Babeş-Bolyai University, Faculty of Mathematics and Computer Science

Bachelor, Computer Science, Groups 911-917, Academic Year 2019-2020

Mathematical Analysis
Seminar 4

1. Find the local extremum points (specifying their type) of the following functions:

a) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x^3 - 3x + y^2$.

b) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x^3 + y^3 - 3xy$.

c) $f : (0, \infty) \times \mathbb{R} \to \mathbb{R}, \quad f(x, y) = x(y^2 + \ln^2 x)$.

d) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x^4 + y^4 - 4(x - y)^2$.

e) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x, y, z) = z^2(1 + xy) + xy$.

f) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x, y, z) = x^3 - 3x + y^2 + z^2$.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined for all $(x, y) \in \mathbb{R}^2$ by

$$f(x, y) = (x^2 - y)(x^2 - 3y).$$

a) Prove that 0_2 is a stationary point of f.

b) Study whether 0_2 is a local minimum point of f.

c) Prove that the restriction of f to any line through 0_2 attains a local minimum at 0_2.

3*. Let $D = [0, 2] \times [0, 4] \subseteq \mathbb{R}^2$ and let $f : D \to \mathbb{R}$ be a function defined for all $(x, y) \in D$ by

$$f(x, y) = x^2 - 2xy + 2y.$$

a) Prove that f has at least one global minimum point and at least one global maximum point.

b) Find all global extremum points of f.