Assignment 1

Honors program, TU/e - High tech systems track

2017-2018

Goal: Build a simulation model of the quadcopter in simulink.

Start date: September 21th, 2017.

Question hour: September 28, 2017, 17h30-18h30h, GEM-Z 0.05.

Deadline: October 5th, 2017.

References: [1–5]

Assignment Build a simulation model for the quadcopter in simulink/Matlab.

1. Write an S-Function in simulink/Matlab, as depicted in Figure 1, implementing the equations of motion of the quadcopter (see slides of Lecture 1)

$$
\begin{aligned}
\dot{x} &= V_x \\
\dot{y} &= V_y \\
\dot{z} &= V_z \\
\dot{V}_x &= -(\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta)\frac{T}{m} \\
\dot{V}_y &= -(\cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi)\frac{T}{m} \\
\dot{V}_z &= -(\cos\phi\cos\theta)\frac{T}{m} + g \\
\dot{\phi} &= p + \sin\phi\tan\theta q + \cos\phi\tan\theta r \\
\dot{\theta} &= \cos\phi q - \sin\phi r \\
\dot{\psi} &= (\sin\phi/\cos\theta)q + (\cos\phi/\cos\theta)r \\
\dot{p} &= \frac{I_y - I_z}{I_x} rq + \frac{\tau_x}{I_x} \\
\dot{q} &= \frac{I_z - I_x}{I_y} pr + \frac{\tau_y}{I_y} \\
\dot{r} &= \frac{I_x - I_y}{I_z} pq + \frac{\tau_z}{I_z}
\end{aligned}
\tag{1}
$$

   for the following parameters

   $m = 0.429$

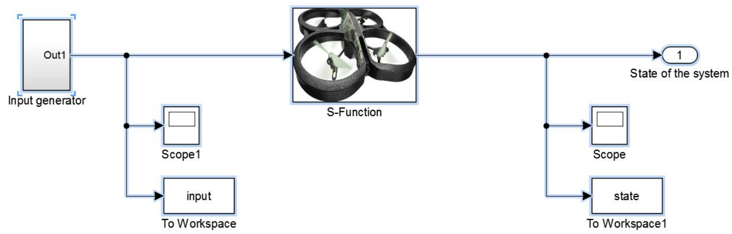

Figure 1: Simulink diagram

$$I_x = 0.002237568$$
$$I_y = 0.002985236$$
$$I_z = 0.00480374$$
$$g = 9.81.$$

The input to the S-function is $u := (T, \tau_x, \tau_y, \tau_z)$ and the output is the full state

$$X := (x, y, z, V_x, V_y, V_z, \phi, \theta, \psi, p, q, r).$$

Note that the dependence of time t is omitted. Assume that the initial state is zero $X(0) = 0$.

2. Compute the constant values of the input u, denoted by $u_{\text{eq}}, = (T_{\text{eq}}, \tau_{x,\text{eq}}, \tau_{y,\text{eq}}, \tau_{z,\text{eq}})$, that result in $\dot{X} = 0$, i.e., such that the quadcopter hovers.

3. Create a function to plot the quadcopter given a 3D position (x, y, z) and the Euler angles (ϕ, θ, ψ). Two examples are shown in Figure 2.[1]

4. Plot all the components of the state and the input in the time interval $[0, 10]$ [2] when the initial state at time $t = 0$ is zero $X(0) = 0$ and the input takes the following forms, generated with the help of the function $w(t) := h(t) - 2h(t-1) + h(t-2)$, where $h(t)$ is the Heaviside function

$$h(t) := \begin{cases} 1, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

[1] Create something simple and efficient, rather than fancy, because you might want to use this function later in real-time. Moreover, it might be useful to read [4]

[2] suggestion: use subplot.m and use one figure to plot the 12 components of the state with respect to time and one figure to plot the 4 components of the inputs with respect to time

Figure 2: Examples of quadcopter plots

a. $T(t) = T_{\text{eq}} + 0.5w(t-1)$, $\tau_x(t) = 0$, $\tau_y(t) = 0$, $\tau_z(t) = 0$ for every $t \geq 0$.

b. $T(t) = T_{\text{eq}}$, $\tau_x(t) = 0.1w(10(t-1))$, $\tau_y(t) = 0$, $\tau_z(t) = 0$ for every $t \geq 0$.

c. $T(t) = T_{\text{eq}}$, $\tau_x(t) = 0$, $\tau_y(t) = 0.1w(10(t-1))$, $\tau_z(t) = 0$ for every $t \geq 0$.

d. $T(t) = T_{\text{eq}}$, $\tau_x(t) = 0$, $\tau_y(t) = 0$, $\tau_z(t) = 0.1w(10(t-1))$ for every $t \geq 0$.

5. Use the function you created in 3. to make an animation of the quadcopter performing the manuevers specified by the control inputs in 4. Interpret the state response to each of the inputs in 4.

References

[1] Peter Corke, Robotics, Vision, and Control, Springer, 2011 (free digital assess from TU/e library).

[2] Qianying Li. Grey-Box System Identification of a Quadrotor Unmanned Aerial Vehicle. Master's thesis, Delft University of Technology, Aug. 2014 (available online)

[3] Matlab documentation. S-Functions:
`http://nl.mathworks.com/help/simulink/slref/sfunction.html`
`http://nl.mathworks.com/help/simulink/sfg/maintaining-level-1-matlab-s-functions.html`

[4] Matlab documentation. Animation techniques:
`http://nl.mathworks.com/help/matlab/creating_plots/animation-techniques.html`

[5] Example of an S-Function for a double-pendulum system (available in the dropbox folder).