Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Bachelor, Computer Science, Groups 911-917, Academic Year 2019-2020

Mathematical Analysis
Seminar 7

1. Compute the following integrals:

a) $\iint_A (\sin x + \sin y)\,dx\,dy$, where $A = [0, \pi/2] \times [0, \pi/4]$.

b) $\iint_A \frac{1}{(x+y)^2}\,dx\,dy$, where $A = [3, 4] \times [1, 2]$.

c) $\iint_A \frac{x}{x^2+y^2}\,dx\,dy$, where $A = [1, 2] \times [0, 1]$.

d) $\iint_A \frac{xy^2}{x^2+1}\,dx\,dy$, where $A = [1, 2] \times [-3, 3]$.

e) $\iint_A \frac{1}{x^2+y^2}\,dx\,dy$, where $A = [1/a, a] \times [0, 1]$ with $a > 1$.

f) $\iint_A \min\{x, y\}\,dx\,dy$, where $A = [0, 1] \times [0, 1]$.

g) $\iiint_A \frac{x^2z^3}{1+y^2}\,dx\,dy\,dz$, where $A = [-1, 0] \times [-1, 1] \times [0, 1]$.

h) $\iiint_A \frac{1}{(x+y+z)^3}\,dx\,dy\,dz$, where $A = [1, a] \times [1, a] \times [1, a]$ with $a > 1$.

j) $\int \cdots \int_A e^{x_1+\cdots+x_n}\,dx_1 \ldots dx_n$, where $A = [0, 1] \times \cdots \times [0, 1]$

2. Let M be the subset of $\mathbb{R}^2$ bounded by the parabola $y = x^2$ and the lines $x = 2$ and $y = 0$. a) Express M as a simple set first w.r.t. the y-axis and then w.r.t. the x-axis. b) Compute $\iint_M xy\,dx\,dy$ in two ways.

3. Let M be the subset of $\mathbb{R}^2$ bounded by the parabolas $y = 2x^2$ and $y = x^2 + 1$. a) Express M as a simple set w.r.t. the y-axis. b) Study whether M is simple w.r.t. the x-axis. c) Compute $\iint_M (x + 2y)\,dx\,dy$.

4. Compute $\iiint_M z\,dx\,dy\,dz$ where $M = \left\{(x, y, z) \in \mathbb{R}^3 \;\middle|\; 0 \le x \le 2,\ 0 \le y \le 2\sqrt{x},\ 0 \le z \le \sqrt{4x - y^2}\right\}$.

5. Compute the integral $\iiint_M \frac{1}{(1+x+y+z)^3}\,dx\,dy\,dz$ where $M \subseteq \mathbb{R}^3$ is the set bounded by the coordinate planes and the plane $x + y + z = 1$.