Babeş-Bolyai University, Faculty of Mathematics and Computer Science

Bachelor, Computer Science, Groups 911-917, Academic Year 2019-2020

Mathematical Analysis
Seminar 11

1. Analyze (without using convergence tests) the convergence of

$$\text{a)} \sum_{n=0\text{or}1}^{\infty} q^n, \quad |q|<1, \quad \text{b)} \sum_{n=1}^{\infty} \frac{1}{n}, \quad \text{c)} \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

2. Discuss (w.r.t. $a>0$) the convergence of the generalized harmonic series $\sum\limits_{n=1}^{\infty} \frac{1}{n^a}$ using

Theorem (Cauchy's condensation test) Let $(a_n)_{n\in\mathbb{N}^*}$ be a decreasing sequence with positive terms, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \sum_{n=1}^{\infty} 2^n a_{2^n}$$

are both convergent or both divergent.

3. Prove that the following series are divergent: a) $\sum\limits_{n\geq 1} \operatorname{arctg} n$ and b) $\sum\limits_{n\geq 1} \sin n$.

4. Study if the following series are convergent or divergent:

$$\text{a)} \sum_{n\geq 1} \frac{e^n}{n+3^n}; \qquad \text{b)} \sum_{n\geq 1} \frac{1}{n^2-\ln n+\sin n};$$

$$\text{c)} \sum_{n\geq 1} \frac{\sqrt{n+1}}{1+2+\cdots+n}; \qquad \text{d)} \sum_{n\geq 1} \frac{2^n\cdot n!}{n^n};$$

$$\text{e)} \sum_{n\geq 1} \frac{5^{n/2}}{n2^n}; \qquad \text{f)} \sum_{n\geq 1} (\arctan n)^n;$$

$$\text{g)} \sum_{n\geq 1} \frac{n^2}{2^{n^2}}; \qquad \text{h)} \sum_{n\geq 1} \frac{(n+1)^n}{n^{n+2}};$$

$$\text{i)} \sum_{n\geq 1} \ln\left(1+\frac{1}{n}\right); \qquad \text{j)} \sum_{n\geq 2} \frac{1}{n\ln n}.$$

5. Find the sum of the following series:

$$\text{a)} \sum_{n=1}^{\infty} (-\pi/4)^n; \qquad \text{b)} \sum_{n=1}^{\infty} 3^{1-2n};$$

$$\text{c)} \sum_{n=1}^{\infty} \binom{n+2}{3}^{-1}; \qquad \text{d)} \sum_{n=1}^{\infty} \frac{1}{1^2+2^2+\ldots+n^2};$$

$$\text{e)} \sum_{n=1}^{\infty} \left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right); \qquad \text{f)} \sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right);$$

$$\text{g)} \sum_{n=0}^{\infty} \operatorname{arctg}\frac{1}{n^2+n+1}; \qquad \text{h)} \sum_{n=0}^{\infty} \frac{n+1}{2^n}.$$