Mathematical Analysis
Seminar 5

1. Find the local extrema of $f : \mathbb{R}^3_+ \to \mathbb{R},\ f(x, y, z) = xyz$ subject to $x + y + z = 1$.

2. Let $a = (1, 2) \in \mathbb{R}^2$. Find the point on the unit circle (2d-sphere) $S := \big\{x = (x_1, x_2) \in \mathbb{R}^2 : \|x\| = 1\big\}$ which lies closest to a. Formulate and solve this as an Optimization problem.

3. The box of fixed volume and minimal surface.
Find the minimum of $f : \mathbb{R}^3_+ \to \mathbb{R},\ f(x, y, z) = 2xy + 2yz + 2zx$ subject to $xyz = 1$.

4. Maximize $f(x, y, z) = 2x + 3y + 5z$ on the sphere $x^2 + y^2 + z^2 = 1$.

5. A large container in the shape of a rectangular solid must have a volume of $480m^3$. The bottom of the container costs $\$5/m^2$ to construct whereas the top and sides cost $\$3/m^2$ to construct. Use Lagrange multipliers to find the dimensions of the container that has minimum cost.