

# Neural Precedence Recommender

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# Outline

① Introduction

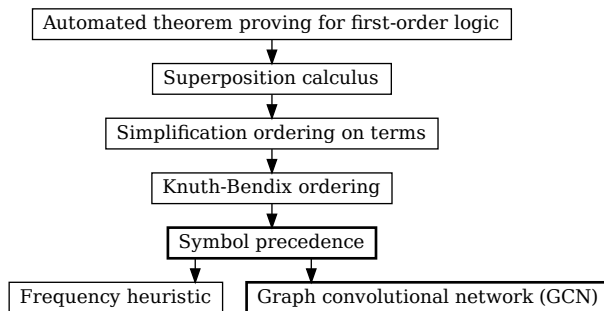
② Architecture

③ Training

④ Evaluation

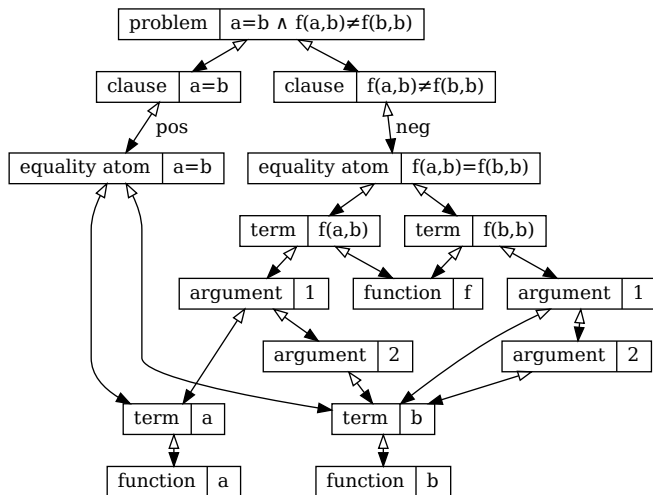
⑤ Summary

# Context



# Graph representation of a CNF problem

Input problem:  $a = b \wedge f(a,b) \neq f(b,b)$



# Graph convolutional network (GCN)

Initial embedding of node  $d$ :

$$h_d^{(0)} = (\text{feature vector}) \oplus (\text{trainable vector})$$

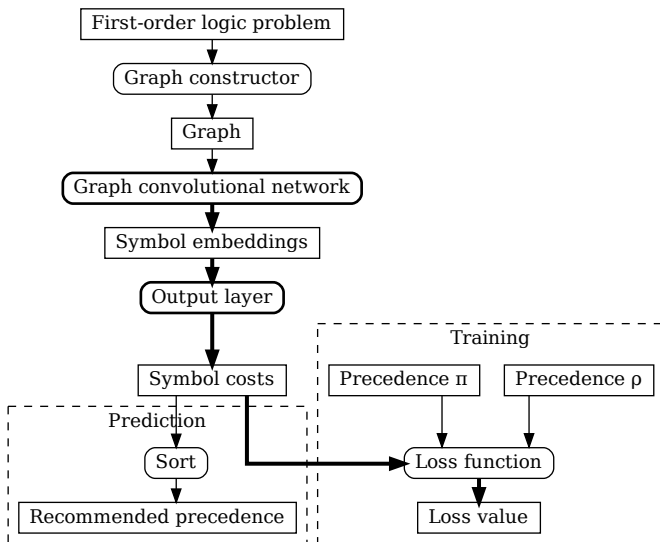
Feature vector:

- Clause: role (axiom, assumption, negated conjecture)
- Symbol: introduced in preprocessing, in conjecture

Propagation rule for layer  $l$ :

$$h_d^{(l+1)} = \sum_{r \in \mathcal{R}} \sigma \left( \sum_{s \in \mathcal{N}_d^r} \frac{1}{\sqrt{|\mathcal{N}_s^r|} \sqrt{|\mathcal{N}_d^r|}} (W_r^{(l)} h_s^{(l)} + b_r^{(l)}) \right)$$

# Precedence recommender overview



# Training: Precedence cost

Reminder:  $c_i$  is the cost of the  $i$ -th symbol.

Cost of symbol precedence  $\pi$  over signature of length  $n$

$$C(\pi) = \frac{2}{n(n+1)} \sum_{i=1}^n i \cdot c_{\pi(i)}$$

Lemma (Precedence cost minimization)

*The precedence cost  $C$  is minimized by any precedence that sorts the symbols by their costs in non-increasing order:*

$$\operatorname{argmin}_{\rho \in \operatorname{Perm}(n)} C(\rho) = \operatorname{argsort}^-(c_1, \dots, c_n)$$

# Training: Loss function

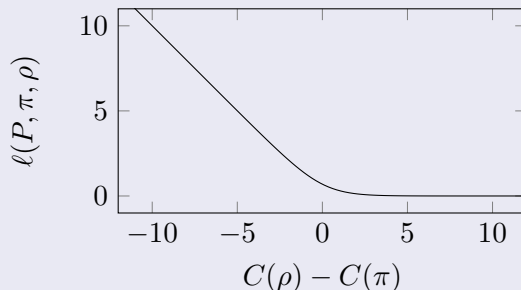
Training example  $\pi \prec_P \rho$

Precedence  $\pi$  is better than precedence  $\rho$  for problem  $P$ .

Reminder:  $C(\pi)$  is the predicted cost of precedence  $\pi$ .

Loss on training example  $\pi \prec_P \rho$

$$\ell(P, \pi, \rho) = -\log \text{sigmoid}(C(\rho) - C(\pi))$$





# Evaluation

Symbol cost model	Success count <sup>1</sup>		Improvement	
	Mean	Std	Absolute	Relative
GCN (pred. and func.)	3951.6	1.62	+182.0	1.048
GCN (predicate only)	3923.6	2.24	+154.0	1.041
GCN (function only)	3874.2	1.83	+104.6	1.028
Frequency (baseline)	3769.6	3.07	0.0	1.000

<sup>1</sup>Total number of validation problems: 7648. Number of repetitions: 5.

# Summary

- GCN over a graph representation of a FOL problem predicts symbol costs
- Sorting symbols by symbol costs yields a precedence
- Training is performed on oriented precedence pairs " $\pi \prec_P \rho$ " ("Precedence  $\pi$  is better than precedence  $\rho$  in problem  $P$ .")
- Combination of two GCNs outperforms the "frequency" heuristic by 4.8 %

Thank you for your attention!

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# Notation overview

$\pi(i)$	index of the $i$ -th symbol in precedence $\pi$
$c$	vector of symbol costs (output of the GCN)
$c_i$	cost of the $i$ -th symbol
$C(\pi)$	cost of symbol precedence $\pi$
$\ell(P, \pi, \rho)$	loss for training example $\pi \prec_P \rho$

# Prediction of symbol precedence $\pi$

**Require:** Problem  $P$

**Ensure:** Symbol precedence  $\pi$

$c \leftarrow \text{GCN}(P)$  {Forward pass through the GCN to obtain vector of symbol costs  $c$ }

$\pi \leftarrow \text{argsort}^-(c_1, \dots, c_n)$  {Sort symbols by  $c$  in non-increasing order to obtain precedence  $\pi$ }

**return**  $\pi$

# Proof sketch: Precedence cost minimization

$$C(\pi) = \frac{2}{n(n+1)} \sum_{i=1}^n i \cdot c_{\pi(i)}$$

Lemma (Precedence cost minimization)

$$\operatorname{argmin}_{\rho \in \operatorname{Perm}(n)} C(\rho) = \operatorname{argsort}^-(c_1, \dots, c_n)$$

Example

$$C\left(\begin{array}{|c|c|c|c|} \hline \text{■} & \text{■} & \text{■} & \text{■} \\ \hline \end{array}\right) < C\left(\begin{array}{|c|c|c|c|} \hline \text{■} & \text{■} & \text{■} & \text{■} \\ \hline \end{array}\right)$$

Proof:

$$\begin{aligned} C\left(\begin{array}{|c|c|c|c|} \hline \text{■} & \text{■} & \text{■} & \text{■} \\ \hline \end{array}\right) - C\left(\begin{array}{|c|c|c|c|} \hline \text{■} & \text{■} & \text{■} & \text{■} \\ \hline \end{array}\right) &\propto ((2 \cdot 3 + 3 \cdot 4) - (2 \cdot 4 + 3 \cdot 3)) \\ &= 18 - 17 = 1 > 0 \end{aligned}$$

# Concise representation of training examples

Cost of symbol precedence  $\pi$  over signature of length  $n$

$$C(\pi) = \frac{2}{n(n+1)} \sum_{i=1}^n i \cdot c_{\pi(i)} = \frac{2}{n(n+1)} \sum_{i=1}^n c_i \cdot \pi^{-1}(i)$$

Loss of training example  $\pi \prec_P \rho$

$$\begin{aligned} \ell(P, \pi, \rho) &= -\log \text{sigmoid}(C(\rho) - C(\pi)) \\ &= -\log \text{sigmoid} \frac{2}{n(n+1)} \sum_{i=1}^n i(c_{\rho(i)} - c_{\pi(i)}) \\ &= -\log \text{sigmoid} \frac{2}{n(n+1)} \sum_{i=1}^n c_i(\rho^{-1}(i) - \pi^{-1}(i)) \end{aligned}$$

Concise representation of  $\pi \prec_P \rho$ :  $\rho^{-1} - \pi^{-1}$