are more likely to invest).

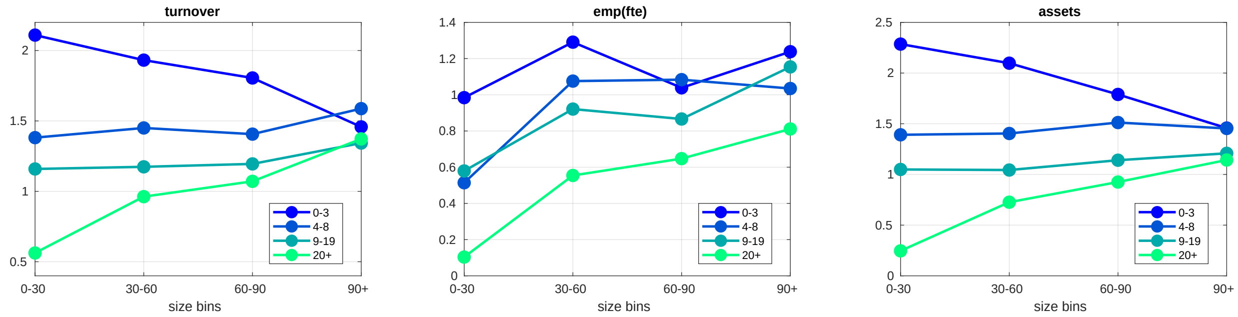
Quantitatively, the distribution of sales growth and profits are similar, but they are unlike the investment. The share of firms is that is achiving large sales growth and high profitability is decreasing is both size and age, with the largest difference being between the entrants and others. In other words, there are more highly growing and profitable small entrants compater to any other size-age combination.

3.2 Cyclicality

In this section we investigate the cyclical sensitivity of firms by size and age. We do so without any reference to financial frictions, or other underlying causes of the differing levels of cyclicality. Thus, the results in this section are meant to be interpreted as theory free, and provide us with our basic stylised facts about firm cyclicality in the Danish economy. Following Crouzet and Mehrotra (2020) "cyclical sensitivity" refers to the extent that a worsening in aggregate conditions is systematically associated with declines in outcomes at firms of various groups. We will informally use "excess sensitivity" to refer to a group of firms having particularly higher cyclical sensitivity relative to a baseline comparison group. Compared to Crouzet and Mehrotra (2020), we disentangle the effect of size and age by allowing the sensitivity to differ for each combination of size and age.

We are primarily interested in the effect of firm age and size on cyclicality, and so will not display the sectoral coefficients, which we treat as control variables. We present our results from regression specification given by equation (2) and we also plot the relevant coefficients graphically in figure 3. We have a separate cyclicality coefficient, $\beta_{j,k}$, for every age-size pair. Indeed we detect a distinct non-linearity in the age-size relationship. In particular, for smaller firms, those in the 0-30th size percentile, the difference in cyclicality between young-small firms and old-small firms is dramatic. The difference in cyclicality between young and old firms remains, but is increasingly less dramatic, within groups of larger firms.

Figure 3: Predicted cyclicality with full interaction of size and age (regression (2)). Apart from the youngest firms, cyclicality increases with age and the level is shifted down by as firms get older. For the youngest firms, the increase in cyclicality by size is much less apparent than for older firms (in the case of employment) or it even falls with firm size (turnover).



(a) Sales (b) Employment (c) Assets

Note: Cyclicality is computed as coefficient β_{lk} from regression (2), which predicts (average) response in sales or employment (in pp) to 1pp change in aggregate gdp growth. Different lines correspond to firms of different age bins whereas firm size bin is on x axis (note that the axes are reversed compared to the previous graphs).

The results show two general patterns. First, younger firms are more cyclical across most of size distribution (the results for the largest size bins are mixed). Second, the effect of size is different for entrants and for firms of all other sizes. For the entrants, the larger firms are on average less cyclical. In contrast, among all the older firms, larger firms tend to be more cyclical and gradient is the largest for the firms in the oldest age bin. This pattern is clear for sales and assets and is somewhat muted for employment. The corollary is that the dispersion in cyclicality among the smallest firms is much larger than among the largest firms. The small young first are the most cyclical and the small old first are the least.

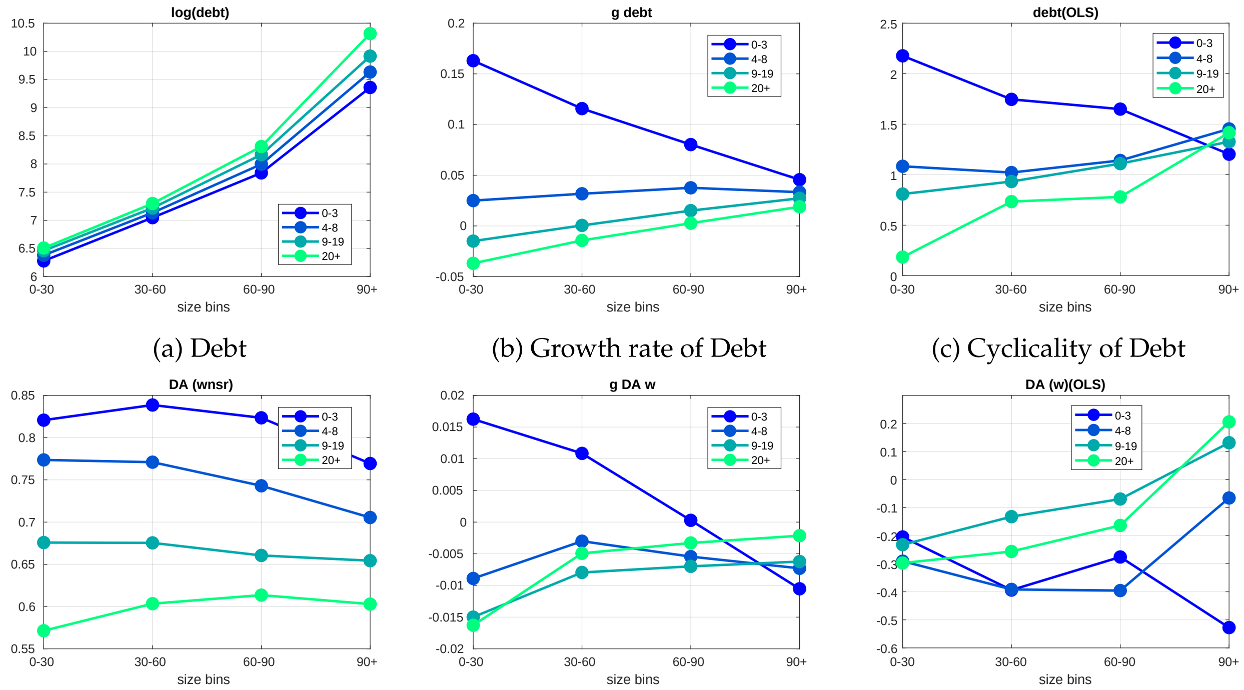
These results explain why size was found to be a less important predictor of cyclicality than age, since size is only an independent predictor of cyclicality amongst a subset of younger firms. This shows that size alone cannot be the sole factor driving the results. In other words, the reason why some old firms are small is likely to be different to why many young firms are small. Consequently, policies that might target all small firms, might have different effects on small firms that are old and those that are young.

One reason why firms grow is that they accommulate consumers. Larger customer base might provide insurance against fluctuations in demand, as long as the demand coming from individual customers is not perfectly correlated. This is consistent with our empirical findings, because we measure correlation with the business cycle, so a shock that is correlated among customers, not uncoditional volatility.

3.3 Suggestive evidence on financial frictions

To gain more insight about which firms are more likely to be more severly credi constraint, we examine firm debt and leverage usign the same lenses as before, studing the differences in levels, growth rates and cyclicality for different sizes and ages. The rusults are captured in figure 4.

Figure 4: Predicted age × size profile of log levels, growth rate and cyclicality of debt and assets. Obtained from the speciffication with full interaction of size and age (regression (1)).



(a) Debt | (b) Growth rate of Debt | (c) Cyclicality of Debt

(d) DA | (e) Growth rate of DA | (f) Cyclicality of DA

Starting with the levels of leverage (panel d), we find that the younger firms are one average more leveraged. These firms are the likely candiates for being the most constrained, both because they already have the most (in relative terms) debt and the least credibility among lenders. Note that the gradient with respect to size is dominated by the effect of age. For most firms the larger size seems to decrease the leverage (the exeption being the the smallest entrants and top 10% of firms in terms of size). Turning to the levels of debt (panel a), we see the reverse; older firms having more debt across all size groups. However, given what we already know about debt/assets, it must be the case that oder firm just have even more assets than debt and hence one can hypothesise that they are probably less credit contraint than the young firms, despite higher debt levels.

This conjecture is further supported by the growth rate of debt and leverage (panels b and e), which shows the young and non-large firms experiencing much high growth rate of both. For large entrants the growth rate of both debt and leverage is in line with other large firms, which suggests entrants of this size are not being treated by banks worse than older firms and are likely not more (or less) financially constrained.

Finally, turning to the cyclicality results, we see that debt (panel c) follows the same pattern as sales and other variables from figure 3, i.e. entrants have more cyclical debt and its cyclicality decreases with size and for everybody else it increases with size. The leverage seems to be mostly countercyclical, possibly because the assets values decline more than debt in recessions

To summarize, the evidence we find is at least consistent with young *and* small firms are unable to borrow as much as they want, as their debt seems be growing faster than their assets, which is suggesting that these firms might want to borrow more than they are allowed to. At the same time, *young and small* firms are accumulating cash fast, suggesting that these firms are building up a buffer against bad shocks, indicating that these firms expect not to be able to borrow instead. If first were allowed to borrow, the need to build such a buffer would be lower, especially given that these firms want to grow so keeping the cash is particularly costly.

3.4 Summary of empirical evidence

We find young firms more cyclical than old. For entrants, increasing size reduces cyclicality, for everybody else, increasing size increases it. The patterns that we observe for financial variables such as debt or cash as well as rations like leverage are in line with financial frictions affect *young and small* firms more than the rest.

Simple firm dynamics models have no problem explaining why we see two types of firms. *Large and old* firms are the firms that survived until they reached their optimal size, and *Small and young* are the entrants, who have not grown out of their financing contraints yet. This begs the question, who are the other firms?

First, we observe large heterogeneity in starting size of firms (over 4% of firms in the lowest age group belong to the top size group). One concern is that these might be just missreported mergers, i.e. two firms merging and creating a new entity, but the DST tries to fix such cases and provided consistent firm identifies. At least for some questions, whther these are trully new firms or not does not matter, for example, they would likely qualify for any support for new enterprises.

Second, what about the *small and old* firms? The large dispersion in long run optimal size of firms is not driven by sectoral differences (we control for sectors in our empirical specifications). Sterk et al. (2021) suggest that the small long term size of these firms is not just a results of a series of bad shocks, but at least to a significant degree, something what is decided at the point of entry, such as a business plan that is build on a small scale of production.

4 Quantitative model

In this section we build a dynamic, quantitative heterogeneous-firm model and assess its ability to replicate our new stylised facts. The model builds on existing work by Khan and Thomas (2013), and we extend this class of model by considering heterogeneity in returns to scale, a notion of collateral prices, and building a model able to match key moments of the firm age and size distributions.

4.1 Description of the model

The model features a continuum of heterogeneous firms, with both ex-ante and ex-post heterogeneity. There is a representative consumer, who owns firms and supplies labour. The model also features a final goods aggregator and simple IO structure and a representative capital producing firm. The key features that we use to connect the model to our empirical work are financial frictions at the firm level, and differences in returns to scale across different classes of firm.

The model is set in continuous time with an infinite horizon. Let $t \in [0, \infty)$ denote time. We focus on the case without aggregate uncertainty, and conduct business-cycle experiments using unanticipated one-time shocks. The model is presented below in steady state, for expositional simplicity, and we therefore drop the time subscript, t, in most of what follows.

4.1.1 Market structure

A continuum of heterogeneous firms are our firms of interest. These are technically intermediate goods producing firms, and we will refer to them simply as "firms" where it does not cause confusion. These firms produce a firm-specific good using capital and labour. Their goods are all sold to a representative final goods producer ("Final Goods Producer") who combines them to produce a composite final good, which is used for consumption and investment. The final good is the numeraire, with price normalised to one in all periods.

4.1.2 Final goods producer and accounting definitions

The representative final goods producer purchases the output of the heterogeneous intermediate goods firms to produce the composite final good. Let q_i be production (gross output) of firm i and p_i the price of its good in terms of the numeraire. The production function is

$$Q = \left(\int_0^G q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}}. \tag{3}$$

where Q is units of production of the final good. G is the mass of active intermediate firms, which can be endogenous or exogenous. Define $\epsilon = 1/(1-\theta)$ as the elasticity of substitution between varieties and restrict to gross substitutability ($\epsilon \geq 1 \Leftrightarrow 0 \leq \theta \leq 1$). This ensures that firms have decreasing returns to scale in revenue, even if they have constant returns to scale in production.

The final-goods firm is a price taker in both the final and intermediates markets. Their profit is given by $\pi = \left(\int_0^G q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}} - \int_0^G p_i q_i \, \mathrm{d}i$. Profit for the final goods producer, and indeed all firms, is denoted in units of the numeraire final good. The final good producer's first order condition for good purchase from firm i gives

$$q_i = p_i^{-\epsilon} Q \tag{4}$$

This is the demand curve for our main firms of interest. Also note that zero profit implies that Q must also equal the sum of purchases from firms: $Q = \int_0^G p_i q_i \, \mathrm{d}i$.

Accounting definitions The resource constraint for the final good states that $Q = C + I$, where C is consumption and I investment. GDP is the sum of value added across firms, which we define as Y. We can show that this is equal to $Y = Q$, giving the usual accounting equation

$$Y = C + I \tag{5}$$

To see this, note that the final good producer makes zero profit or value added, so GDP is the sum of value added across the normal firms. For each firm let $y_i \equiv p_i q_i$ denote value added. GDP is therefore

$Y \equiv \int_0^G y_i di$. Using the definition of y_i and $Q = \int_0^G p_i q_i \, \mathrm{d}i$ gives $Y = Q$.

4.1.3 Intermediate goods firms (a.k.a. "Firms")

There is a mass G of firms which arises via firm entry and exit. Firms have both ex-ante and ex-post heterogeneity. Firms face downwards facing demand curves (i.e. have well defined optimal size) and financial frictions. Additionally, to match the data that a large fraction of employment is contained in a few very large old firms, there is a very small chance a firm will at some point receive a shock which transforms it into a superstar, removing financial frictions and drastically increasing productivity. Firms are owned by the representative household, and discount the future at the interest rate, r.

Production function, demand, and static optimization Firms produce goods with capital and labour. They have a generic production function

$$q_i = z_i f(k_i, l_i)^{\eta_i} \tag{6}$$

Here, $f(k_i, l_i)$ is a constant returns to scale function common across firms, and $\eta_i > 0$ controls the degree of returns to scale at that firm, which can be heterogeneous. If all firms had $\eta_i = 1$ then all firms would have constant returns to scale in production, and $\eta_i < 0$ denotes decreasing returns to scale, and $\eta_i > 1$ denotes increasing returns to scale. Note that the downwards sloping demand curve additionally pushes firms towards having decreasing returns in revenue. For concreteness, consider a Leontief production function:

$$q_i = z_i \min\left\{k_i, \frac{l_i}{\alpha}\right\}^{\eta_i} \tag{7}$$

This is the production function we will use in our quantitative application, and α controls the labour share in production. The demand curve is (4). A firm's revenue is therefore $p_i q_i = z^\theta f(k_i, l_i)^{\eta_i \theta} Q^{1-\theta}$. Value added is equal to sales: $y_i = p_i q_i$. Idiosyncratic physical productivity is z_i, which follows a Poisson process: at rate α_z a new value z' is drawn from distribution $F(z', z)$.

At the firm level, all factors of production can be adjusted freely without cost. We are in continuous time, and there is no time to build for capital. It is convenient to first optimise labour for a given level of capital.

From now on, remove i subscript and let z index discrete firm types. Let z control both productivity and the returns to scale by setting $\eta = \eta_z$. Static profit is

$$\pi(k, z) = \max_{l \geq 0} z^\theta f(k, l)^{\eta_z \theta} Q^{1-\theta} - wl \tag{8}$$

Firm profit depends on the aggregate state of the economy, from general equilibrium effects from two sources. Firstly, the real wage w affects the price of labour. Secondly, aggregate output, Q, affects the demand for a firm's variety since lower output means less demand for all varieties. Solving this optimization gives policy functions for labour $l(k, z)$. In the Leontief case we have simply that $l(k) = \alpha k$, and $\pi(k, z) = z^\theta k^{\eta_z \theta} Q^{1-\theta} - \alpha w k$.

Capital evolution A firm's capital stock evolves through a standard accumulation equation. Given investment i per unit of time and depreciation rate δ we have: $\dot{k} = i - \delta k$. One unit of investment costs p_k units of final good, which is potentially endogenous. Old capital and investment are perfect substitutes for firms, so capital also trades at the price p_k.

Financial frictions and net worth evolution Firms can borrow using a risk-free short-term bond b with interest rate r. Firms face a borrowing constraint which limits the amount they can borrow according to the amount they can post as collateral. Specifically, we assume that borrowing is limited by the constraint $b \leq \lambda p_k k$, where recall that k is a firm's physical capital. The parameter λ controls the tightness of the borrowing limit, with smaller λ making the constraint tighter and restricting the amount a firm can borrow for a given quantity of collateral. In the business cycle experiments, we allow λ to evolve as an aggregate financial shock.

A firm's net worth, n is defined as its assets less its liabilities: $n = p_k k - b$. Combining this with the borrowing limit gives $k \leq \frac{n}{p_k(1-\lambda)}$. Define a firm's leverage, ϕ, as $\phi \equiv p_k k/n$. Combining this with the borrowing limit gives the constraint as a constraint on leverage instead: $\phi \leq \bar{\phi}$, where $\bar{\phi} \equiv \frac{1}{1-\lambda}$ is the endogenous leverage limit.

A firm's net worth evolves according to

$$\dot{n} = \left(\frac{\pi(k,z)}{k} - (\delta + r)p_k \right) k + rn - d \tag{9}$$

where the first term is the net return on leveraged investment, and d denotes the dividend payout flow. We assume that firms cannot raise equity at all after the moment of birth, and so impose $d \geq 0$. We simplify the dividend payout policy, and impose that firms payout dividends exogenously whenever net worth exceeds an exogenous level $\bar{n}$, and payout such that net worth remains at $\bar{n}$. Firms therefore pay no dividends while they are young, but then start paying out dividends when they are older and have achieved sufficient scale.

Transition to superstar status In the data, there is significant dispersion in size across firms. This leads to a skewed firm distribution, where a very small number of very large firms account for a very large share of total employment. Our calibration aims to replicate this feature of the data, using heterogeneity in productivity and returns to scale across firms.

As part of this calibration procedure, we allow a small number of firms to become "superstar" firms. In the calibration this is a very small number of firms (less than 1% of the total) with very high employment (more than 500 times larger than the smallest firm in the economy). Additionally, in the data these superstar firms tend to be very old, and therefore do not appear to grow to this size until later in their lifecycle. We accordingly assign all firms a very small probability of becoming a superstar, which happens at rate α_s. When a firm becomes a superstar, it switches to a special superstar state with productivity z_s and returns to scale η_s. Given the enormous change in firm optimal size that happens at this point, we allow firms to raise equity at the moment they become superstars, and allow them continuous access to equity from then on. They therefore become Modigliani Miller firms and their financial structure becomes undefined and they follow the efficient investment and production policies. We assume superstars hold a constant leverage rate, calibrated to that of the largest firms in the economy.

Firm HJB The firm's problem can be stated recursively using a Hamilton Jacobi Bellman (HJB) equation. For any firm which has not transitioned to superstar status, denote optimized firm value as

$v(n,z)$. This can be expressed as

$$rv(n,z) = \max_{0 \leq p_k k \leq \bar{\phi} n} d(n) + v_n(n,z)\left(\left(\frac{\pi(k,z)}{k} - (\delta + r)p_k\right)k + rn - d(n)\right) + \\ + \alpha_z \sum_{z'} \eta_{z,z'}\left(v(n,z') - v(n,z)\right) + \zeta_z\left(n - v(n,z)\right) + \alpha_s\left(v^s + n - v(n,z)\right) \quad (10)$$

Here, $d(n)$ is the exogenous dividend payout policy for the current level of net worth. The v_n term is the drift in net worth, which depends on the capital choice and dividend payout. The α_z term is the change in value when productivity jumps to a new value. $\eta_{z,z'}$ is the probability of drawing z' when current productivity is z. The α_s captures the transition to superstar status. Since superstars face no financial frictions, superstar value can be expressed as $v^s + n$ for some constant v^s. ζ_z is an exogenous firm exit rate. Notice that we allow the exit rate to differ across firm productivity types. We will use this to exogenously create a declining firm exit rate with firm age, as seen in the data.

Investment policy The firm investment policy in this setting can be expressed as an unconstrained optimal capital stock, which firms will achieve only if they are financially unconstrained. The first order condition with respect to capital is $v_n(n,z)\left(\pi_k(k,z) - (\delta + r)p_k\right) = \mu_k$, where $\mu_k \geq 0$ is the multiplier on the borrowing constraint. If a firm hits its borrowing constraint then we know that $k = \bar{\phi} n / p_k$. If a firm is rich enough to be unconstrained, then $\mu_k = 0$ and the capital FOC gives us $\frac{\pi_k(k,z)}{p_k} = \delta + r$. This is the unconstrained investment policy in the absence of financial frictions. This typically has an analytical solution, and in the Leontief case we have

$$k^{unc}(z) = \left(\frac{\eta_z \theta z^\theta Q^{1-\theta}}{\alpha w + p_k(\delta + r)}\right)^{\frac{1}{1-\eta_z \theta}} \quad (11)$$

The overall investment policy can then be simply expressed as $k(n,z) = \min\left\{\bar{\phi} n / p_k, k^{unc}(z)\right\}$.

4.1.4 Firm entry and exit

As previously mentioned, we restrict the model to only allow for exogenous firm exit, but with heterogeneous exit rates across firms in order to match the age profile of exit rates in the data. Denote by μ_0 the flow rate at which new firms enter. New firms draw an initial productivity level z from a distribution $\gamma_{0,z}$.

New entrants are endowed with some initial amount of net worth, n, from an initial equity injection by their owners. We suppose that firms of productivity z start life with net worth equal to $n_0(z)$. This is allowed to differ by initial productivity, which will be important for matching the data on cyclicality by joint age-size bin.

4.1.5 Closing the model

Given the solution to the firm problem, we can simulate the endogenous firm distribution in steady state or in transitions. We can then calculate aggregates such as output and employment, and moments of the firm size and age distribution. We close the model by specifying how the prices that firms face (real wage, interest rate, and capital prices) are determined in a quasi-general equilibrium setting.

Interest rate and wages We focus on general equilibrium in all of the prices apart from the real interest. This is partially a simplification, but also allows us to study monetary policy in our framework in a simple way. Specifically, we suppose that the central bank simply controls the real interest rate, r. Monetary policy will then be conducted by choosing different paths for the real interest rate. This can be motivated by a simple structure where the nominal price level is fully rigid, and thus inflation does not react to monetary policy. This means that the central bank fully controls the real interest rate by changing the nominal interest rate.

In order to determine the equilibrium real wage, we assume Greenwood Hercowitz Huffman preferences, which mean that labor supply is independent of the level of consumption. We assume preferences such that labor supply is a constant elasticity function of the wage: $L^s = \chi w^\eta$.

Price of new investment and capital We assume aggregate investment adjustment costs, so that the price of capital depends on the aggregate quantity of investment. We assume quadratic adjustment costs to give a standard equation for the price of capital: $p_k = 1 + \psi_k \left(\frac{I}{K} - \delta\right)$. This ensures that the price of capital is normalized to one in steady state (where $I = \delta K$) and falls whenever investment demand falls. ψ_k controls the sensitivity of capital prices to changes in investment.

4.2 Calibration and steady state

In this section we first describe our calibration strategy, and then discuss the fit and properties of our model in steady state. We take one unit of time to be one year.

A "standard" calibration In this section we describe the calibration of what we call the "standard" model. This model is meant to represent a simple heterogeneous-firm model with financial frictions, without any added features. We will then build on this model in later exercises, in order to explain our new facts.

Steady state parameters We start by describing our relatively standard parameters. We set the interest rate r to a 2% annual real interest rate, in line with the lower real interest rates seen in recent years. We set θ to 0.9 to give a 10% markup in a frictionless model, as is standard in the New Keynesian literature. We choose α to control the equilibrium quantity of employment. Employment is set to match the average firm size (total employment over total number of firms) in Denmark. The depreciation rate δ is set to a 10% annual rate. The labour supply disutility χ is chosen to match a labour share of income of 60%. The entry rate of firms μ_0 is chosen to normalize the mass of firms in steady state to one.

Firm distribution parameters A major goal of our calibration is to match well both the firm age and size distributions, and we now discuss how we achieve this. We specialize the model to have four standard productivity levels, z_1 to z_4, in addition to the fifth superstar productivity level, z_s. These productivity levels are chosen to match the cross sectional firm size distribution. In particular, we assume that productivity is permanent, and so abstract from firm-level productivity shocks during the life of a firm. A firm is assigned a productivity state from 1 to 4 at birth. We choose these levels to mimic the binning strategy in our empirics. Specifically, we suppose the probability of being born in states 1 to 3 ($\gamma_{0,i}$) is 30% each, and state 4 is 10%. By choosing the productivity levels appropriately, each productivity type is therefore assigned to form the predominant mass of firms in each of the 0-30%, 30-60% 60-90%, and 90%+ size bins respectively.

We choose the productivity levels themselves, z_i, to match the average employment inside each firm size bin in our data. Note that the returns to scale parameter, η_z, also affects firm size. Thus, the choice of z_i allows us to control firm size exactly for a given choice of returns to scale in each bin. We discuss returns to scale in the next section, and for now note that we choose the average returns to scale to ensure that the economy has constant returns to scale on average in the steady state. The fraction of firms in each age bin is exactly matched, by definition. We are able to exactly match the fraction of employment in each size bin, meaning the model provides a good match to the empirical firm size distribution in Denmark.

Moving on to the firm age distribution, we target both the distribution of firms by age (i.e. the exit rates) and the distribution of total employment by firm age. To target the exit rate we use exit rate data from Andersen and Rozsypal (2019) for the Danish economy. We suppose that the exit rate is independent of the permanent firm type, but introduce a two-state process to allow it to fall with age as in the data. In particular, firms are born with a high exit rate ζ_y, which then falls to $\zeta_o < \zeta_y$ at some rate α_ζ. These three values are chosen to match an overall exit rate of 8.34% per year and the ratio of the exit rates of firms aged 0 and 6 to firms aged 16+. The exit rate of superstars is set to the low value of 1% per year, in line with the low exit rate of very old firms in the data. This provides a very good match to the distribution of firms by age, as shown in Figure 5.

For the distribution of employment by firm age, we target this in two ways. Firstly, we follow Khan and Thomas (2013) and use the initial net worth of entrants to target the average size of firms at age 0. This sets the parameter α_0, which gives that entrant firms start with only 33% of the unconstrained net worth level, in order to match the average size of entrant firms of 4.7 employees, relative to the 12.9 average in the whole economy. Secondly, we use the superstar firms to target the high employment share of very old firms. Their productivity level, z_s, is used to target the employment share of firms aged 15+ years old. Intuitively, the superstar shock is very rare, and therefore only occurs for firms on average when they are very old. We target that only 0.5% of firms become superstars in the ergodic distribution. Without the very productive superstar firms, the employment share of the oldest age bin in the economy would be far too low, due to the relatively low number of old firms in the economy. Together, these features provide a very good match to the distribution of employment and the number of firms by firm age, as shown in Figure 5.

Finally, we discuss the financial frictions. We set the borrowing limit so that the maximum leverage (capital to net worth) in the steady state is 3. This implies that firms remain financially constrained only until around age 3 on average, and therefore represents a relatively loose borrowing limit. We set the level of net worth at which firms start paying out dividends to a very large number. Above the minimum saving policy, exactly whether or not firms pay out dividends has no effect on the steady state. We therefore choose that firms pay out for some $\bar{n}$ such that even the most productive firms can fund their unconstrained optimal capital with leverage significantly below the borrowing limit (during both steady state and all transition experiments). In Figure 6 we plot the average policies by firm productivity and age.

Business cycle parameters Several parameters related to the business cycle are changed across different experiments, so we discuss the details throughout the text as they change. The labor supply elasticity η is set to 0.5, which implies that wages fall by around 50% for a given change in employment. We currently set the capital price parameter ψ_k to zero so that the capital price is constant.

Table 2: Calibration

	Interpretation	Value	Source
r	Discount rate	0.0202	2% yearly real interest rate
z	Productivity distribution	-	See text
η_z	Returns to scale distribution	-	See text
ζ_z	Exit rates	-	See text
θ	Substitution across varieties	0.9	10% markup in frictionless model
α	Labor-capital ratio in prod fun	7.208	Aggregate L
μ_0	Firm entry rate	0.0834	Normal total mass of firms to one
$\bar{\phi}$	S.s. collateral limit	3	Maximum leverage
δ	Depreciation rate	0.1054	10% annual rate
$\bar{n}$	Net worth where start paying dividends	38.78	Normalisation
α_s	Rate transition to superstar firm	5e-5	0.5% of firms are superstar
z_s	Superstar productivity	1.2803	Employment share of firms age 20+
χ	Labor disutility shifter	0.0128	Labor share of income
η	Labor supply elasticity	0.5	Real wage flexibility

4.3 Results 1: Model features needed to match the data

Our first set of model results investigate the features required for a model to replicate our empirical results. In Section 3.2 we showed that cyclicality varied by firm age, size, and joint age-size bin. In this section we show the additions required to a more standard model in order to generate these results.

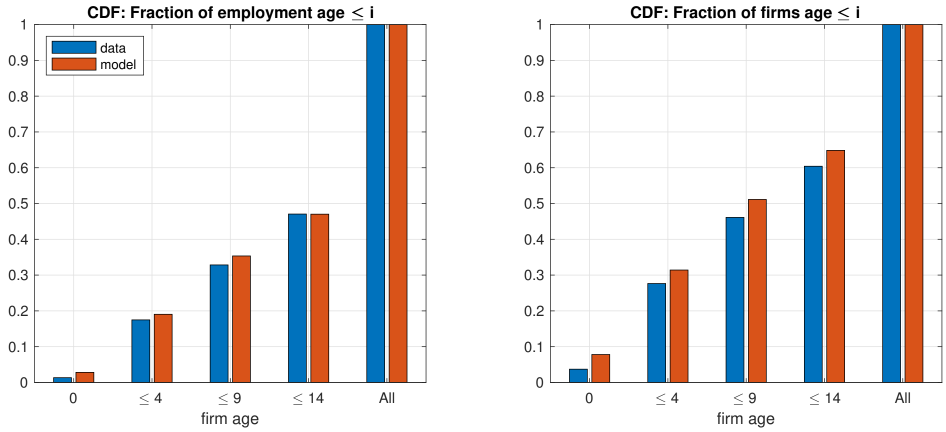
4.3.1 Effect of different aggregate shocks in the "standard" financial frictions model

We begin our exploration of the financial frictions model using our "standard" calibration. As the cyclicality of different firm groups may depend on the shock hitting the economy, we begin by exploring the response of the economy to three different shocks. Given that we are primarily studying the Financial Crisis, we choose two financial shocks, and one real shock. In all cases, the size of the shock is not directly calibrated, but chosen to be of a reasonable size, with the focus instead being on how the response to the shock differs across different firm groups.

A leverage constraint shock The first shock that we consider is an exogenous tightening of the leverage constraint, represented as a reduction in the parameter $\bar{\phi}$. This shock was used in Khan and Thomas (2013), for example, to represent a financial crisis. We consider a 10% reduction in $\bar{\phi}$, which gradually returns to steady state such that it has mostly died out within two years.

The results of this exercise are plotted in Figure 7(a). The left panel shows the aggregate output response to the shock, showing that output falls by more than 1% before recovering. In the remaining panels we plot the results of running regressions on simulated firm level data taken in a 15 year window around our recession. The third panel shows that the collateral shock affects young firms much more than old firms, with a clear downwards gradient in the cyclicality by firm age. This feature is present in the data, indicating that a standard collateral shock can match the age profile seen in the data. However, one area that a collateral constrain fails is that it generates no cyclicality of old firms, with firms age 9+ being essentially acyclical. This is in contrast to the data, while even older firms are

Figure 5: Age distribution



Panels plot the firm age distribution in the model and data. Left panel is total employment in firms of a given size, and right panel is total numbder of firms of a given size.

cyclical during the financial crisis.

Moreover, this single shock also generates problems in the firm size dimension, as shown in the second panel. There, we see that small firms are more cyclical than large firms, while the opposite is true in the data. This follows directly from the fact that young firms are more cyclical in this experiment, and that young firms tend to be smaller as they are still outgrowing their financial constraints. Finally, the fourth panel shows the regression results by joint age-size bin, where we again see a mismatch with the data, which is of course already implied by the failure in the pure firm size regressions.

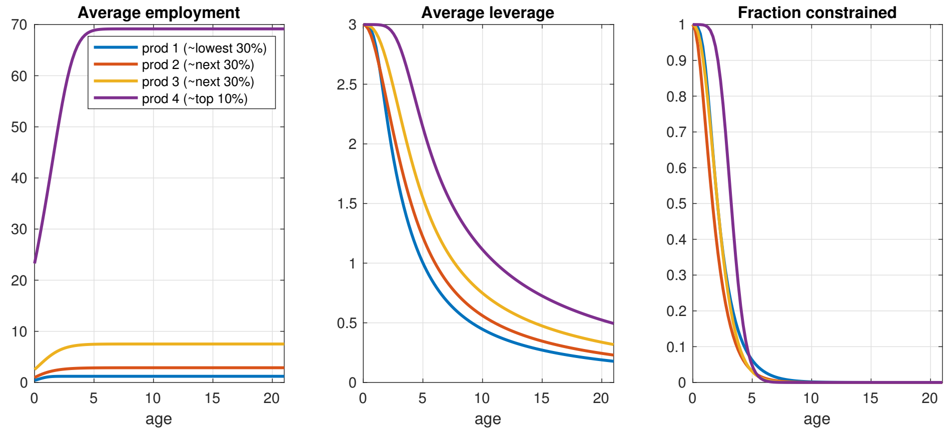
In summary, while a collateral shock can match the excess responsiveness of young firms well, it does not generate sufficient cyclicality of old firms, and misses important responses in the firm size dimension.

A discount rate / spread shock We next investigate a discount rate or spread shock. The idea here is to investigate financial shocks which are modelled in a different way. In particular, note that the collateral shock above does not change the cost of debt to firms, which is held at r, but just the availability by reducing the maximum that firms can borrow. In our next experiment we instead ask what happens if debt becomes more expensive for all firms, while holding the maximum leverage that firms are allowed constant. As motivation for this kind of shock, consider that the financial crisis created many problems in the financial sector which raised the cost of funds. This can be seen in the increase in spreads charged to firms even without any default risk. XXX add some evidence here

In our model, we represent this shock as an increase in the interest rate r, which we could consider as the sum of a risk-free rate plus a spread charged to firms. We consider a 15% increase in r, and plot the results in Figure 7(b). This shock leads to a drastically different firm age response to the collateral shock. Old firms are now more responsive than young firms, in contrast to the data which shows the opposite. Intuitively, this follows from the fact that young firms are financially constrained, and up against their (now unchanged) collateral constraint $\bar{\phi}$. They are therefore unresponsive to an increase in the cost of finance, r, as they simply choose to remain up against the constraint.[8]

[8]One important thing to note here is that in a different model, it could be possible that an increase in the cost of credit

Figure 6: Average outcomes by age and productivity bin



Panels plot average firm policies by age and productivity state. These are averages across the ergodic distribution. Left panel gives employment, central panel leverage (assets of net worth) and right panel the fraction of firms who are financially constrained.

However, this shock does add one feature which the collateral constraint is missing, which is that old firms do react to the shock. To see this, note that firms aged 9+ have a regression coefficient of around 1 in response to the spread shock, but close to zero for the collateral constraint shock. Intuitively, the spread shock raises the cost of funds, which encourages older firms to scale back their investment plans despite being financially unconstrained. Finally, the firm size response remains counterfactual in response to this shock: while larger firms are now more cyclical than small, the effect is rathe small compared to the data.

A TFP shock The final shock we consider is a simple aggregate TFP shock, which we model as a 1% reduction in TFP at all firms. We include this shock in order to consider non-financial shocks, and how they compare to the two financial shocks considered above. We plot the results in Figure 7(c).

The firm-level responses to this shock are essentially identical to the spread shock considered above: old firms are more cyclical than young, and large marginally more so than small. The intuitions are the same, with firms now adjusting their production in response to a change in TFP rather than their financing costs. This highlights an important point, which is that the simple firm-level comparisons we consider do not separately identify all shocks from each other. For example, comparing the firm-level cyclicalities by age and size in Figures 7(b) and (c) cannot distinguish a spread shock from a TFP shock. Identifying which of these shocks occurred in the real world would require other data, such as aggregate data on TFP, or average spreads.

Summary In summary, in this section we investigated the firm-level responses to three aggregate shocks in a standard heterogeneous firm model with financial frictions. By comparing the cyclicalities of firms by age and size, we are able to i) partially identify the role of aggregate shocks in driving the

could *endogenously* tighten the collateral constraint, for example by raising default risk at low net worth firms. This could allow an increase in r to make young firms more cyclical than old firms, in line with the data. While we abstract from this possibility, a benefit of our approach is that it identifies the need for this credit supply tightening, independently from an increase in cost of funding.

business cycle in our sample, and ii) identify facts that the standard model is not able to replicate. Our main results are that, in a standard model:

1. Only the collateral constraint shock is able to replicate the excess cyclicality of younger firms. Increases in the cost of credit or an aggregate TFP shock instead lead old firms to be more cyclical.
2. A collateral constraint shock alone cannot generate that older firms are cyclical, and other shocks are needed.
3. None of the shocks considered can generate the fact that large firms are more cyclical than small, or the more complicated joint age-size cyclicality patterns in the data.

4.3.2 Building a model which can match our empirical results

Having investigated the performance of a standard model in response to several shocks, we now turn to building and calibrating an extended model which can replicate all of the facts from our empirical work.

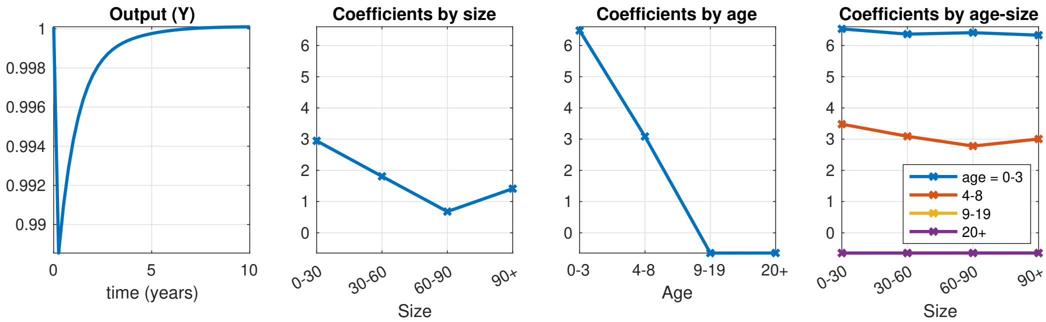
As we noted in the last section, using the relative cyclicality data alone does not identify an aggregate TFP shock from a discount rate shock. In this section, we choose to focus only on financial shocks, and assume that the economy is subject only to the collateral and discount rate shock. Therefore, the results can be interpreted as asking what features are needed for a purely financial shock to explain our results. In the final section we discuss robustness and how policy implication depend on this choice.

Building the full model: shocks and model "twists" Our calibration exercise targets three facts about relative cyclicality of firms by age and size, following our regression results from Section 3.2. We choose one shock process and two parameters of the model to match these firm level outcomes in this exercise. To understand the link between these features and the data, we later discuss how the model fails if these features are switched off.

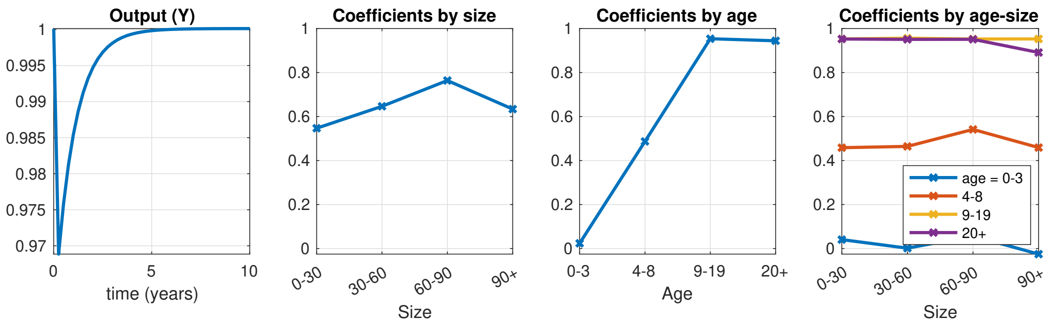
Firstly, in the data young firms are more cyclical than old firms, as shown by their relative regression coefficients in Figure 3. We choose the size of the collateral constraint shock in order to match the response of young firms relative to old firms to the recession. Given that the linear regression coefficients are invariant to the scale of the recession, we can normalize the size of the recession for this exercise, which we do by fixing the size of the discount rate shock to a 15% rise. We thus identify the relative importance of the collateral versus discount shock using this channel. A financial shock equal to a 10% tightening in the borrowing constraint matches this data well.

Secondly, in the data large firms are more cyclical than small firms, as shown by their relative regression coefficients in Figure 3. We calibrate the differing degrees of the returns to scale across size groups in order to match the relative cyclicalities of firms of different sizes. Crucially, differing degrees of returns to scale also imply different responsiveness to shocks, in a way that very natural meshes with our empirical findings. In particular, firms with more decreasing returns to scale (who would tend to be smaller) are also endogenously less responsive to shocks. This follows naturally from the curvature of their profit functions. Intuitively, one can think of small firms as having decreasing returns (making them optimally smaller), and hence large profits and less incentive to adjust their size in response to the cycle. We assign each of our four productivity types a given returns to scale parameter η_z, and assign the superstar firms the same parameter as the most productive of the four

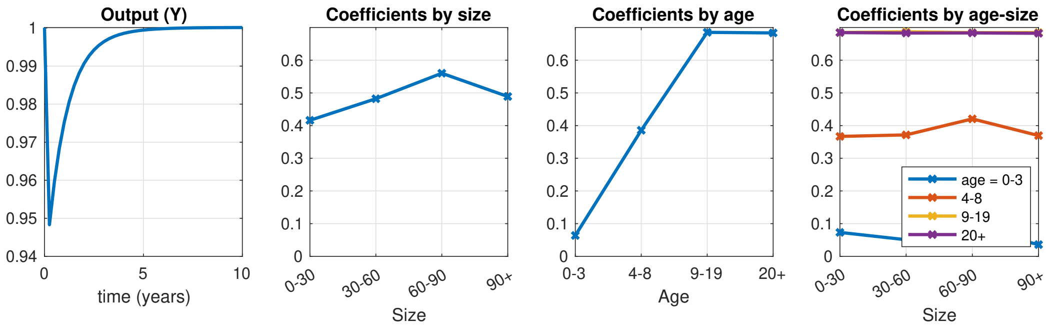
Figure 7: Effect of various shocks in a standard model



(a) Collateral constraint shock



(b) Spread / discount rate shock



(c) TFP shock

Panels give regression coefficients from regressions of firm-level growth rates of employment on aggregate GDP growth, computing using the model simulation in Experiment 1. The regressions are on firm-level data aggregated to the yearly level and treated in the same way as the data.

groups. To reduce the number of free parameters we assign a minimum and maximum returns to scale, for bins 1 and 4 respectively, and equally space the parameters in bins 2 and 3 between the extremes. The returns to scale in group 4 are chosen to normalize the average returns to scale to one, leaving only a single free parameter. We choose this minimum returns to scale to match the relative cyclicality of small firms relative to large. We find that setting $\eta_1 = 0.867$ and $\eta_4 = 1.020$ matches this data well. This implies that smaller firms have much more decreasing returns to scale than larger firms.

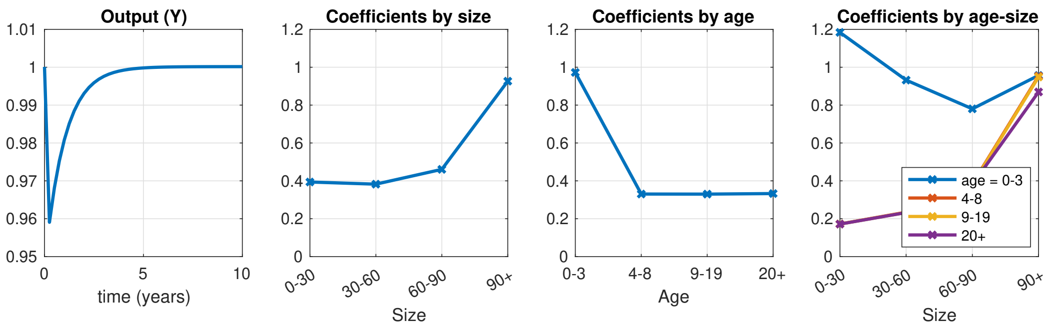
Finally, in the data large firms are equally cyclical regardless of their age, as shown by the similar regression coefficients of all firms in the top 10th percentile regardless of age bin in Figure 3. This is in contrast to smaller firms, where cyclicality declines as firms age. In order to match that age is not relevant for the cyclicality of older firms, we allow for differing net worth at birth for firms of different sizes. In particular, we impose that the largest firm group are not financially constrained at any point in their lifecycle, by assuming that they are born with sufficient net worth to reach their financially-unconstrained optimum scale in steady state. Smaller firms are born with relatively less net worth, and will be financially constrained early in life. Specifically, we let $n_0(z_i) = \alpha_i \bar{n}(z_i)$, where $\bar{n}(z)$ is the net worth required to become unconstrained in steady state and $\alpha_i \leq 1$. We set $\alpha_4 = 1$ so that the largest group are unconstrained from birth. For the smallest firm group, we set $\alpha_1 = 0.1$ which ensures that they are constrained. To reduce free parameters, we set α_2 and α_3 as equally spaced between α_1 and α_4.

We plot the results of our calibration in Figure 8(a). The first panel shows that the two aggregate shocks combined lead to a fall in GDP of over 4%. In the remaining three panels we plot the firm-level regressions for the full calibration. The model fits the firm-level data well. The second and third panels show the regression coefficients by size and age respectively. These are targeted, and show that the model can simultaneously match that 1) large firms are more cyclical than small, while also 2) young firms are more cyclical than old. As in the data, this finding could be considered initially surprising, given that young firms tend to be smaller on average. However, the resolution to the puzzle is the same here as it is in the data. In the final panel we plot the coefficients by joint-age size bin, which yields the same pattern as the data. In particular, within a given age bin, large firms are more cyclical than small firms. At the same time, within a given size bin, young firms are more cyclical than old firms (except for the largest size bin, as targeted). It is these within group relationships which allow the model to jointly match the results by age and size at the same time.

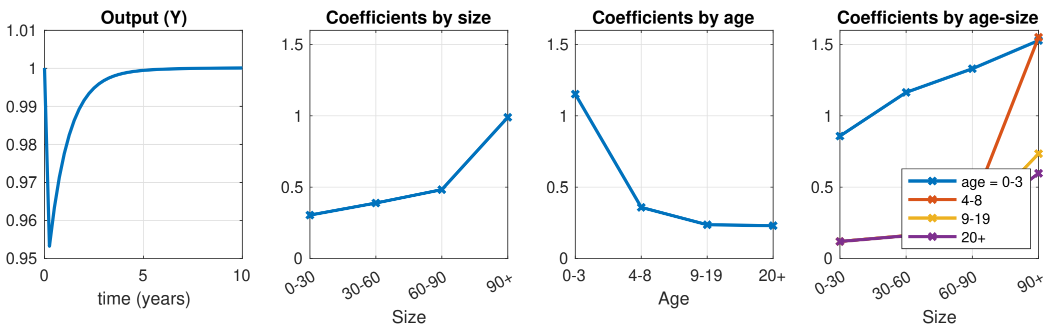
Understanding the mechanisms Having built our full model, we now turn to explaining how the various features allow it to match our three calibration targets from the data. The first feature is the combination of the collateral constraint and spread shocks, in calibrated the the appropriate relative sizes. As shown in Figure 7, each shock has different effects on the age distribution, and combining them in the right ratio allows the model to match the higher cyclicality of young firms. This insight is helpful in understanding exactly how financial shocks should be modelled in a heterogeneous firm model.

The remaining two forces needed to match the firm level data are "twists" to the basic heterogeneous-firm financial frictions model. The first twist is the addition of heterogeneous returns to scale by firm size. To see the effect of this twist on firm-level cyclicality, in Figure 8(c) we plot the results of a counterfactual where we turn off heterogeneous returns to scale, and impose that all firms have constant returns to scale, as in our standard model from the last section. This removes the model's ability to generate that large firms are more cyclical than small. In fact, small firms again become more cyclical

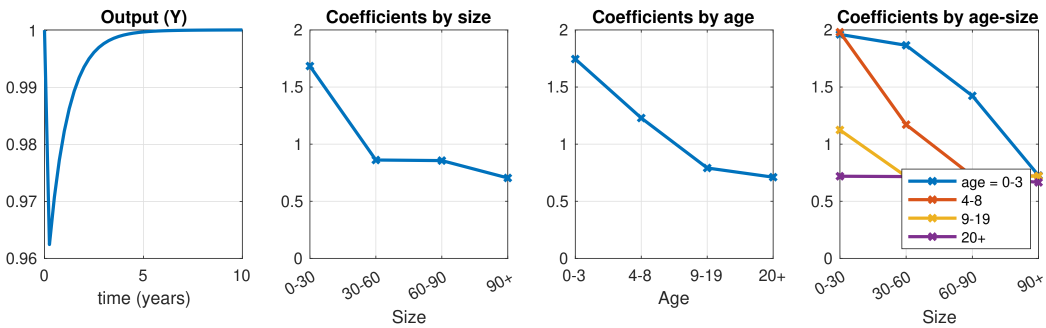
Figure 8: Effect of calibrated shock combination in various models



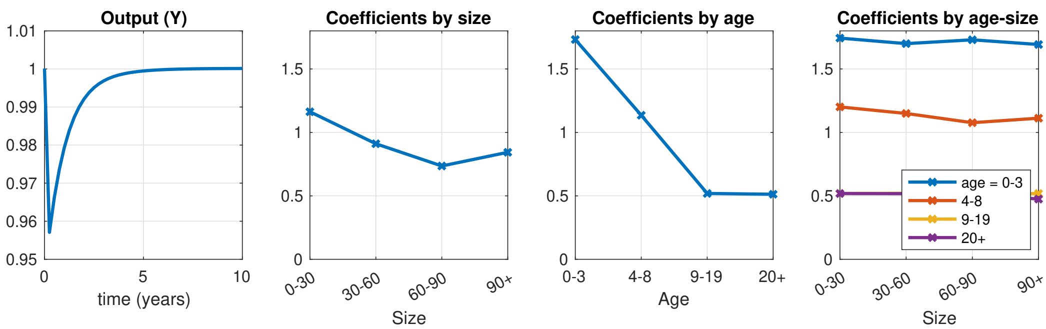
(a) Adding both heterogeneous initial net worth and RTS



(b) Adding heterogeneous returns to scale



(c) Adding heterogeneous initial net worth



(d) Standard model

Panels give regression coefficients from regressions of firm-level growth rates of employment on aggregate GDP growth, computing using the model simulation in Experiment 1. The regressions are on firm-level data aggregated to the yearly level and treated in the same way as the data.

than large, as in the standard model.

The second twist is allowing for differing levels of initial net worth by firm size, meaning that larger firms are less financially constrained while young. To see the effect of this twist on firm-level cyclicality, in Figure 8(b) we plot the results of a counterfactual where we turn off heterogeneous initial net worth. Here we impose that all firms start with 30% of the net worth needed to become financially unconstrained, as in the standard model. In the second and third panels, we see that removing this twist does not qualitatively worsen the model's ability to generate the basic patterns that larger firms are more cyclical than small, and young firms are more cyclical than old. It is only in the joint age-size relationships that this twist becomes important for helping the model match the data. The right panel shows that removing the twist now means that within the largest size bin (90+ percentile) young firms are more cyclical than old firms, with the coefficient for the youngest firm being roughly twice that of the older. This is in contrast to the data in Figure 3, where we found that within the largest firms, firm age does not matter for cyclicality. Intuitively, the fact that age does not matter for cyclicality among the largest firms suggests that financial frictions are not a major concern for these firms. If firms of this size started life constrained, they would be more cyclical when young, and cyclicality would decline with age (as it does for smaller firms).

Summary In summary, in this section we built a model capable of explaining our empirical findings on the cyclicality of firms by age, size, and joint age-size bin. Our main results are that:

1. In order to explain why young firms are more cyclical than old firms using financial type shocks (e.g. collateral constraint or discount rate type shocks), a sufficient tightening of the collateral constraint is required.

2. Heterogeneous returns to scale can explain why large firms are more cyclical than small, if large firms have less decreasing returns to scale.

3. To explain the joint age-size patterns – and in particular that the largest firms are equally cyclical regardless of their age – requires that larger firms enter with relatively more net worth than smaller firms.

By calibrating three features, we are able to bring the model in line with the key firm-level cyclicality findings from our data. These features represent departures from a standard heterogeneous-firm model, and hence our data is helpful in suggesting how these models may need to be modified to match certain features of the data.

4.4 Results 2: Policy implications of firm age/size distribution and responsiveness

Our final exercise is to investigate the implications of our results and model for business cycle policy. These are informed both by our empirical findings on the cyclicality of different firm groups, and the twists they implied to our calibrated model. In particular, we compare policy exercises in both our "standard" financial frictions model, and in the final "calibrated" model, which introduced the two new features of i) heterogeneous entrant net worth by size, and ii) heterogeneous returns to scale. We will consider two policies, and show how their effects are modified by these new model features.

We consider two simple policies, meant to capture two different styles of possible policy intervention during a recession. The first policy we call an "incentive" type policy, which consists of a temporary subsidy to the firm's wage bill. In particular, we introduce a subsidy so that the government pays a fraction τ of all firms' wage bills, with $\tau = 0$ in steady state. We consider a temporary increase of the subsidy to 1% of the wage bill, which fades within two years. We call this policy an incentive type policy because it changes the effective marginal cost of production for firms, and hence incentivises them to expand production. The second policy we call a "balance sheet" policy, which consists of giving debt relief to firms. In this policy, at time 0 the policymaker pays off a fraction x of all firms' debts, reducing their debt from b to $(1 - x)b$, and hence increasing their net worth. This policy does not affect the marginal cost of finance for firms, but it does increase the net worth of firms, and thus the access to debt for financially constrained firms. We consider a one-off 20% debt forgiveness at time 0.

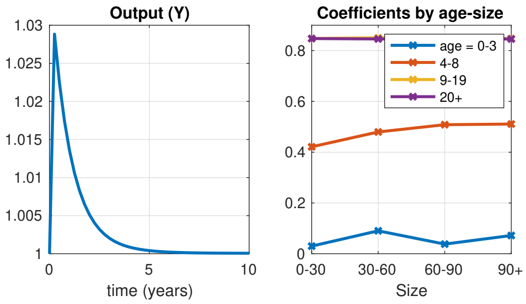
The results are given in Figure 9. The labor subsidy is investigated in the top row and the balance sheet policy in the bottom row. The standard model is investigated in the left column and the calibrated model in the right. For each configuration, we plot both the response of aggregate output, and the cyclicality of firms by joint age-size bin, as measured by the regression coefficients.

We start with the labor subsidy policy. In the standard model, shown in Figure 9(a), this leads to a nearly 3% increase in output on impact, which fades as the policy is withdrawn. The regression coefficients reveal that large firms are more responsive to this policy than young firms. This follows from the fact that young firms are financially constrained, and hence unable to expand in response to the labor subsidy because it would require taking on more debt to purchase capital, which they do not have access to. The regression results also reveal that the policy does not have differential effects by firm size, once age is conditioned for. The effect of this policy is drastically changed in our new calibrated model, which is shown in Figure 9(b). At an aggregate level, the policy is more effective in our new model, with the increase in output rising to 3.8% on impact, as opposed to 3% in the standard model. The reasons behind this relate to the firm age-size responses, which are markedly different from the standard model. The policy now has a clear firm size dimension, with large firms being more responsive to the policy even conditioning on age. This creates a composition effect which boosts the aggregate response, as the firms who respond the most also happen to be large and hence more important for aggregate output. As with earlier results, this follows directly from the fact that large firms having less decreasing returns to scale makes them more sensitive to changes in their marginal costs. Finally, this effect is also amplified by the heterogeneous entry net worth assumption. As large entrants are now less financially constrained, this allows them to be as responsive to the policy as older large firms, which increases the total number of large firms who are more responsive to the policy.

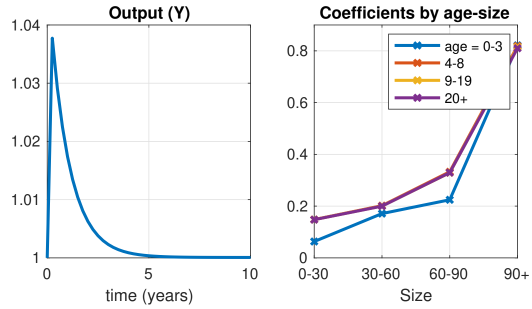
We now turn to the debt forgiveness policy. In the standard model, shown in Figure 9(c), this leads to a roughly 3% increase in output on impact, which gradually fades over the next five years. This persistence is fully endogenous, as the policy is only in place at time 0. It leads to higher output even many years in the future because firms financial positions are slow moving, and so by helping firms at time 0 they remain less financially constrained for the rest of their lifecycle. The regression results reveal that this policy has the largest effect on young firms, as these firms are more likely to be financially constrained. As with the labor subsidy policy, there is no difference in the strength of the policy by firm size, once age is controlled for. The effect of this policy is again drastically changed in our new calibrated model, which is shown in Figure 9(d).

At an aggregate level, the policy is less effective in our new model, with the increase in output reduced by a factor of 10 on impact. The reasons behind this relate to the firm age-size responses, which are markedly different from the standard model. The policy still affects young firms more than old, but the effect is stronger for "young and small" firms, and declines as firm size increases. This

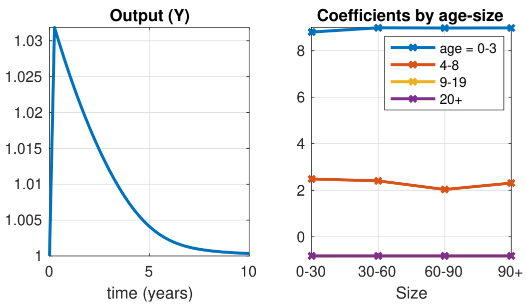
Figure 9: Effect of two policies in standard vs calibrated model



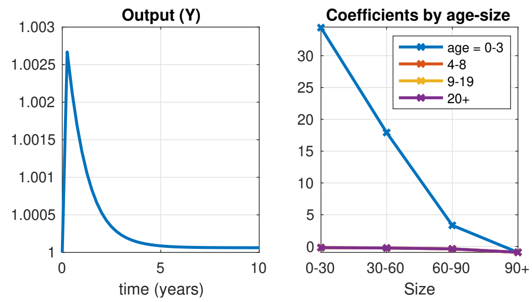
(a) Labor subsidy: standard model



(b) Labor subsidy: calibrated model



(c) Debt relief: standard model



(d) Debt relief: calibrated model

Panels give regression coefficients from regressions of firm-level growth rates of employment on aggregate GDP growth, computing using the model simulation in Experiment 1. The regressions are on firm-level data aggregated to the yearly level and treated in the same way as the data.

creates a composition effect which dampens the aggregate response, because the firms responding to the policy are now smaller on average. This follows because entrant net worth is heterogeneous by firm size, with larger entrants being less financially constrained, and hence less responsive to a debt forgiveness policy.

In conclusion, our empirical results inform changes to a standard financial frictions model which have important policy implications. Some policies (incentive based) are more effective, and others are less (debt forgiveness), with the changes due to the fact that firms of different ages and sizes now respond differently to these policies.

5 Conclusion

In this paper we documented novel facts about the cyclicality firms by age and size, with particular attention paid to the interaction between the two, and the role of finance. Using high quality registry data from the universe of Danish firms, we first document that employment and turnover are more sensitive to the business cycle at younger firms than older firms, but that the relationship between size and cyclicality is more complicated. Among older firms, large firms are more cyclical than small, while among young firms, small firms are more cyclical than large.

These results are possible because our dataset contains explicit information about when firms are formed, allowing us to construct a high quality measure of firms actual age from legal inception. This distinguishes our dataset from other sources where it is either not possible to measure age, or only to do so from the age that firms go public. This allows us to look at the cyclicality of very young firms, which is where we find the strongest excess cyclicality. We additionally have data for firms of all sizes, allowing us to investigate cyclicality for even the smallest of firms. We use this data to additionally provide a detailed investigate of firm outcomes and growth over the lifecycle.

Given that our dataset contains detailed financial variables, we then investigate the role of finance in driving the excess cyclicality of different firm groups. We find that young firms have higher leverage than old firms, and hence are more likely to be financially constrained. They additionally are typically trying to expand their leverage, while leverage is typically shrinking at older firms. On the other hand, leverage ratios are remarkably similar across firm size groups, after controlling for firm age. Hence, we argue that the excess cyclicality of young firms is plausibly linked to financial frictions, while the same is less likely to be true for larger old firms.

We then use these insights to build a quantitative heterogeneous firm model, and investigate the extensions to a standard calibration needed to replicate our new facts. We find that standard calibrations struggle to match cyclicalities across age, size, and joint age-size bins at the same time. Part of the problem is that in standard models age and size are too closely linked, as young firms tend to be financially constrained and, hence, smaller. Two extensions bring the model closer to the data. Firstly, we introduce heterogeneous returns to scale, so that large firms have less decreasing returns to scale. This can parsimoniously explain why they are larger and more cyclical. Secondly, we allow larger firms to be born richer, and hence less financially constrained. This explains why, among large firms, cyclicality does not depend on firm age, as in the data. Together, these extensions bring the model's implications for cyclicality by joint age-size bin in line with the data. We finally use our model to investigate the effect of recession-fighting policies, and how they transmit through the firm age and size distributions. A key implication of these exercises is that properly matching the responsiveness of firms by age and size can have large effects on the policy implications of our models.

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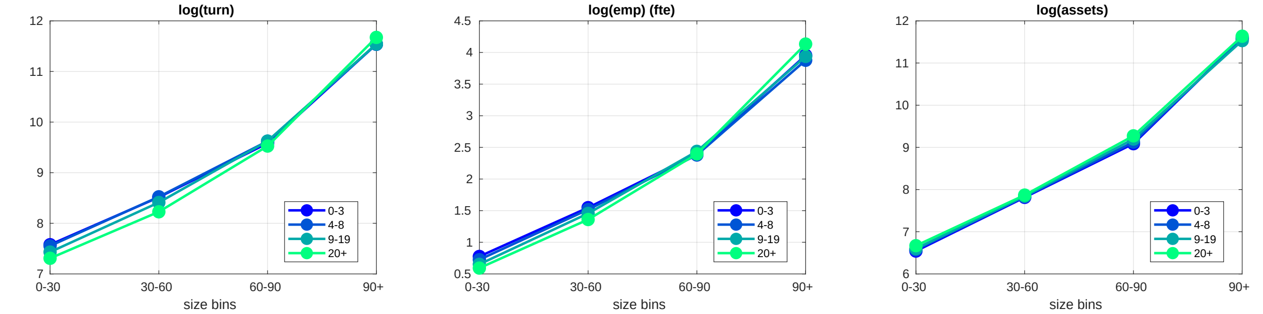
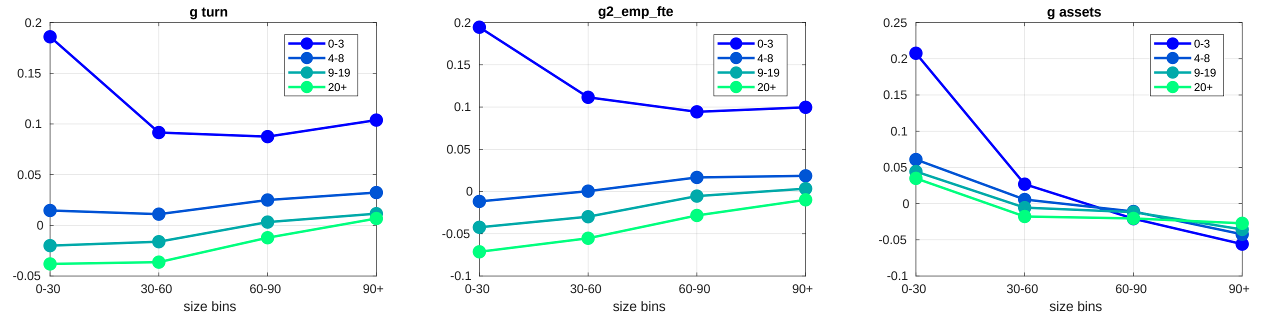
A Results with assets as size sorting variable

Figure 10: Average levels with full interaction of size and age (regression (1)).



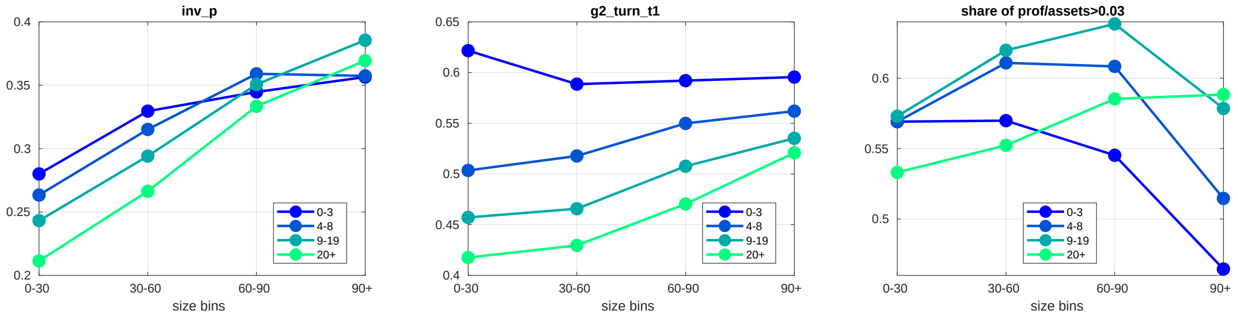
(a) Sales (b) Employment (c) Assets



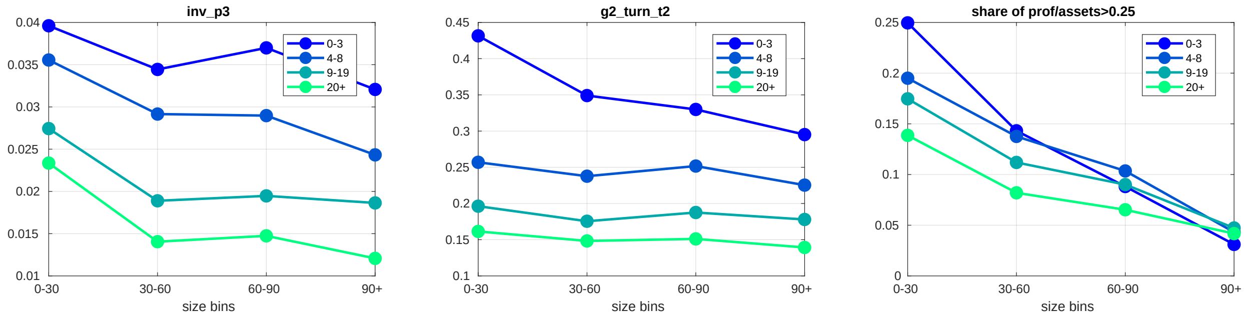
(d) Growth rate of sales (e) Growth rate of employment (f) Growth rate of assets

Note: Average level is computed as coefficient α_{lk} from regression (1). Different lines correspond to firms of different size bins whereas firm age bin is on x axis.

Figure 11: Average growth rates, full interaction of size and age (regression (1)).



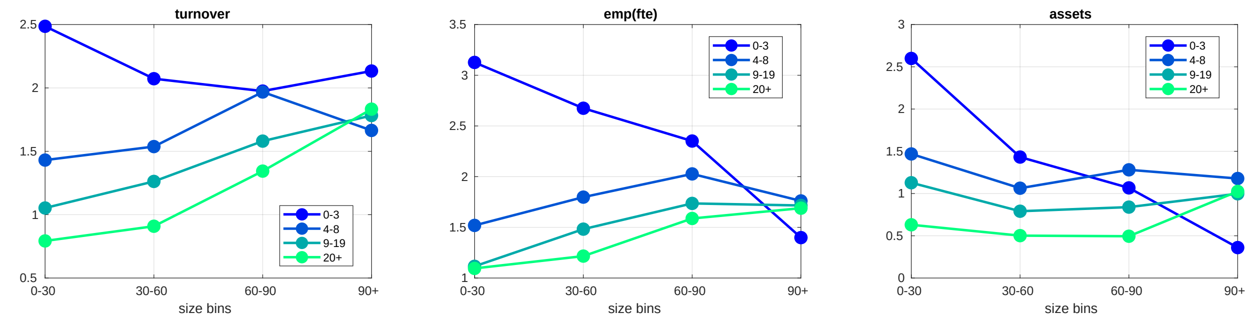
(a) Share with positive investment (b) Share with above median growth in sales (c) Share with midly positive profits



(d) Share with high investment (e) Share with high sales growth (f) Share with high profits

Note: high investment defined as investment/assets>0.25. Mild profits are defined as profits/assets>0.03, high profits are defined as profits/assets>0.25. High sales growth is defined as sales growth larger than 75th percentile of sales growth distribution.

Figure 12: Predicted cyclicality with full interaction of size and age (regression (2)). Apart from the youngest firms, cyclicality increases with age and the level is shifted down by as firms get older. For the youngest firms, the increase in cyclicality by size is much less aparent than for older firms (in the case of employment) or it even falls with firm size (turnover).



(a) Sales (b) Employment (c) Assets

Note: Cyclicality is computed as coefficient β_{lk} from regression (2), which predicts (average) response (in pp) in tunrnover or employment to 1pp change in aggregate gdp growth. Different lines correspond to firms of different age bins whereas firm size bin is on x axis (note that the axes are reversed compared to the previous graphs).