

XMS3GU050 - EARTH AND PLANETARY INTERIORS

GEOMAGNETISM PRACTICAL

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GEOMAGNETIC FIELD REPRESENTATION Outside the region of geomagnetic field generation (i.e just above the core-mantle boundary) and considering the mantle as an insulator hence $\vec{J} = 0$ and the Ampère's law $\nabla \times \vec{B} = \mu_0 \vec{J}$ becomes $\nabla \times \vec{B} = 0$ where \vec{B} is the magnetic field, \vec{J} the current density and μ_0 the vacuum magnetic permeability. The Earth's magnetic field can therefore be represented as the gradient of a scalar potential:

$$\vec{B} = -\nabla V$$

Applying the non-divergence of magnetic fields (non existence of magnetic monopole) $\nabla \cdot \vec{B} = 0$ then:

$$\nabla^2 V = 0$$

the part of V generated by internal sources is given by an analytical solution in spherical coordinates, the so called spherical harmonics solution:

$$V(r, \theta, \phi) = a \sum_{\ell=1}^{\ell_{max}} \left(\frac{a}{r}\right)^{\ell+1} \sum_{m=0}^{\ell} (g_{\ell}^m \cos m\phi + h_{\ell}^m \sin m\phi) P_{\ell}^m$$

where a is Earth's radius, r , θ and ϕ are the spherical radial, co-latitude and longitude coordinates, g_{ℓ}^m and h_{ℓ}^m are the Gauss coefficients of degree ℓ and order m and P_{ℓ}^m are the Schmidt semi-normalized associated Legendre polynomials. Hence the radial component of Earth's magnetic field can be written:

$$B_r(r, \theta, \phi) = -\partial_r V(r, \theta, \phi) = \sum_{\ell=1}^{\ell_{max}} \left(\frac{a}{r}\right)^{\ell+2} \sum_{m=0}^{\ell} (g_{\ell}^m \cos m\phi + h_{\ell}^m \sin m\phi) P_{\ell}^m$$

SYNTHETIC Manipulating the spherical harmonic coefficients (Gauss coefficients) fields that are purely dipolar, quadrupolar, octopolar, zonal, non-zonal, sectorial and tesseral can be produced.

- What g_2^0/g_1^0 ratio gives $B_r = 0$ at the CMB at latitude 30°N ?
- What g_1^1/g_1^0 ratio gives dipole tilt of 10° ? Does the radial distance matter?

REAL Using a model based on observations of Earth's magnetic field model (COV-OBS.x2, Huder et al., (2020)) for the historical and modern era:

- Plot B_r at the CMB for different ℓ_{max} and calculate $\max(|B_r|)$ vs. ℓ_{max} for $\ell_{max} = 5-14$.
- Plot $B_{r_{dip}}$ ($\ell = 1$) and $B_{r_{ndip}}$ ($\ell > 1$). Which are the largest non-dipole contributions?
- Plot $B_{r_{zon}}$ ($m = 0$) and $B_{r_{nzon}}$ ($m \neq 0$). Which are the largest non-zonal contributions?
- Plot $B_{r_{sym}}$ ($\ell + m$ even) and $B_{r_{nsym}}$ ($\ell + m$ odd). Field more equatorial symmetric or anti-symmetric?

SPECTRUM The magnetic field spectrum R_ℓ at the CMB can be expressed as a function of spherical harmonic degree ℓ in terms of the Gauss coefficients of the core field as (Lowes, 1974):

$$R_\ell = (\ell + 1) \left(\frac{a}{c}\right)^{2\ell+4} \sum_{m=0}^{\ell} ((g_\ell^m)^2 + (h_\ell^m)^2)$$

where c is the radius of Earth's outer core.

- Plot R_ℓ in function of ℓ at the core-mantle boundary. How would you describe Earth's magnetic field based on its spectral analysis?

TIMESERIES: Studying the time-dependence of the strength of the axial dipolar component of Earth's magnetic field.

- Plot the axial dipolar component of the field in function of time. What is the average decrease rate? If it continues when will the field reverse?