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Main equations of the HESEM code:

1) Incorporating the embedding variable α:

$$\alpha S_k = \hat{V}_k^*(\alpha^*)\left[\sum_{m=1}^{N} Y_{km}^{tr}\hat{V}_m(\alpha) - \alpha Y_k^{sh}\hat{V}_k(\alpha)\right] \tag{2}$$

$$\alpha S_{km}^* = \hat{V}_k^*(\alpha^*)\left[y_{km}a_{km}(a_{km}\hat{V}_k(\alpha) - \hat{V}_m(\alpha)) + jb_{km}^{sh}\hat{V}_k(\alpha)\right] + S_{km}^{inc}(\alpha) \tag{3}$$

$$\alpha I_{km} = y_{km}a_{km}(a_{km}\hat{V}_k(\alpha) - \hat{V}_m(\alpha)) + jb_{km}^{sh}\hat{V}_k(\alpha) + I_{km}^{inc}(\alpha) \tag{4}$$

2) Calculating each term $[n]$:

$$\sum_{m=1}^{N} Y_{km}^{tr}\hat{V}_k[n] = S_k^*\hat{W}_k^*[n-1] - Y_k^{sh}\hat{V}_k[n-1] \quad n \geq 1 \tag{5}$$

$$y_{km}a_{km}(a_{km}\hat{V}_k[n] - \hat{V}_m[n]) + jb_{km}^{sh}\hat{V}_k[n] + S_{km}^{inc}[n] = S_{km}^*\hat{W}_k^*[n-1] \quad n \geq 1$$

$$S_{km}^{inc}[n] = \begin{cases} -a_{km}^2 y_{km} + a_{km}y_{km} - jb_{km}^{sh} & n = 0 \\ a_{km}^2 y_{km} - a_{km}y_{km} + jb_{km}^{sh} & n = 1 \\ 0 & n > 1 \end{cases} \tag{6}$$

$$y_{km}a_{km}(a_{km}\hat{V}_k[n] - \hat{V}_m[n]) + jb_{km}^{sh}\hat{V}_k[n] + I_{km}^{inc}[n] = \delta_n(I_{km}), \quad n \geq 1$$

$$I_{km}^{inc}[n] = \begin{cases} -a_{km}^2 y_{km} + a_{km}y_{km} - jb_{km}^{sh} & n = 0 \\ a_{km}^2 y_{km} - a_{km}y_{km} + jb_{km}^{sh} & n = 1 \\ 0 & n > 1 \end{cases} \tag{7}$$

$$\hat{V}_k[n] = \delta_n(\hat{V}_k - 1), \quad n \geq 1 \tag{8}$$

$$\delta_n = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases} \tag{9}$$

3) **HESEM** main equation:

$$(\mathrm{H}^T\mathrm{H}) \cdot \hat{\mathbf{x}}[n] = (\mathrm{H}^T) \cdot \mathbf{h}[n-1] \tag{10}$$

$$(\mathrm{H}^T\mathrm{R}^{-1}\mathrm{H}) \cdot \hat{\mathbf{x}}[n] = (\mathrm{H}^T\mathrm{R}^{-1}) \cdot \mathbf{h}[n-1] \tag{11}$$

4) Matrix solution: **H . x[n] = h[n-1]**

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & G_{kk} & -B_{kk} & \cdots & G_{km} & -B_{km} & \cdots & G_{kN} & -B_{kN} \\ \cdots & B_{kk} & G_{kk} & \cdots & B_{km} & G_{km} & \cdots & B_{kN} & G_{kN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & a_{km}^2 g_{km} & -a_{km}^2 b_{km} - b_{km}^{sh} & \cdots & -a_{km}g_{km} & a_{km}b_{km} & \cdots & 0 & 0 \\ \cdots & a_{km}^2 b_{km} + b_{km}^{sh} & a_{km}^2 g_{km} & \cdots & -a_{km}b_{km} & -a_{km}g_{km} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & a_{km}^2 g_{km} & -a_{km}^2 b_{km} - b_{km}^{sh} & \cdots & -a_{km}g_{km} & a_{km}b_{km} & \cdots & 0 & 0 \\ \cdots & a_{km}^2 b_{km} + b_{km}^{sh} & a_{km}^2 g_{km} & \cdots & -a_{km}b_{km} & -a_{km}g_{km} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \cdots & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ \hat{V}_k^{re}[n] \\ \hat{V}_k^{im}[n] \\ \vdots \\ \hat{V}_m^{re}[n] \\ \hat{V}_m^{im}[n] \\ \vdots \\ \hat{V}_N^{re}[n] \\ \hat{V}_N^{im}[n] \end{bmatrix} = \begin{bmatrix} \vdots \\ \mathrm{Re}\left\{S_k^*\hat{W}_k^*[n-1] - Y_k^{sh}\hat{V}_k[n-1]\right\} \\ \mathrm{Im}\left\{S_k^*\hat{W}_k^*[n-1] - Y_k^{sh}\hat{V}_k[n-1]\right\} \\ \vdots \\ \mathrm{Re}\left\{S_{km}^*\hat{W}_k^*[n-1] - S_{km}^{inc}[n]\right\} \\ \mathrm{Im}\left\{S_{km}^*\hat{W}_k^*[n-1] - S_{km}^{inc}[n]\right\} \\ \vdots \\ \delta_n\left(\mathrm{Re}\left\{I_{km} - I_{km}^{inc}[n]\right\}\right) \\ \delta_n\left(\mathrm{Im}\left\{I_{km} - I_{km}^{inc}[n]\right\}\right) \\ \vdots \\ \delta_n\left(\mathrm{Re}\left\{V_k - 1\right\}\right) \\ \delta_n\left(\mathrm{Im}\left\{V_k - 1\right\}\right) \\ \vdots \end{bmatrix} \quad n \geq 1 \tag{1}$$