Computer Assignment 2

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In this case we are going to choose five different stocks, and the corresponding market index. The next step will be to construct an optimal portfolio. The stocks that have been chosen is, Assa Abloy, H&M, Kinnevik, Swedbank and Volvo and the corresponding index OMX 30.

```
## Hämtar hem all data som kommer användas ##

# Mha getSymbols hämtas respektive aktie och index
getSymbols(c('ASSA-B.ST','HM-B.ST','KINV-B.ST','SWED-A.ST','VOLV-B.ST','^OMX'),
           scr = 'yahoo', from = '2011-04-20', to = '2021-04-20', periodicity = "weekly")
```

```
## [1] "ASSA-B.ST" "HM-B.ST"   "KINV-B.ST" "SWED-A.ST" "VOLV-B.ST" "^OMX"
```

Weekly return

In the first part we want to determine the simple return for each stock. This is done by using the following formula:

$$K_i = \frac{S(T) - S(0)}{S(0)}$$

This will then give us the following return-vector, K_i where $i \in \{1, 2, 3, 4, 5\}$.

```
ASSA_R <-weeklyReturn(`ASSA-B.ST`$`ASSA-B.ST.Close`)
HM_R <-weeklyReturn( `HM-B.ST`$`HM-B.ST.Close`)
KINV_R <-weeklyReturn(`KINV-B.ST`$`KINV-B.ST.Close`)
SWED_R <-weeklyReturn(`SWED-A.ST`$`SWED-A.ST.Close`)
VOLV_R <-weeklyReturn(`VOLV-B.ST`$`VOLV-B.ST.Close`)
OMX_R <-weeklyReturn(`OMX`$`OMX.Close`)

return_vector <- as_tibble(cbind(ASSA_R,HM_R,KINV_R,SWED_R,VOLV_R))
```

Next step will to calculate the mean-vector and the covariance matrix which will be done by the code bellow.

```
# Beräknar mean_vector

mean_vector <- c(mean(ASSA_R),mean(HM_R),mean(KINV_R),mean(SWED_R),mean(VOLV_R))


# skapar en funkion som kan beräkna cov-matrisen för olika aktier

cov_matrix <- function(stocks){
  cm <- c()  # skapar en lista som alla värden kan sparas i
  for(i in stocks){ # gör en loop för varje aktie där vi:
    z <- c() # skapar en provisorisk lista som vi ska spara cov-värdet i
    for(j in stocks){ # sedan tar vi kombination och beräknar covariansen det vill säga c_ij
      z <- append(z,cov(i,j))
```

```
    }
    cm<-rbind(cm,z) # sparar varje kombinations värden som en egen rad.
  }
  return(cm)
}


C <- cov_matrix(return_vector)
```

Weights of the minimum variance portfolio w_{mvp}

In order to calcultate the weight of the minimum variance portfolio, that is w_{mvp}, we are using the following formula:

$$\frac{UC^{-1}}{UC^{-1}U^T}$$

where $U = (1, \ldots, 1)$ and C is the covariance matrix.

```
U <- as.matrix(c(1,1,1,1,1))
w_mvp <- (t(U)%*%solve(C)/as.numeric((t(U)%*% solve(C) %*% U)))
```

From the code above we get that $w_{mvp} = (0.44, 0.2, 0.14, 0.21, 0.017)$. This information gives us that we should most of our investment in Assa Abloy, 44%, and the least invested in Volvo 1.7%.

Weights of the market portfolio w_M

The formula to calculate the weights of the market portfolio is given by:

$$w_M = \frac{(m - RU)C^{-1}}{(m - RU)C^{-1}U^T}$$

where $m = (\mu_1, \ldots, \mu_n)$ and $\mu_i = E[K_i]$. The variabel R is the risk-free rate, and in this case we consider the risk-free rate to be 2% per year using continuous compounding.

```
 # eftersom vi använder oss av en risk-free rate per year och en weekly return,
# konverterar vi till risk-free rate per week.
R <- (1+0.02)^(1/52)-1

w_M <- (((mean_vector)-R%*%t(U))%*%solve(C))/as.numeric((((mean_vector)-R%*%t(U))%*%solve(C)%*%U)
```

By using the code above we get that weights, $w_M = (1.16, -0.4, 0.48, -0, 23, -0, 015)$. From this we can see that we should have most of the stock Assa Aloy, in fact we should have 115% of this asset in our porfolio. We should also have 48% of our portfolio invested in Kinnevik, the rest of the stocks we should short, that is H&M, Swedbank and Volvo.

Minimum variance line

In this part we will calculate the optimal portfolios by using the fact that the minimum variance line satifiy. That is the follwing:

$$w = s \cdot w_{min} + (1 - s)w_m \text{ for } s \in \mathbb{R}.$$

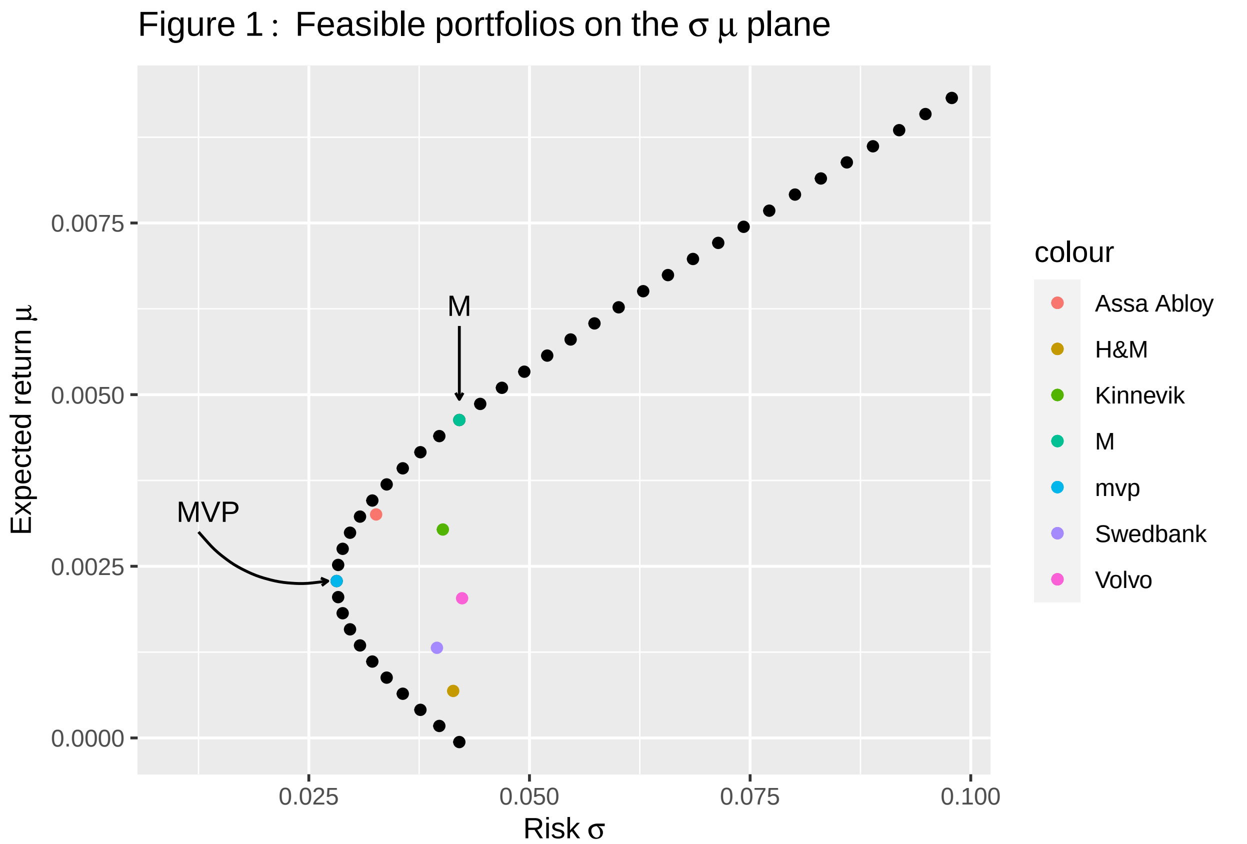
In this case we compute the optimal portfolios for $s \in [-2, 2]$ with step 0.1. This is done by the code below.

```
w=c()
s<-seq(-2,2,0.1)
for (i in s){
  vec <- ((i*w_mvp)+((1-i)*w_M))
  w <- rbind(w,vec)
}
```

From the information above we can now plot the portfolios, the result is seen in *Figure 1.* This gives us that the share Assa Abloy has the lowest risk and highest expected return of all the securities It can also be analyzed that the shares Kinnevik, H&M, Volvo and Swedbank has approximately the same risk, however Kinnevik has the highest return of them, therefore you invest in Kinnevik. Investing in a securitie with the same risk but lower expected return would be a bad decision. We can also see in the figure a line of black dots, this is the minimum variance line. The Market portfolio and the Minimum variance portfolio is located on this line, in the figure these porfolios is marked with an color and with an arrow. The shape the of the dotted line is knowed as the Markowitz bullet, and securities inside the boundary is all feasible portfolios.

```
mu <- mean_vector%*%t(w)

std <- c()
for(i in 1:41){
  k <- sqrt(t(w[i,])%*%C%*%w[i,])
  std <- append(std,k)
}


# expected and std for w_MVP
mu_mvp <- w_mvp%*%mean_vector
std_mvp <- sqrt(w_mvp%*%C%*%t(w_mvp))

# expected and std for w_M
mu_M <- w_M%*%mean_vector
std_M <- sqrt(w_M%*%C%*%t(w_M))

# std for the stocks

std_stocks <-  c(sd(ASSA_R),sd(HM_R),sd(KINV_R),sd(SWED_R),sd(VOLV_R))


ggplot()+
  geom_point(data=tibble(t(mu),std),aes(y=mu,x=std)) +
  geom_point(data=tibble(std_mvp,mu_mvp),aes(x=std_mvp,y=mu_mvp,color="mvp")) +
  geom_point(data=tibble(std_M,mu_M),aes(x=std_M,y=mu_M,color="M")) +
  geom_point(data=tibble(std_stocks,mean_vector),
             aes(x=std_stocks,y=mean_vector,
                 color=c("Assa Abloy","H&M","Kinnevik","Swedbank","Volvo"))) +
  annotate(
    geom = "curve", x = 0.0125, y = 0.003, xend = std_mvp - 0.001, yend = mu_mvp,
    curvature = .3, arrow = arrow(length = unit(1, "mm"))
  ) +
  annotate(geom = "text", x = 0.01, y = 0.0033, label = "MVP", hjust = "left") +
  annotate(
    geom = "curve", x = std_M, y = 0.006, xend = std_M , yend = mu_M+0.0003,
    curvature = 0, arrow = arrow(length = unit(1, "mm"))
  ) +
```

```
annotate(geom = "text", x = std_M, y = 0.006+0.0003, label = "M", hjust = "center") +
labs(y = expression(Expected~return~mu), x = expression(Risk ~ sigma)) +
ggtitle(expression(Figure~ 1: ~Feasible~ portfolios ~on ~the~sigma~mu ~plane))
```



β for each stock following CAPM

Here we will calculate β for each stock using CAPM. There are two ways to calculate the factor β, first one is giving by the follwing equation:

$$\beta_V = \frac{\mu_V - R}{\mu_M - R}.$$

As previously we assume that the risk-free rate is equal to 2% per year using continuous compounding. The calculation is done by the code below

```
beta_1 <-  (mean_vector-R)/(as.numeric(mu_M-R))
```

The second way to calculate the factor β is giving by the follwing equation:

$$\beta_V = \frac{cov(K_V, K_M)}{\sigma_M^2}.$$

The calculation is done by the code below.

```
beta_2 <-(cov(return_vector,OMX_R)/as.numeric(sd(OMX_R)^2))
```

We can now compare the two ways to calculate the factor β, by analyzing *Table 1*. We can see that it's a big diffrence between the two types of β. When calculating β_1 we only consider the five stocks as a market, but for β_2 we using the whole index which includes 25 more stocks. This is the reason why we can a diffrence in *Table 1*. If we had included all 30 stock when calculating β_1 we would have a closer result.

```r
betas <- cbind(beta_1,as_tibble(beta_2))
rownames(betas) <- c("Assa Abloy", "H&M", "Kinnevik", "Swedbank", "Volvo")
knitr::kable(betas,
             col.names = c("$\\beta_1$","$\\beta_2$"),
             caption = "Comparison between $\\beta_1$ and $\\beta_2$")
```

Table 1: Comparison between β_1 and β_2

	β_1	β_2
Assa Abloy	0.6762840	0.9320741
H&M	0.0714313	0.9995191
Kinnevik	0.6246918	1.0219500
Swedbank	0.2192221	1.0844144
Volvo	0.3890134	1.3117935