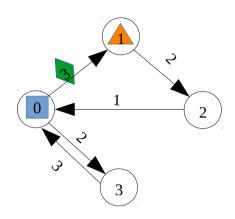
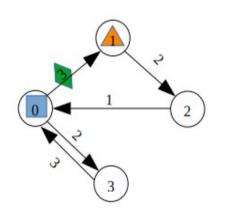
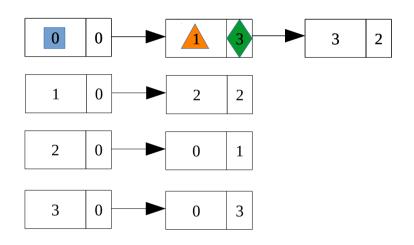
Grafos - Matriz de Adjacência



Vertices	0	1	2	3
0	0	3	0	2
1	0	0	2	0
2	1	0	0	0
3	3	0	0	0

Grafos - Lista Ligada





Algoritmo de Dijkstra Pseudocódigo

}

```
Function Dijkstra(G, source, target):
    for each vertex v in G
        dist[v] = infinity
    dist[source] = 0
    Q has the set of all nodes in G
    while Q is not empty:
        u = vertex in Q with smallest dist
        remove u from Q
        if u = target
             break
        for each arc (v,u) in G
             if dist[v] > dist[u] + dist_between(v, u)
                 dist[v] = dist[u] + dist_between(v, u)
    return dist
Código
void dijkstra(int graph[][MAX_NODES], int num_nodes, int source, int target){
    for (int i=0; i<num_nodes; i++) graph[i][i] = __INT_MAX__;</pre>
    graph[source][source] = 0;
    int node_set[num_nodes];
    for (int i=0; i<num_nodes; i++) node_set[i] = 1;</pre>
    while (set_is_empty(node_set, num_nodes) \neq 1){
        int smallest = smallest_dist(graph, node_set, num_nodes);
        node set[smallest] = -1;
        if (smallest = target) break;
        for (int i=0; i<num_nodes; i++){</pre>
            if (graph[smallest][i] \neq 0 \ 86 \ node\_set[i] \neq -1){
                 if(graph[i][i] > graph[smallest][smallest]+graph[smallest][i]
                      & graph[smallest][smallest] ≠ __INT_MAX_
                                                  _INT_MAX__
                      \delta \theta graph[smallest][i] \neq
                     graph[i][i]=graph[smallest][smallest] + graph[smallest][i];
                 }
            }
        }
    }
}
int smallest_dist(int graph[][MAX_NODES], int node_set[], int num_nodes){
    int min = __INT_MAX__, node=-1;
    for (int i=0; i<num_nodes; i++){</pre>
        if (node_set[i] \neq -1){
            if (graph[i][i] < min){</pre>
                 min = graph[i][i];
                 node = i;
            }
        }
    }
    return node;
}
int set_is_empty(int node_set[], int num_nodes){
   int is_empty = 1;
   for (int i=0; i<num_nodes; i++){</pre>
       if (node\_set[i] \neq -1){
           return -1;
   return is_empty;
```

Algoritmo de Bellman-Ford

Pseudocódigo

}

```
Function BelmannFord(G, source):
    for each vertex v in G
        dist[v] = infinity
    d[source]=0;
    for(i=0; i<|V|-1; i++)
        for each arc (u,v) in G
            if dist[v] > dist[u]+ dist_between(u, v)
                d[v] = d[u] + dist\_between(u, v)
    // Verificação de ciclos negativos
    for each arc (u, v) in G
        if dist[v] > dist[u]+ dist_between(u, v)
            return false // Ciclo negativo!!
    return true
Código
int bellman_ford(int graph[][MAX_NODES], int num_nodes, int source, int target){
    for (int i=0; i<num nodes; i++) graph[i][i] = INT MAX ;
    graph[source][source] = 0;
    /*Iterate |V| - 1, i.e, number of nodes - 1 */
    for (int i=0; i<num_nodes-1; i++){</pre>
        for (int j=0; j<num_nodes; j++){</pre>
            for (int k=0; k<num_nodes; k++){</pre>
                 if (k=j \parallel graph[j][k] = 0) continue;
                 if (graph[k][k] > graph[j][j] + graph[j][k]
                     & graph[j][j] \neq INT_MAX_ & graph[j][k] \neq INT_MAX_){
                     graph[k][k] = graph[j][j] + graph[j][k];
            }
        }
    }
    /*Iteration number |V| serves to detect any negative cycles*/
    for (int j=0; j<num_nodes; j++){</pre>
        for (int k=0; k<num_nodes; k++){</pre>
            if (k=j \parallel graph[j][k] = 0) continue;
            if (graph[k][k] > graph[j][j] + graph[j][k]
                 & graph[j][j] \neq INT_MAX_ & graph[j][k] \neq INT_MAX_){
                 return -1;
            }
        }
    }
    return 1;
```

Algoritmo de Floyd-Warshall

Pseudocódigo

```
void floyd warshall(int graph[][MAX NODES], int num nodes){
    /*Set the diagonal to 0*/
    for (int i=0; i<num_nodes; i++) graph[i][i] = 0;</pre>
    /*Set 0 values to infinity (INT MAX)*/
    for(int i=0; i<num_nodes; i++){</pre>
         for(int j=0; j<num_nodes; j++){
    if (i\neqj 86 graph[i][j] = 0) graph[i][j] = __INT_MAX__;
    }
    for (int k=0; k<num nodes; k++){</pre>
         for (int i=0; i<num_nodes; i++){</pre>
             for (int j=0; j<num_nodes; j++){</pre>
                  /*Don't calculate if right side values are INT_MAX, overflow*/
                  if (graph[i][j] > graph[i][k] + graph[k][j]
                     & graph[i][k] \neq __INT_MAX__ & graph[k][j] \neq __INT_MAX__)
                      graph[i][j] = graph[i][k] + graph[k][j];
             }
        }
    }
}
```

Algoritmo BFS (Breadth-first Search)

Pseudocódigo

```
void bfs(int graph[][MAX NODES], int num nodes, int current, int target, int
visited[], int previous[]){
    /*Queue that holds neighbor nodes of current that haven't yet been visited*/
    queue <int> bfs_queue;
    /*Push the first node (the source node)*/
    bfs_queue.push(current);
    previous[0] = -1;
    /*While stack has nodes to visit*/
    while(!bfs queue.empty()){
        /*Update current node and pop*/
        current = bfs_queue.front();
        bfs_queue.pop();
        /*We have visited this function node*/
        visited[current] = 1;
        /*For each edge that goes out of current node*/
        for (int j=0; j<num_nodes; j++){</pre>
            if (graph[current][j] \neq 0){
                /*If one of these nodes is our target*/
                if (j=target){
                    previous[j] = current;
                    return;
                }
                /*If we haven't visited node yet*/
                else if(visited[j]=0){
                    previous[j] = current;
                    bfs_queue.push(j);
                }
           }
       }
   }
}
```

Algoritmo DFS (Depth-first Search)

/*Haven't found target in this recursive step*/

Pseudocódigo

return 0;

}

```
Function DFS(G, v):
    if v is the goal:
        exit
    label v as visited
    for each neighbor u of v:
        if u is not labeled as discovered:
             u.parent = v
             DFS(G, u)
Código
int dfs(int graph[][MAX_NODES], int num_nodes, int current, int target, int visited[],
int previous[]){
    previous[0] = -1;
    /*We have visited this node*/
    visited[current]=1;
    if (current=target){
        return 1;
    }
    else{
        /*For each edge*/
        for (int j=0; j<num_nodes; j++){</pre>
            /*If an edge exists*/
            if (graph[current][j] \neq \emptyset)
                /*Unvisited node and in recursion we found target,update previous*/
                if (dfs(graph, num_nodes, j, target, visited, previous)=1
                      & visited[j]\neq 1){
                    previous[j] = current;
                    return 1;
                }
            }
        }
```

Algoritmo de Kruskal - Estrutura União-Busca (Disjoint-set)

Pseudocódigo - MakeSet

```
function MakeSet(x)
  if x is not already present:
    add x to the disjoint-set tree
    x.parent = x
    x.rank = 0
    x.size = 1
```

Pseudocódigo - Find

Path compression	Path halving	Path splitting
<pre>function Find(x) if x.parent!= x x.parent:= Find(x.parent) return x.parent</pre>	<pre>function Find(x) while x.parent!= x x.parent:= x.parent.parent x:= x.parent return x</pre>	<pre>function Find(x) while x.parent!= x next := x.parent x.parent := next.parent x := next return x</pre>

Pseudocódigo - Union

```
Union by rank
                                                                    Union by size
function Union(x, y)
                                                 function Union(x, y)
 xRoot:= Find(x)
                                                   xRoot:= Find(x)
 yRoot:= Find(y)
                                                   yRoot:= Find(y)
 //x and y are already in the same set
                                                   //x and y are already in the same set
 if xRoot == yRoot
                                                   if xRoot == yRoot
      return
                                                        return
 //x and y are not in same set,so merge them
                                                   //x and y are not in same set,so merge them
  if xRoot.rank < yRoot.rank</pre>
                                                   if xRoot.size < yRoot.size</pre>
    // swap xRoot and yRoot
                                                     // swap xRoot and yRoot
    temp := xRoot
                                                     temp := xRoot
    xRoot := yRoot
                                                     xRoot := yRoot
    yRoot := temp
                                                     yRoot := temp
 // merge yRoot into xRoot
                                                   // merge yRoot into xRoot
 yRoot.parent:= xRoot
                                                   yRoot.parent:= xRoot
 if xRoot.rank == yRoot.rank:
                                                   xRoot.size:= xRoot.size + yRoot.size
    xRoot.rank:= xRoot.rank + 1
```

Algoritmo de Kruskal - Estrutura União-Busca (Disjoint-set)

```
void make_set(int parent_set[], int rank_set[], int size_set[], int num_nodes){
    /*Create a new set with num nodes nodes, all of them are their own parents*/
    for (int i=0; i<num nodes; i++){</pre>
        parent set[i] = i;
        rank_set[i] = 0;
        size set[i] = 1;
    }
}
int find compress(int parent set[], int x){
    if (parent_set[x] \neq x){
        parent set[x] = find compress(parent set, parent set[x]);
    }
    return parent_set[x];
}
int find_halve(int parent_set[], int x){
   while (parent_set[x] \neq x){
        parent_set[x] = parent_set[parent_set[x]];
        x = parent_set[x];
    }
    return x;
}
int find_split(int parent_set[], int x){
    while (parent_set[x] \neq x){
        int next = parent_set[x];
        parent set[x] = parent set[next];
        x = next;
    }
    return x;
}
```

Algoritmo de Kruskal - Estrutura União-Busca (Disjoint-set)

Código (cont.)

```
void union_rank(int parent_set[], int rank_set[], int x, int y){
    int xRoot = find_compress(parent_set, x);
    int yRoot = find compress(parent set, y);
    /* x and y are already in the same set */
    if (xRoot = yRoot) return;
    /* x and y are not in same set, so we merge them */
    if (rank_set[xRoot] < rank_set[yRoot]){</pre>
        int temp = xRoot;
        xRoot = yRoot;
        yRoot = temp;
    }
    /*Merge vRoot into xRoot*/
    parent_set[yRoot] = xRoot;
    if (rank_set[xRoot]=rank_set[yRoot])
       rank set[xRoot] = rank set[xRoot] + 1;
}
void union size(int parent set[], int size set[], int x, int y){
    int xRoot = find compress(parent set, x);
    int yRoot = find_compress(parent_set, y);
    /* x and y are already in the same set */
    if (xRoot = yRoot) return;
    /* x and y are not in same set, so we merge them */
    if (size set[xRoot] < size set[yRoot]){</pre>
        int temp = xRoot;
       xRoot = yRoot;
        yRoot = temp;
    }
    /*Merge yRoot into xRoot*/
    parent set[vRoot] = xRoot;
   size set[xRoot] = size set[xRoot] + size set[yRoot];
}
/************
* Structure that represents a graph edge.
* u: node u;
* v: node v;
**********************************
typedef struct Edge{
   int u, v;
   int weight;
} Edge;
```

Algoritmo de Kruskal - Algoritmo

Pseudocódigo

```
Function Kruskal(G):
    A = \emptyset
    foreach v E G.V:
        MAKE-SET(v)
    foreach (u, v) in G.E ordered by weight(u, v), increasing:
        if FIND-SET(u) ≠ FIND-SET(v):
            A = A \cup \{(u, v)\}
            UNION(u, v)
    return A
Código
```

```
void kruskal(int graph[][MAX_NODES], int num_nodes){
    /*Empty set of edges, will hold result*/
    Edge *spanning_tree = (Edge*) malloc(MAX_NODES*(MAX_NODES)-1 * sizeof(int));
    int spanning_tree_size = 0;
    /*Set that will hold all edges in graph sorted by weight*/
    Edge *sorted_edges = (Edge*) malloc(MAX_NODES*(MAX_NODES)-1 * sizeof(int));
    int sorted_edges_size = 0;
    /*Create new set*/
    int parent_set[num_nodes]; int rank_set[num_nodes]; int size_set[num_nodes];
    make_set(parent_set, rank_set, size_set, num_nodes);
    /*Add the edges and sort them*/
    for (int i=0; i<num_nodes; i++){</pre>
        for (int j=0; j<num_nodes; j++){</pre>
            if (graph[i][j] \neq 0){
                sorted_edges[sorted_edges_size].u = i;
                sorted_edges[sorted_edges_size].v = j;
                sorted_edges[sorted_edges_size].weight = graph[i][j];
                sorted_edges_size+=1;
            }
        }
    }
    qsort(sorted_edges, sorted_edges_size, sizeof(struct Edge), comparator);
    /*For each sorted edge*/
    for (int i=0; i<sorted_edges_size; i++){</pre>
        int u = sorted_edges[i].u; int v = sorted_edges[i].v;
        if (find_compress(parent_set, u) \neq find_compress(parent_set, v)){
            /*Add the edge to the spanning tree*/
            spanning_tree[spanning_tree_size] = sorted_edges[i];
            spanning_tree_size+=1;
            union_rank(parent_set, rank_set, u, v);
        }
    }
    /*Print results*/
    printf("Spanning tree: \n");
    for (int i=0; i<spanning_tree_size; i++){</pre>
        printf("Edge (%d, %d): %d\n", spanning_tree[i].u, spanning_tree[i].v,
spanning_tree[i].weight);
    }
    /*Free allocated memory*/
    free(spanning_tree);
    free(sorted_edges);
}
```

Algoritmo para calcular pontos de articulação de um grafo

Pseudocódigo - DFS modificada

```
GetArticulationPoints(i, d)
    visited[i] = true
    depth[i] = d
   low[i] = d
    childCount = 0
    isArticulation = false
    for each ni in adj[i]
        if not visited[ni]
           parent[ni] = i
           GetArticulationPoints(ni, d + 1)
           childCount = childCount + 1
           if low[ni] ≥ depth[i]
               isArticulation = true
           low[i] = Min(low[i], low[ni])
       else if ni \neq parent[i]
           low[i] = Min(low[i], depth[ni])
    if (parent[i] \neq null and isArticulation) or (parent[i] = null and childCount > 1)
       Output i as articulation point
Run GetArticulationPoints() for every unvisited node in main()
Código
void articulation points(int graph[][MAX NODES], int num nodes, int current, int
depth, int visited[], int parent[], int depths[], int low[], int points[]){
    visited[current] = 1;
    depths[current] = depth;
    low[current] = depth;
    int child count = 0;
    bool is articulation = false;
    /*For each neighbor*/
    for (int j=0; j<num nodes; j++){
        /*If a connection exists*/
        if (graph[current][j] \neq \emptyset){
             if (visited[j]=0){
                 parent[j] = current;
                 articulation points(graph, num nodes, j, depth+1, visited,
parent, depths, low, points);
                 child count += 1;
                 if (low[j] ≥ depths[current]) is_articulation = true;
                 low[current] = min(low[current], low[j]);
             else if (j \neq parent[current])
                low[current] = min(low[current], depths[j]);
        }
    }
    /*If node isn't the root node and is an articulation point, or it is root
node and has more than a child, mark it as an articulation point of graph*/
    if ((parent[current] \neq -1 & is articulation) || (parent[current] = -1 &
            child count>1))
        points[current] = 1;
}
```

Algoritmo de Tarjan para encontrar componentes fortemente conexas

Pseudocódigo

```
algorithm tarian is
  input: graph G = (V, E)
  output: set of strongly connected components (sets of vertices)
  index = 0
 S = empty stack
  for each \boldsymbol{v} in \boldsymbol{V} do
    if (v.index is undefined):
      strongconnect(v)
  function strongconnect(v)
    // Set the depth index for v to the smallest unused index
    v.index = index
    v.lowlink = index
    index = index + 1
    S.push(v)
    v.onStack = true
    // Consider successors of v
    for each (v, w) in E:
      if (w.index is undefined):
        // Successor w has not yet been visited; recurse on it
        strongconnect(w)
        v.lowlink = min(v.lowlink, w.lowlink)
      else if (w.onStack):
        // Successor w is in stack S and hence in the current SCC
        //If w isn't on stack, then(v,w) is cross-edge in DFS tree and ignored
        // Note: The next line may look odd - but is correct.
        // It says w.index not w.lowlink; that is deliberate
        v.lowlink = min(v.lowlink, w.index)
    // If v is a root node, pop the stack and generate an SCC
    if (v.lowlink = v.index):
      start a new strongly connected component
      do
        w = S.pop()
        w.onStack = false
        add w to current strongly connected component
      while (w \neq v)
      output the current strongly connected component
```

Algoritmo de Tarjan para encontrar componentes fortemente conexas

```
void tarjan(int graph[][MAX_NODES], int num_nodes, int current, stack <int> *S,
int index[], int lowlink[], bool onStack[], set <int> strong_component[], int
*index_global, int *num_components){
    // Set the depth index for v to the smallest unused index
    index[current] = *index global;
    lowlink[current] = *index_global;
    (*index_global) += 1;
    (*S).push(current);
    onStack[current] = true:
    // Consider successors of v
    for (int j=0; j<num_nodes; j++){</pre>
        /*If there's a connection*/
        if (graph[current][j] \neq \emptyset){
            if (index[j] = -1){
                // Successor w has not yet been visited; recurse on it
                tarjan(graph, num_nodes, j, S, index, lowlink, onStack,
strong_component, index_global, num_components);
                lowlink[current] = min(lowlink[current], lowlink[j]);
            else if (onStack[j]){
                // Successor w is in stack S and hence in the current SCC
                // If w is not on stack, then (v, w) is a cross-edge in the DFS
tree and must be ignored
                lowlink[current] = min(lowlink[current], index[j]);
            }
        }
    }
    // If v is a root node, pop the stack and generate an SCC
    int j;
    if (lowlink[current] = index[current]){
        do{
            j = (*S).top();
            (*S).pop();
            onStack[j] = false;
            strong_component[(*num_components)].insert(j);
        } while (j \neq current);
        (*num components)+=1;
    }
}
```

Ordenação topológica

Pseudocódigo - DFS

```
L 	Empty list that will contain the sorted nodes
while exists nodes without a permanent mark:
    select an unmarked node n
    visit(n)

function visit(node n):
    if n has a permanent mark then return
    if n has a temporary mark then stop (not a DAG)
    mark n with a temporary mark
    for each node m with an edge from n to m:
        visit(m)
    remove temporary mark from n
    mark n with a permanent mark
    add n to head of L
```

```
int sort(int graph[][MAX_NODES], int num_nodes, int current, vector<int>
*sorted, int visited[]){
    /*If current node was already visited*/
    if (visited[current]=1) return 1;
    /*If current node is already being visited in another stack call*/
    if (visited[current]=-1) return -1;
    /*Mark the node with a temporary mark*/
    visited[current] = -1;
    /*For each neighbor of current node*/
    for (int j=0; j<num nodes; j++){
        /*If a connection exists*/
        if (graph[current][j] \neq \emptyset){
            if (sort(graph, num nodes, j, sorted, visited)=-1) return -1;
    }
    /*Mark this node as visited*/
    visited[current] = 1;
    /*Add the node to the list*/
    (*sorted).emplace back(current);
    return 1;
}
```

Input

```
Exemplo - Um caso de teste <source>
```

```
<target>
<matriz>
int main(int argc, char **argv){
    char temp[MAX_TEMP];
    int graph[MAX NODES][MAX NODES];
    int num nodes=0, i=0, j=0;
    int source, target;
    int maxval; //For nice printing only, ignore
    /*Source and target nodes*/
    fgets(temp, MAX_TEMP, stdin);
    source = atoi(temp);
    fgets(temp, MAX_TEMP, stdin);
    target = atoi(temp);
    /*Read actual graph*/
   while(fgets(temp, MAX_TEMP, stdin) ≠ NULL){
        char *token = strtok(temp, " ");
        while (token \neq NULL){
            if(atoi(token)>maxval) maxval=atoi(token); //For nice printing only,
ignore
            graph[i][j] = atoi(token); //Nodes updated here
            token = strtok(NULL, " ");
            j+=1;
        }
        i+=1:
        num_nodes+=1; //Count overall number of nodes
    }
    /*Print received input*/
    //int width = round(1+log(maxval)/log(10)); //Adjust for different sized
numbers in output
    //print input(graph, num nodes, source, target, width);
    /*Use algorithm*/
   something();
   return 0;
}
```

Input

}

}

return 0;

```
Exemplo - Vários casos de teste
<n> <n_v> Numero de vertices / Numero de ligações
n v linhas
<v u> Conexao
<v u>
. . .
int main(int argc, char **argv){
    char temp[MAX_TEMP];
    int graph[MAX NODES][MAX NODES];
    int num_nodes=0, num_connections;
    //int maxval; //For nice printing only, ignore
    /*Read actual graph*/
   while(fgets(temp, MAX_TEMP, stdin) ≠ NULL){
        char *token = strtok(temp, " ");
        /*Number of nodes*/
        num_nodes = atoi(token); token = strtok(NULL, " ");
        /*Get number of connections*/
        num connections = atoi(token);
        /*Set graph values to 0*/
        for (int i=0; i<num nodes; i++){</pre>
            for (int j=0; j<num nodes; j++){
                graph[i][j] = 0;
            }
        }
        /*Graph nodes*/
        for (int t=0; t<num_connections; t++){</pre>
            /*Get source and target node*/
            fgets(temp, MAX_TEMP, stdin);
            token = strtok(temp, " ");
            int source = atoi(token); token = strtok(NULL, " ");
            int target = atoi(token);
            /*Update the graph*/
            graph[source][target] = 1;
        }
        /*Print received input*/
        //int width = round(1+log(maxval)/log(10));
        //print input(graph, num nodes, width);
        /*Use algorithm*/
       something();
```

C++ STL (Standard Template Library)

Iterators

```
<tipo>::iterator <nome>
using namespace std;
int main(){
    vector<int>::iterator it1;
    set<int>::iterator it2;
    /*To print in reverse*/
    set<int>::reverse iterator it3;
    set<int> list; list.insert(0); list.insert(2);
    /*To get an iterator*/
    it2 = list.begin(); //Pointing to the first element
    it3 = list.rbegin(); //Pointing to last element (r - reverse)
    /*Example*/
    for (it2 = list.begin(); it2 ≠ list.end(); ++it2){
        printf("%d\n", *it2);
    return 1;
}
Queue FIFO - #include <queue>
queue<tipo> <nome>
using namespace std;
int main(){
    queue<int> queue1;
    /*Add elements*/
    queue1.push(1);
    queue1.push(10);
    /*Read the element at the front of the queue*/
    int number = queue1.front();
    /*Remove the element at the front of the queue*/
    queue1.pop();
    /*Check if it's empty*/
    while(!queue1.empty()){
    return 1;
}
```

C++ STL (Standard Template Library)

```
Stack - #include <stack>
stack<tipo> <nome>
using namespace std;
int main(){
    stack<int> stack1:
    /*Insert elements*/
    stack1.push(1);
    stack1.push(10);
    /*Get element at the top of the stack*/
    int number = stack1.top();
    /*Remove the top element*/
    stack1.pop();
    /*Check if the stack is empty*/
    while(!stack1.empty()){
    return 1;
}
Vector - #include <vector>
vector<tipo> <nome>
using namespace std;
int main(){
    vector<int> vector1;
    /*Add elements at beggining*/
    vector1.insert(vector1.begin(), 1);
vector1.insert(vector1.begin(), 10);
    /*Add elements at specific index*/
    vector1.insert(vector1.begin() + index, 1);
    vector1.insert(vector1.begin() + index, 10);
    /*Add elements at the end*/
    vector1.push_back(1);
    /*Delete element at specific index*/
    vector1.erase(vector1.begin() + index);
    /*Check its size and if empty*/
    int size = vector1.size();
    vector1.empty();
    /*Swap two elements in the vector*/
    iter_swap(vector1.begin() + index1, vector1.begin() + index2);
    return 1;
}
```

Alocação dinâmica do grafo

```
#define MAX_NODES 5001
int main(){
    /*Matrix that contains adjacency matrix*/
    int **graph = (int**)malloc((MAX_NODES) * sizeof *graph);
    int *data = (int*)malloc((MAX_NODES) * (MAX_NODES) * sizeof *data);
    for (int i=0; i<MAX_NODES; i++, data+=MAX_NODES) graph[i] = data;

    while(1){
        ... //Use algorithm
    }
    /*Free memory*/
    free(*graph);
    free(graph);
    return 0;
}</pre>
```